

Shadow Banks and Optimal Regulation

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Abstract

We develop a new framework to study regulatory policy in the presence of unregulated financial institutions (“shadow banks”) when there are pecuniary externalities. Using sufficient statistics, we show that optimal regulation in the presence of shadow banks is scaled by a “regulatory arbitrage multiplier.” This multiplier only depends on aggregate shadow banking activity. Our framework provides guidance on how to regulate currently unregulated financial institutions and sectors. To first order, the marginal welfare gain of regulating a shadow bank is large when a notion of its intermediary activity substitution effects across its activities is large. We further characterize optimal activity-based regulation whereby the planner regulates a particular activity across all shadow banks, e.g. a tax on debt. To first order, gains from activity regulation are large when average substitution effects across intermediaries are large for the regulated activity. We show how our results extend to broader classes of non-pecuniary externalities.

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1 Introduction

In the wake of the 2008 financial crisis, a new regulatory regime for financial stability has emerged.¹ A distinctive feature of this new regime is that conventional banks now face tighter regulatory requirements while other financial institutions that conduct similar activities remain unregulated.² These unregulated institutions are often proverbially referred to as “shadow banks.” This asymmetry in regulatory standards has raised questions about both the efficiency and efficacy of current regulatory policies. As conventional banks curb regulated activities – so the conventional wisdom goes – unregulated institutions could step in, thus diminishing the intended effect of regulation in the first place. However, currently unregulated institutions differ from conventional banks not only in their regulatory status but also in many other fundamental characteristics and activities. It is therefore not obvious what implications their presence has for bank regulation. An active debate has ensued about whether and how to start regulating the shadow banking sector.

We develop a new framework to study financial regulation in the presence of unregulated shadow banks, allowing for rich flexibility in the types of and relationships between different financial intermediaries. In our setting, heterogeneous sets of financial institutions interact with households and with one another in a general environment.³ Their interactions are governed by equilibrium market prices that also enter constraint sets, thus giving rise to pecuniary externalities that warrant regulation. For example, occasionally binding collateral constraints may lead to a fire sale externality.⁴ Relative to previous work on shadow banking, our model remains agnostic to the fundamental differences between banks and shadow banks, not only in their fundamental characteristics but also in the extent to which they are affected by externalities. It allows us not only to consider the effect of the presence of shadow banks on optimal regulation of banks, but also to offer guidance to policymakers on how best to go about regulating shadow banks.

Our main result contrasts optimal financial regulation in the presence of shadow banks with a baseline where all banks are regulated. The optimal tax formulas in both cases are identical up to a “regulatory arbitrage multiplier”. The regulatory arbitrage multiplier scales the tax formula for banks when shadow banks are unregulated. Using a sufficient statistics approach, we show that this multiplier depends only on the activities

¹In the domestic U.S. context, the Dodd-Frank Wall Street Reform and Consumer Protection Act of 2010 strengthened regulation of bank holding companies and established an orderly resolution regime. At an international level, the Basel III accords strengthened international regulatory standards.

²See [Financial Stability Board \(2013\)](#) for a policy perspective, and [Financial Stability Board \(2020\)](#) for an annual empirical overview of the shadow banking system.

³Our notation is sufficiently general that “households” can also include firms.

⁴As in [Dávila and Korinek \(2018\)](#) for example.

of the aggregate shadow bank sector, rather than on the entire cross-sectional distribution, even when shadow banks are heterogeneous. Intuitively, a change in bank regulation induces a response in the behavior of regulated banks and thus a change in the total market demand for financial activities. This change in aggregate demand must be absorbed by other market participants, resulting in an equilibrium price adjustment. When prices enter constraints, such an equilibrium price adjustment generically generates a pecuniary externality. In absence of shadow banks, the planner sets regulation of banks to account for this pecuniary externality.

In the presence of unregulated shadow banks, however, the change in prices induced by changes in regulated bank demand induces a further response in shadow bank demand. This augments the direct change in aggregate banking sector demand that must be absorbed, and results in further changes in equilibrium prices. The regulatory arbitrage multiplier is the fixed point of this feedback loop: the total change in the aggregate demand of the shadow banking sector that arises from a change in the aggregate demand of the banking sector through induced changes in equilibrium prices. Because market clearing and hence equilibrium prices are determined by the aggregate demand that the household sector must absorb, the regulatory arbitrage multiplier is determined by aggregate responses of the shadow bank sector, and does not depend on the cross-section of responses.

The regulatory arbitrage multiplier may amplify or dampen optimal bank regulation relative to the baseline without shadow banks. Intuitively, one might expect that shadow banks would substitute into activities that banks are restricted from conducting due to changes in equilibrium prices for those activities. We show that the magnitude of the regulatory arbitrage multiplier can be determined from a general set of income and substitution effects of price changes on shadow bank activities. Despite the presence of substitution effects, we show that strong income effects can lead the regulatory arbitrage multiplier to amplify, rather than dampen, regulation of banks. Strong income effects arise when pecuniary externalities strongly affect shadow banks. The intuition can be understood from a standard fire sale example with a binding collateral constraint. When the fire sale price rises, the collateral constraint is relaxed, which is tantamount to a positive income effect. When this effect is sufficiently strong – that is, there is a strong pecuniary externality on shadow banks – total shadow bank fire sale risk in fact decreases when banks become safer, even though shadow banks might respond by taking actions, such as increases in debt issuance, that increase their risk. This leads regulation of banks to be more effective, and leads to stronger regulation of banks.

The regulatory arbitrage multiplier emerges as the key sufficient statistic to character-

ize how the presence of shadow banks affects optimal bank regulation in our setting. In this exercise, we hold the set of shadow banks fixed. The flexibility of our environment also enables us, however, to consider the welfare implications of shadow bank regulation. In Section 4, we ask what kind of financial institution the planner would benefit most from being able to regulate. We again frame our results in terms of directly observable sufficient statistics. Our model also allows for a direct comparison with *activity-based regulation*, which we study in Section 5. Under activity-based regulation, the planner enacts regulation that applies uniformly to a particular financial activity regardless of which institutions perform the activity.⁵ The activity-based approach has recently garnered interest from practitioners because it circumvents the daunting challenge of having to classify shadow banks.

Our main result in Section 3 is that, to first order, the welfare gain from regulating a previously unregulated shadow bank is related to the substitution effects of that shadow bank to changes in prices, holding fixed collateral values. In principle, these substitution effects are micro-estimable for a regulator. The intuition is that shadow banks with strong substitution effects have particularly large demand responses to changes in regulation. These larger demand responses generate greater movements in equilibrium prices, generating larger welfare gains. For example, if only a single activity is regulated in equilibrium, welfare gains are largest from regulating the shadow bank with the greatest own-price substitution effect (at fixed collateral values) for the regulated activity. This welfare analysis provides general guidance on which shadow banking institutions a policymaker should prioritize when looking to expand institution-based regulation.

A key difficulty in practice of regulating shadow banks is that the shadow banking system comprises a number of different non-bank financial intermediaries. As a result, an institution-based regulatory framework might involve tailoring regulatory requirements to each specific type of institution, and might further admit regulatory arbitrage around the designation of type of institution. An alternative approach that might help to circumvent these concerns is activity-based regulation, which focuses on regulating a specific activity or set of activities across all shadow banks. For example, proposals for activity-based regulation have included taxes on leverage ([Federal Reserve Bank of Minneapolis \(2017\)](#)). Our next contribution is to study activity regulation, where the planner has a set of wedges on a limited set of activities that must be applied equally across all shadow banks. We first characterize optimal activity regulation for a fixed set of instruments. Optimal activity regulation balances the changes in equity value at shadow banks against changes in

⁵For example, proposals include implementing taxes on broad activities such as leverage ([Federal Reserve Bank of Minneapolis \(2017\)](#)).

pecuniary externalities, and in particular weights these externalities by changes in shadow bank activities, which are determined once again by the (in principle micro-estimable) shadow bank substitution effects, holding fixed collateral values. We further study the welfare gains from regulation of a specific activity, which helps to inform the most valuable activities for a planner to regulate. To first order, the welfare gain from regulation of an activity across shadow banks is related to the *average* substitution effect across all shadow banks from changes in the price of that activity (holding fixed collateral values). This implies that activity regulation should be targeted towards activities with particularly strong substitution effects. Moreover, this offers guidance for policymakers weighing whether to extend institution-based regulation to specific shadow banks, or to apply activity-based regulation across shadow banks. In particular, activity-based regulation is valuable when a single activity price is associated with strong substitution effects across intermediaries, whereas institution-based regulation is valuable when a particular type of shadow banks has strong substitution effects across activities.

Although our main analysis is conducted assuming complete instruments and pecuniary externalities, we show that our qualitative insights extend to environments with incomplete instruments and non-pecuniary externalities in Section 6. We allow for a standard (e.g. [Dávila and Korinek \(2018\)](#)) form of incomplete instruments where a regulator sets ex-ante macroprudential policy at an initial date, and then allows the economy to unfold from there, and show that a similar characterization of optimal policy arises. The regulatory arbitrage multiplier may now depend on the cross-section of shadow banks, however, if different shadow banks contribute differently to future price determination or to non-pecuniary externalities. Moreover, we show these results also extend to a broader class of non-pecuniary externalities, such as externalities from deposit insurance or broader non-banking externalities (e.g. environmental externalities). This provides a more general theory for regulators concerned about the possibility of regulatory arbitrage by unregulated agents.

Finally, in Section 7, we specialize our general model to a canonical application of fire sales and collateral constraints.⁶ In this simple model, we show that if shadow banks contribute to fire sales by liquidating assets in tandem with banks, the presence of shadow banks may amplify or dampen bank regulation depending on the relative strength of income and substitution effects. If the probability of a crisis is not too likely, the income effect dominates and the presence of unregulated shadow banks is a force for stronger regulation of banks. By contrast, if shadow banks *alleviate* fire sales by purchasing fire-sold assets from banks, we show that in fact the regulatory arbitrage multiplier is dampening. In this

⁶Models of this type include [Kiyotaki and Moore \(1997\)](#) and [Dávila and Korinek \(2018\)](#).

case, shadow banks mute the price impact of bank regulation, dampening the externality effect of bank regulation.

Related Literature. First, we relate to a growing literature on the role of shadow banks and their effects on bank regulation. On the positive side, a growing literature has looked to understand the differences between traditional and shadow banks, including regulatory arbitrage (Acharya et al. (2013), Claessens et al. (2012), Gorton and Metrick (2010)), combinations of regulatory arbitrage and technological differences (Buchak et al. (2018)), implications of deposit insurance access for asset holdings (Hanson et al. (2015)) or side of the asset resale market (Chretien and Lyonnet (2020)), and different forms of liquidity creation (Moreira and Savov (2017)). On the normative side, papers have studied the implications of shadow banks for regulation of banks, including Plantin (2014), Ordoñez (2018), Huang (2018), He et al. (2018), Grochulski and Zhang (2019), Bengui and Bianchi (2019), Begenau and Landvoigt (2020), and Farhi and Tirole (2020). This literature commonly studies a particular form of regulation (e.g. capital requirements, liquidity requirements, taxes on debt), and often models shadow banks either as unregulated banks or as an outside option for regulated banks. For example, Bengui and Bianchi (2019) show that the optimal debt tax on banks may be larger in the presence of otherwise identical unregulated shadow banks because the magnitude of their pecuniary externality increases, even though shadow banks increase risk due to safer banks. We contribute by providing a general framework in which shadow banks may differ from banks in both regulatory status and fundamentals, in which shadow banks and banks may differ endogenously in the activities they conduct, in which there is a general class of pecuniary externality, and in which the regulator has a large set of possible instruments. We use this framework not only to characterize the response of regulation to shadow banks, but also to characterize welfare consequences of shadow banks and optimal regulatory policies for previously unregulated shadow banks.

Second, we relate to a large literature on financial regulation, particularly in the presence of pecuniary externalities such as fire sales.⁷ We contribute to this literature by considering regulation with pecuniary externalities and a class of unregulated shadow banks, and show how optimal policy differs from the case without shadow banks.

⁷For example, Bianchi (2011), Bianchi and Mendoza (2018), Caballero and Krishnamurthy (2001), Chari and Kehoe (2016), Dávila and Korinek (2018), Farhi et al. (2009), Farhi and Tirole (2012), and Lorenzoni (2008).

2 Model

The economy is populated by banks and households (or general non-financials). Banks are indexed by their type $i \in \mathcal{I}$, each of measure μ_i , while households are indexed by $h \in \mathcal{H}$, each of measure μ_h , with $\sum_{i \in \mathcal{I}} \mu_i + \sum_{h \in \mathcal{H}} \mu_h = 1$. The economy has $M + 1$ goods available for trade: a numeraire, denoted by $z \in \mathbb{R}$, as well as a vector of M goods, denoted by $x \in \mathbb{R}^M$, which are traded at a price vector p . Separating out the numeraire will be notationally convenient. Because z and x correspond to trades of goods between agents, they will be in zero net supply.

In the model, traded goods can correspond to trades of different types of goods, such as consumption goods, investment goods, state contingent securities, labor supply, and so on. They can also correspond to trades of a single type of good in different states or periods. For example, the set M might capture trades of a consumption good and a capital good over a set of histories over a set of periods.

2.1 Banks

Banks have a vector of actions $a_i \in \mathcal{A}_i$ available. Examples of actions include consumption, production, and capital stock adjustments. In addition, banks can trade goods. Bank i can engage in trades z_i of the numeraire and x_i of other goods, with $x_i(m) > 0$ denoting bank i receiving good m .

The decision problem of bank i is given by

$$\max_{a_i, z_i, x_i} \omega_i U_i(a_i)$$

subject to

$$z_i + px_i \leq w_i \tag{1}$$

$$\Gamma_i(a_i, z_i, x_i, p) \geq 0 \tag{2}$$

Equation (1) is the budget constraint of bank i over all goods trades. We allow for banks to have different initial wealth levels. Note that because the budget constraint is defined over trades, the average (tradeable) wealth across agents in the economy must be zero. Equation (2) is a general constraint set, which includes technological constraints (e.g. production functions and capital evolution equations), financing constraints (e.g. collateral constraints), incomplete markets constraints, and so on. We pre-multiply bank utility $U_i(a_i)$ by a social welfare weight ω_i without loss of generality.

From here, we define the private Lagrangian of bank i as

$$\mathcal{L}_i = \omega_i U_i(a_i) + \lambda_i(w_i - z_i - px_i) + \Lambda_i \Gamma_i, \quad (3)$$

which is also the indirect utility function when substituting in demand functions. Notice that $\lambda_i > 0$ is the marginal value of wealth to bank i .

Shadow Banks. The economy contains both banks and shadow banks. Whereas banks can be regulated, we will generally consider shadow banks to not be subject to regulation. Denote $\mathcal{S} \subset \mathcal{I}$ to be the set of shadow banks, while $\mathcal{B} \subset \mathcal{I}$ is the complementary set of regulated banks. We keep the set of banks \mathcal{I} in the economy constant, so that an economy without shadow banks ($\mathcal{S} = \emptyset$) implies that $\mathcal{B} = \mathcal{I}$.

Example (Capital and Collateral). Although p is the price vector of traded goods x , it is easy to see that it can be applied to stocks held by the bank. Suppose that we define an element $a_i(m+1) \in a_i$ by $a_i(m+1) = (1 - \delta)a_i(m) + x_i(m+1)$, which defines capital accumulation as the combination of depreciation of existing capital and trades of capital. The collateral value of this capital stock at “date $m+1$ ” is given by $p(m+1)a_i(m+1)$, which is the total value it could be sold for on the market.

2.2 Households

Households have a vector of actions $a_h \in \mathcal{A}_h$, such as consumption and labor supply, and can also engage in trades z_h and x_h . As a result, household h solves

$$\max \omega_h U_h(a_h)$$

subject to

$$z_h + px_h \geq w_h \quad (4)$$

$$\Gamma_h(a_h, z_h, x_h, p) \geq 0 \quad (5)$$

We have adopted, without loss of generality, the notational convention in household budget constraints of reversing the side of the budget constraint that x_h appears on, so that for a household $x_h(m) > 0$ corresponds to supplying good m . This is to simplify the expression of the fact that under market clearing, the aggregate household sector must take the opposite position of the aggregate banking sector in equilibrium. Under this convention, w_h is the “debt” level of households, with $w_h < 0$ indicating that household h has positive initial wealth.

As with banks, we define the private Lagrangian of household h as

$$\mathcal{L}_h = \omega_h U_h(a_h) + \lambda_h(z_h + px_h - w_h) + \Lambda_h \Gamma_h, \quad (6)$$

where $\lambda_h > 0$ is once again the marginal value of household h wealth.

Example (Labor Supply). Suppose that household h supplies labor and consumes. We define household actions $a_{h1}, a_{h2} \geq 0$ to be hours worked and consumption, respectively, so that utility is $\max u_h(a_{h2}) - v_h(a_{h1})$. Let z_h be the consumption good and let $x(\tilde{m})$ be the labor supply market, so that the budget constraint is $z_h + p(\tilde{m})x_h(\tilde{m}) \geq 0$. Finally, Γ_h consists of two constraints: (1) $a_{h2} \leq -z_h$, which states that consumption is the amount of the consumption good purchased; and (2) $x(\tilde{m}) \leq a_{h1}$, which states that hours worked is how much labor supply can be sold.

Example (Second-Best Capital Users). Suppose that household h is a second best user of bank capital (“arbitrageur”). Let good m correspond to sales of the capital good and let z be the consumption good. We can represent a simple second best production user by $U_h(a_h) = \mathcal{F}(a_h(1)) - a_h(2)$, with $a_h(1) = -x_h(m)$ and $a_h(2) = z_h$.

2.3 Market Clearing and Competitive Equilibrium

Market clearing is defined for the set of traded goods.⁸

We adopt the notational convention that $X_{\mathcal{N}}$ is aggregate traded goods positions summed over a set \mathcal{N} of agents. Under this convention, $X_{\mathcal{I}} = \sum_{i \in \mathcal{I}} \mu_i x_i$ is the aggregate demand of banks and $X_{\mathcal{H}} = \sum_{h \in \mathcal{H}} \mu_h x_h$ is the aggregate supply of households. $X_{\mathcal{I}(m)} < 0$ indicates negative bank demand, that is banks supply good m in aggregate, while $X_{\mathcal{H}(m)} < 0$ indicates negative household supply. As a result, market clearing is given by

$$X_{\mathcal{I}} = X_{\mathcal{H}}. \quad (7)$$

There is also a market clearing condition for the numeraire, $Z_{\mathcal{I}} = Z_{\mathcal{H}}$. We adopt the convention of using Walras’ law to omit the numeraire market clearing condition. Because actions a are not direct objects of trade, there is no market clearing condition for actions.

From here, a competitive equilibrium is a set of allocations $\{a_i, z_i, x_i\}_{i \in \mathcal{I}}, \{a_h, z_h, x_h\}_{h \in \mathcal{H}}$ and vector of prices p such that banks and households maximize utility, given p , and such that traded goods market clearing (7) holds.

⁸The economy also satisfies $\sum_{i \in \mathcal{I}} \mu_i w_i = \sum_{h \in \mathcal{H}} \mu_h w_h$, but this is a restriction on the initial wealth distribution.

2.4 Sufficient Statistics

We now characterize two objects that will play a role in many of the results to come.

Definition. Let $\Xi_i = \nabla_p x_i$ denote a Jacobian, ordered such that $\Xi(m, m') = \frac{\partial x_i(m')}{\partial p(m)}$.

1. The *aggregate supply response* is given by

$$\Xi_{\mathcal{H}} \equiv \nabla_p X_{\mathcal{H}} = \sum_{h \in \mathcal{H}} \mu_h \Xi_h \quad (8)$$

which measures the response of aggregate household supply to price changes.

2. The *aggregate shadow bank demand response* is given by

$$\Xi_{\mathcal{S}} \equiv \nabla_p X_{\mathcal{S}} = \sum_{i \in \mathcal{S}} \mu_i \Xi_i \quad (9)$$

which measures the response of aggregate shadow bank demand to price changes.

The object $\Xi_{\mathcal{H}}$ provides a mapping of the total change $\Delta X_{\mathcal{H}}$ in traded goods positions across households that arises in response to a change Δp in prices. Considering households to be suppliers of traded goods, this measures the total supply response of traded goods to a change in the vector of prices. Conversely, $\Xi_{\mathcal{H}}^{-1}$ provides the inverse mapping, telling us how a change in the aggregate supply of households induces a change in equilibrium prices.⁹

Likewise, $\Xi_{\mathcal{S}}$ maps a change in prices into a change in the aggregate traded goods position of the shadow bank sector. Considering shadow banks to be on the demand side of traded goods markets, then this is the total change in goods demand by the shadow banking sector in response to a vector of traded goods price changes.

3 Optimal Regulation

In this section, we consider optimal bank regulation, both with and without shadow banks. In both cases, household supply functions must be taken as given by the social planner when designing regulation. When there are shadow banks, the planner must additionally take as given shadow bank demand functions when designing regulation. We show that optimal regulation with shadow banks introduces a regulatory arbitrage multiplier into the equation characterizing optimal regulation.

⁹Recall that we have omitted the numeraire from $X_{\mathcal{H}}$ and hence from the matrix $\Xi_{\mathcal{H}}$.

3.1 Optimal Regulation without Shadow Banks

We begin by characterizing optimal regulation when all banks are regulated, but households are unregulated. In this economy, $\mathcal{S} = \emptyset$ and $\mathcal{B} = \mathcal{I}$, so that there are no shadow banks.

In order to solve this problem, the planner must take as given the household response functions $(a_h, z_h, x_h)(p)$, but has direct control over all allocations of banks. As a result, we can solve the regulatory problem as follows: given demand functions, a planner chooses a pricing vector p by absorbing residual demand in the banking sector. In other words, we adopt a solution strategy whereby the planner directly chooses p , and then must set banking sector quantities (i.e. wedges) in such a way that markets clear. The Lagrangian of the social planning problem is therefore given by

$$\mathcal{L}|_{p, \{a_i, z_i, x_i\}_{i \in \mathcal{I}}} = \sum_{h \in \mathcal{H}} \mu_h \mathcal{L}_h + \sum_{i \in \mathcal{I}} \mu_i \mathcal{L}_i + (X_{\mathcal{H}} - X_{\mathcal{I}})' \lambda,$$

where we have internalized the demand functions of households, and where as usual we have used Walras' law to drop market clearing for the numeraire good.

We characterize optimal regulation by its implementation. In particular, we define the wedges τ_i on goods trades x_i , which appear in budget constraints and are remitted lump sum to the banks they are collected from.¹⁰

Proposition 1. *Suppose there are no shadow banks ($\mathcal{S} = \emptyset$). Then, optimal regulation is given by*

$$\tau_i = -\frac{1}{\lambda_i} \Xi_{\mathcal{H}}^{-1} \mathcal{E} \quad (10)$$

where \mathcal{E} is an externality vector given by

$$\mathcal{E} = \underbrace{\sum_{h \in \mathcal{H}} \mu_h \lambda_h x_h - \sum_{i \in \mathcal{I}} \mu_i \lambda_i x_i}_{\text{Distributive Externalities}} + \underbrace{\sum_{h \in \mathcal{H}} \mu_h \nabla_p \Lambda_h \Gamma_h + \sum_{i \in \mathcal{I}} \mu_i \nabla_p \Lambda_i \Gamma_i}_{\text{Collateral Externalities}}.$$

Moreover, τ_i does not depend on i .

Proposition 1 tells us that optimal regulation is determined by an interaction between two objects: the aggregate supply response and an externality vector. To understand this interaction, suppose that a bank increases its activities by dx_i . Because all banks are

¹⁰Optimal policy sets all wedges on a_i and on the numeraire z_i to zero.

regulated, the consequence of this increase is an increase in excess demand, which must then be absorbed by households. In other words, we have $dX_{\mathcal{H}} = dx_i$. In order to induce households to absorb this excess demand, prices must adjust accordingly. $\Xi_{\mathcal{H}}^{-1}$ gives this inverse mapping from changes in household supply to changes in prices, so that we have $dx_i \rightarrow dp$ via the relationship $dX'_{\mathcal{H}} \Xi_{\mathcal{H}}^{-1} = dp'$.

The externality vector \mathcal{E} in turn describes the social welfare consequences of a vector of price dp . In particular, the externality vector \mathcal{E} comprises two types of externalities: distributive externalities and collateral externalities (e.g. [Dávila and Korinek \(2018\)](#)). Distributive externalities arise because an increase in price $dp(m)$ redistributes money away from buyers of good m towards sellers of good m . This has a net social welfare consequence if and only if the marginal value of wealth is not equated across agents. The second set of externalities are collateral externalities: because the constraint sets of agents depend of prices, the set of feasible allocations is altered by changes in prices. When agents face binding constraints, such as a binding collateral constraint, changes to the feasible allocation set have net social welfare consequences.¹¹

Taken together, $\Xi_{\mathcal{H}}^{-1}\mathcal{E}$ comprises a mapping from changes dx_i in bank activities to changes in social welfare, $d\mathcal{L}$, in three steps. This mapping takes the form

$$dx_i \xrightarrow{I} dX_{\mathcal{H}} \xrightarrow{\Xi_{\mathcal{H}}^{-1}} dp \xrightarrow{\mathcal{E}} d\mathcal{L}$$

and gives us the form of the tax rate.

Finally, to understand why τ_i does not depend on i , the externality here is a pecuniary externality that results from changes in prices that result from changes in excess demand that must be absorbed by households. Because different banks (normalizing by their measure) generate the same changes in excess demand through the same changes in activities, the mapping from changes in activities to changes in welfare is the same for all banks. As a result, optimal regulation sets the same tax rate on all regulated banks.

3.2 Optimal Regulation with Shadow Banks

Suppose now that we have shadow banks in the economy, that is $\mathcal{S} \neq \emptyset$. Recall that the set of banks \mathcal{I} is the same, so that \mathcal{S} represents a change in the set of regulated agents. We solve the problem by a similar method as before, but now must take the demand functions of shadow banks as given. We obtain the following characterization.

¹¹In the general notation adopted, it is possible that the constraint set includes budget constraints, for example date-by-date budget constraints, in which case what we call collateral externalities would also include these distributive externalities.

Proposition 2. *Optimal bank regulation with shadow banks is given by*

$$\tau_i = -\frac{1}{\lambda_i} \mathcal{M} \Xi_{\mathcal{H}}^{-1} \mathcal{E} \quad (11)$$

where $\mathcal{M} = (I - \Xi_{\mathcal{H}}^{-1} \Xi_{\mathcal{S}})^{-1}$ is the regulatory arbitrage multiplier. Moreover, τ_i does not depend on i .

Optimal regulation in the presence of shadow banks is the same formula as without shadow banks, up to an additional term: the *regulatory arbitrage multiplier* \mathcal{M} . Notice that when $\Xi_{\mathcal{S}} = 0$, as is the case without shadow banks, then $\mathcal{M} = I$ and we recover the result of Proposition 1.

To build intuition for the regulatory arbitrage multiplier, consider a first order approximation, given by

$$\mathcal{M} \approx \underbrace{I}_{\text{Direct Impact}} + \underbrace{\Xi_{\mathcal{H}}^{-1} \Xi_{\mathcal{S}}}_{\text{First Round Shadow Bank Arbitrage}}.$$

To first order, \mathcal{M} reflects two effects of an increase in demand dx_i by a regulated bank. The first is the direct increase in excess demand, $dx'_i I$, which is identical to the previous section. However, there is also an indirect change in excess demand from all banks operating through the shadow banking sector. In particular, the direct change $dx'_i I$ fuels price changes $dp' = dx'_i \Xi_{\mathcal{H}}^{-1}$. In turn, these price responses fuel changes in shadow bank demand, given by $dX'_{\mathcal{S}} = dp' \Xi_{\mathcal{S}}$. As a result, to first order the total excess demand that must be absorbed by the household sector is $dx'_i + dX'_{\mathcal{S}} = dx'_i (I + \Xi_{\mathcal{H}}^{-1} \Xi_{\mathcal{S}})$. The response $\Xi_{\mathcal{H}}^{-1} \Xi_{\mathcal{S}}$ thus reflects to first order regulatory arbitrage by the shadow banking sector.

Beyond first order, regulatory arbitrage repeats itself. In response to the change in shadow bank activities, prices change again, which further fuels changes in shadow bank activities. This process generates a multiplier which is a fixed point of this cycle, given by

$$\mathcal{M} = \underbrace{I}_{\text{Direct Impact}} + \underbrace{\sum_{t=1}^{\infty} (\Xi_{\mathcal{H}}^{-1} \Xi_{\mathcal{S}})^t}_{\text{Total Shadow Bank Arbitrage}} = (I - \Xi_{\mathcal{H}}^{-1} \Xi_{\mathcal{S}})^{-1}.$$

As a result, \mathcal{M} dictates the total change in demand that must be absorbed by households after accounting for regulatory arbitrage by the shadow banking sector. In other words,

the total externality mapping is now given by

$$dx_i \xrightarrow{\mathcal{M}} dX_{\mathcal{H}} \xrightarrow{\Xi_{\mathcal{H}}^{-1}} dp \xrightarrow{\mathcal{E}} d\mathcal{L}$$

where \mathcal{M} is the fixed point of the regulatory arbitrage problem.

Direct and Indirect Pecuniary Externalities. The economy features both “direct” and “indirect” pecuniary externalities. A direct pecuniary externality arises because a change in activity m induces price changes dp , which induces a pecuniary externality. An indirect pecuniary externality arises because price changes alter shadow bank behavior, which in turn induces pecuniary externalities.

Taking a first order approximation of the regulatory arbitrage multiplier, we have

$$\tau_i \approx - \underbrace{\frac{1}{\lambda_i} \mathcal{E}|_{\mathcal{H}}}_{\text{Direct Externalities}} - \underbrace{\frac{1}{\lambda_i} \Xi_{\mathcal{H}}^{-1} \Xi_{\mathcal{S}} \mathcal{E}|_{\mathcal{H}}}_{\text{Indirect Externalities}},$$

where we define $\mathcal{E}|_{\mathcal{H}} = \Xi_{\mathcal{H}}^{-1} \mathcal{E}$. A key feature of indirect externalities is that they can lead to regulation of activities that do not contribute to direct externalities. For example, suppose that good m is liquidations of capital (fire sale), and the direct externality vector has $\mathcal{E}|_{\mathcal{H}}(m') \neq 0$ only for $m' = m$. In this model, optimal regulation without shadow banks only regulates liquidations of the capital good, which is the source of the externality.

Now, suppose that there are shadow banks. Although the direct externality is zero for all goods $m' \neq m$, we nevertheless have for $m' \neq m$

$$\tau_i(m') \approx - \underbrace{\frac{1}{\lambda_i} \Xi_{\mathcal{H}}^{-1}(m', m) \frac{\partial X_{\mathcal{S}}(m)}{\partial p(m')} \mathcal{E}|_{\mathcal{H}}(m)}_{\text{Indirect Externalities}}.$$

The intuition behind this effect is as follows. Suppose that the demand of shadow banks for the good generating the externality (“capital liquidations”) depends partly the price of another good (“debt”). When the price of debt goes up, shadow banks reoptimize and adjust their liquidations of capital. As a result, changes in the price of debt, although it has no direct pecuniary externality, generates an indirect pecuniary externality by affecting liquidations and the fire sale price. This indirect effect generates the possibility that the presence of shadow banks extends regulation to previously unregulated activities.

3.3 Magnitude of the Regulatory Arbitrage Multiplier

The regulatory arbitrage multiplier captures whether the presence of shadow banks amplifies or dampens bank regulation. Heuristically, this depends on whether \mathcal{M} is “larger” than 1 and therefore amplifying, or “smaller” than 1 and dampening.

Intuitively, one might expect regulatory arbitrage to dampen bank regulation. The chain of logic might be as follows. Consider a standard case of forced deleveraging leading to fire sales. As bank regulation increases bank stability and reduces fire sales, the liquidation price rises, encouraging shadow banks to take on more risk. As a result, the regulatory arbitrage multiplier is dampening, and bank regulation is less stringent. Intellectually, this line of reasoning amounts to a classical substitution effect: shadow bank *supply* of capital liquidations increases in the price of liquidations.

To first order, the magnitude of the regulatory arbitrage multiplier depends on the properties (including sign) of Ξ_S , which is the matrix of price derivatives of shadow bank Marshallian demand. To understand these demand responses, we perform a price theory decomposition into income and substitution effects.¹² Unlike the standard price theory decomposition, the price p appears in two places. First, as is standard, it appears in the budget constraint. However, it also appears in constraint sets. This appearance in constraint sets impacts the decomposition of income and substitution effects and affects how to think about them.

To this end, we adopt an explicit notational difference between the price \tilde{p} that appears in the budget constraint, and the price p that appears in the constraint set, so that we can explicitly separate out effects arising from prices in constraints and prices in budget constraints. Of course, in equilibrium we have $\tilde{p} = p$. Using this distinction, we can define the Marshallian demand function as $x_i(\tilde{p}, p, w_i)$, with $\Xi_i(m, m') = \frac{\partial x_i(\tilde{p}, p, w_i)}{\partial \tilde{p}} + \frac{\partial x_i(\tilde{p}, p, w_i)}{\partial p} \Big|_{\tilde{p}=p}$. Similarly, define Hicksian demand $h_i(\tilde{p}, p, \bar{U}) \in \mathbb{R}^M$ and the expenditure function $e_i(\tilde{p}, p, \bar{U})$ from the expenditure minimization problem.¹³

As usual, Hicksian demand h_i maintains a constant utility level with a price change through income compensation. As such, price derivatives of Hicksian demand capture substitution effects. In this case, substitution effects arise not only due to changes in the budget constraint price \tilde{p} , but also the constraint set price p . As such, we can write the

¹²The price theory logic that follows is closely related to [Farhi and Gabaix \(2020\)](#), who characterize price theory decompositions when behavioral agents have demand functions that exhaust budget constraints but do not maximize utility.

¹³The expenditure minimization problem for shadow banks is $\min h_i^z + \tilde{p}h_i$ subject to $U(h_i^a) \geq \bar{U}$ and $\Gamma(h_i^a, h_i^z, h_i, p) \geq 0$.

total substitution effect for a price change in both p and \tilde{p} (in the same direction) as

$$\bar{H}_i = \nabla_{\tilde{p}} h_i + \nabla_p h_i \Big|_{\tilde{p}=p}.$$

Similarly, e_i is the income compensation for a price change required to keep the utility level of a shadow bank fixed. In this case, income compensation occurs not only for changes in the budget constraint price \tilde{p} , but also for changes in the constraint set price p . As such, we can write the total income compensation required for a price change in both p and \tilde{p} (in the same direction) as

$$\bar{\nabla}_p e_i = \nabla_{\tilde{p}} e_i + \nabla_p e_i = h_i - \frac{1}{\lambda_i} \nabla_p \Lambda_i \Gamma_i = -\frac{1}{\lambda_i} \mathcal{E}_i.$$

Notice that the total income effect is the externality \mathcal{E}_i . Intuitively, \mathcal{E}_i reflects both sources of required income compensation for a price change. First, the budget constraint price changes, affecting directly the effective wealth of the consumer. However, the change in price also affects the constraint set, altering the feasible set of allocations. The alteration of the feasible set of allocations acts as an effective change in wealth because it leads the consumer to reoptimize within the new feasible set. A tightening of the constraint set is equivalent to a reduction in wealth, and requires positive income compensation to maintain the same utility level.

We can connect these income and substitution effects using the usual relationship between Hicksian and Marshallian demand, given by

$$h_i(\tilde{p}, p, \bar{U}) = x_i \left(\tilde{p}, p, e_i(\tilde{p}, p, \bar{U}) \right).$$

Differentiating in (\tilde{p}, p) in the same direction and evaluating at $\tilde{p} = p$, we have

$$\Xi_i = \underbrace{\bar{H}_i}_{\text{Substitution Effect}} + \underbrace{\frac{1}{\lambda_i} \mathcal{E}_i \nabla_{w_i} x_i}_{\text{Income Effect}}$$

where $\nabla_{w_i} x_i$ is the vector of wealth effects on traded goods, as so is positive for normal goods. Aggregating over the shadow banking sector, we have

$$\Xi_S = \bar{H}_S + \sum_{i \in S} \mu_i \frac{1}{\lambda_i} \mathcal{E}_i \nabla_{w_i} x_i. \quad (12)$$

Recalling that we had $\mathcal{M} - I \approx \Xi_{\mathcal{H}}^{-1} \Xi_{\mathcal{S}}$, this tells us that the sign of $\mathcal{M} - I$ generally depends on the interaction between income and substitution effects. Although the price constraint substitution effect will generally be negative, the constraint set substitution effect is more difficult to sign. For example, an increase in collateral value may encourage selling more capital (budget constraint substitution effect) but may also encourage holding more capital (constraint set substitution effect). The resulting total substitution effect therefore cannot immediately be signed.

On the other hand, there is also an income effect, which is weighted by \mathcal{E}_i , the total wealth-equivalent value to shadow bank i of the price change. When the total externality of a price increase \mathcal{E}_i is positive (and we have a normal good), the total income effect is positive. In this manner, the constraint set externality has the potential to amplify the income effect. This implies that when externalities are particularly large, the income effect is also particularly large.

As a result, even if the total substitution effect is negative, pushing for total shadow bank demand to *decrease* in the price, a pecuniary externality with $\mathcal{E}_i > 0$ generates an offsetting income effect that pushes total shadow bank demand to *increase* in the price due both to prices in budget constraints and prices in constraints. The strength of this income effect scales with the magnitude of the externality \mathcal{E}_i , suggesting that large pecuniary externalities result in large income effects. In the case of the fire sale model, the substitution effect shadow bank liquidations increase in the price due to the substitution effect, but decrease in price due both direct to higher income from liquidations and due to relaxed collateral constraints which reduce pressure for forced sales. This implies that if the income effect dominates the substitution effect, the regulatory arbitrage multiplier will be amplifying rather than dampening, as stronger bank regulation has an additional positive spillover of reducing shadow bank risk.

We now show a simple case in which income effects dominate, and the regulatory arbitrage multiplier is larger than 1.

Corollary 3. *Suppose that all cross-price elasticities are zero for shadow banks and households, so that $\Xi_{\mathcal{H}}$ and $\Xi_{\mathcal{S}}$ are diagonal. Then, \mathcal{M} is diagonal, with*

$$\mathcal{M}(m, m) = 1 + \frac{X_{\mathcal{S}}(m)}{X_{\mathcal{B}}(m)} \quad (13)$$

Corollary 3 shows that when cross-price elasticities are zero and when $\frac{X_{\mathcal{S}}(m)}{X_{\mathcal{B}}(m)} > 0$, that is banks and shadow banks are on the same side of a market m , then $\mathcal{M}(m, m) > 1$

and shadow banks *amplify* regulation of banks. For example, if market m is liquidations of capital during a crisis and both banks and shadow banks are liquidating capital, the regulatory arbitrage multiplier says that the presence of shadow banks nevertheless *increases* regulation against crisis state liquidations by banks. In this case, we have $\frac{\partial X_S(m)}{\partial p(m)} = -\frac{X_S(m)}{p(m)} > 0$, so that shadow bank liquidations *decrease* in the liquidation price $p(m)$ when $X_S(m) < 0$, implying a dominating income effect. To understand why, note that this relationship implies that $p(m)X_S(m) = c(m)$ for some constant $c(m)$, that is total shadow bank expenditures on good m are constant in the price. As such, if shadow banks sell good m , a price increase reduces the amount that must be sold to obtain c , leading shadow banks to reduce total sales. The income effect dominates. Notably, this case has some degree of economic relevance. For example, this might be a case where shadow banks must raise sufficient funds to cover a fixed liquidity shock or to service a debt repayment, but a financial market friction implies that they cannot roll over or issue new debt to cover the need and therefore must liquidate assets. This suggests that a dominating income effect may not be unrealistic in fire sale models. Indeed, in Section 7 we illustrate an application of the general model in the context of fire sales, and show precisely this outcome.

3.4 Effects of Shadow Banking on Optimal Regulation

The tax formula of Proposition 2 characterizes a regulatory arbitrage multiplier distinguishes the tax formulas with and without shadow banks. However, that statement holds fixed an allocation (including prices). We now ask what the total impact of introducing shadow banks into the economy is on bank regulation.

In particular, identify a set of banks \mathcal{S} to become shadow banks. The following result characterizes the first order impact of the set \mathcal{S} of banks becoming unregulated.

Proposition 4. *To first order, introducing shadow banks into the economy induces a change in regulation*

$$\tau(\mathcal{S}) - \tau(\emptyset) \approx \underbrace{\Xi_{\mathcal{H}}^{-1} \Xi_{\mathcal{S}} \tau(\emptyset)}_{\text{Regulatory Arbitrage}} - \underbrace{\nabla_{\mathcal{S}}(\Xi_{\mathcal{H}}^{-1} \mathcal{E})}_{\text{Change in Externalities}}$$

where $\tau = \lambda_i \tau_i$.

Proposition 4 shows that in general, two forces arise from the introduction of shadow banks into the economy. The first force is the introduction of regulatory arbitrage via shadow banks. To first order, the introduction of regulatory arbitrage accounts for the first step in the regulatory arbitrage change: shadow banks adjust activities in response

to the price change, which alters total changes in demand than must be absorbed by the household sector. As such, the first order impact of regulatory arbitrage is simply the first order approximation of the regulatory arbitrage multiplier. If the regulatory arbitrage multiplier tends to be larger (smaller) than 1 following the introduction of shadow banks, then to first order introducing shadow banks into the economy increases (reduces) equilibrium bank regulation.

However, there is also a second force, which is total changes in the externalities caused by bank activities.¹⁴ This term comprises two effects: changes in the aggregate supply response, and changes in the externality vector. The first effect states that because the allocations of trades held by households changes due to shadow banks, their marginal value of traded securities also changes, which affects the mapping from changes in aggregate household positions to prices. To give a concrete example, if households are second-best users of capital and the introduction of shadow banks alters how much liquidated capital they must purchase, then their willingness-to-pay for the marginal unit of liquidated capital also changes. The second effect, changes in the externality vector, arises because the introduction of shadow banks affects equilibrium prices and quantities, which also affects the severity of the pecuniary externality.

4 Welfare Costs of Shadow Banking

Section 3 characterizes optimal regulation with and without shadow banks, and contrasts optimal bank regulation under the two cases. This provides a comparison of regulation in the policy tool space. In this section, we consider the consequences of shadow banking in the welfare space.

In particular, we consider the welfare gain associated with the ability to regulate a previously unregulated shadow bank. Suppose that we begin from an equilibrium with shadow banks ($\mathcal{S} \neq \emptyset$), and the planner becomes able to regulate shadow bank $i \in \mathcal{S}$. The following result characterizes the welfare gains to first order from introducing a vector of taxes τ_i on this previously unregulated shadow bank.

Proposition 5. *Suppose that shadow bank $i \in \mathcal{S}$ is unregulated. Then to first order, the welfare gain of introducing regulation τ_i on shadow bank i is*

$$\Delta\mathcal{L}|_{\tau_i} \approx -\mu_i \tau_i' H_i \mathcal{M} \Xi_{\mathcal{H}}^{-1} \mathcal{E} \quad (14)$$

¹⁴See e.g. [Bengui and Bianchi \(2019\)](#).

where $H_i(m, m') = \frac{\partial x_i(m')}{\partial p(m)} + \frac{\partial x_i(m')}{\partial w_i} x_i(m)$ is the matrix of substitution effects in the budget constraint price.

Proposition 5 can be understood as follows. Because an unregulated shadow bank was previously at a privately optimal allocation, the private cost to that shadow bank of the new regulation is negligible to first order. As a result, the social welfare gain from new regulation is the social value of the change in that shadow banks' activities induced by regulation. The social benefit of a given change in activities is evaluated the same way as before, by the regulatory arbitrage and price adjustment process $\mathcal{M}\Xi_{\mathcal{H}}^{-1}$ which yields a set of price changes, with price changes being weighted by the externality vector \mathcal{E} . This coincides with the final three terms of Proposition 5. The first three terms, $-\mu_i \tau_i' H_i$, reflect (to first order) the change in activities of shadow bank i induced by the change in regulation. In particular, the regulatory changes τ_i , which are effectively changes in perceived prices of shadow banks, are multiplied by a matrix H_i of shadow bank activity responses to these price changes. The matrix H_i reflects a form of substitution effect: holding fixed both the effective income of shadow bank i and the collateral prices faced by shadow bank i , H_i gives the response of shadow bank demand to a change in the traded goods price. In this sense, H_i reflects a matrix of standard substitution effects, but with the caveat that these substitution effects hold fixed the collateral prices that determine shadow bank constraints (e.g. collateral constraints).

Which Shadow Bank to Regulate. Using Proposition 5, we can gain insight on where a social planner should look to regulate previously unregulated shadow banks. Take two shadow banks $i, j \in \mathcal{S}$, and propose regulations τ_i and τ_j , respectively. Proposition 4 tells us that to first order, the welfare gain from regulating i rather than j is given by

$$\Delta \mathcal{L}|_{\tau_i} - \Delta \mathcal{L}|_{\tau_j} \approx - \left(\mu_i \tau_i' H_i - \mu_j \tau_j' H_j \right) \mathcal{M}\Xi_{\mathcal{H}}^{-1} \mathcal{E}. \quad (15)$$

Considering two shadow banks of equal measure and proposing the same regulation ($\tau_i = \tau_j$), equation (15) tells us the choice between regulating different shadow banks rests on differences in the matrices of substitution effects. Intuitively, a shadow bank with substitution effects that are larger in magnitude delivers a larger quantity response to a given change in wedges. Because to first order there is no welfare impact on shadow bank equity value, the first order welfare gain is simply the pecuniary externality resulting from the change in activities.

To see this logic more clearly, let us suppose that in equilibrium, only a single activity

is regulated. That is to say, there exists some m_0 such that $(\mathcal{M}\Xi_{\mathcal{H}}^{-1}\mathcal{E})(m) = 0$ for all $m \neq m_0$. In this case, we obtain the welfare gain

$$\Delta\mathcal{L} - \Delta\mathcal{L}|_{\hat{\tau}} \approx -\hat{\tau}(m_0) \left(\mu_i \frac{\partial h_i(m_0, m_0)}{\partial \tilde{p}(m_0)} - \mu_j \frac{\partial h_j(m_0, m_0)}{\partial \tilde{p}(m_0)} \right) (\mathcal{M}\Xi_{\mathcal{H}}^{-1}\mathcal{E})(m_0),$$

which is directly proportional to the relative gap in (mass-weighted) own-price substitution effects.

5 Activity Regulation

Section 4 asks the question of what the welfare gain would be of regulating a previously unregulated shadow bank would be. Another possibility is for the social planner to regulate a specific activity or set of activities across shadow banks, rather than focusing on comprehensive regulation of specific institutions.¹⁵ For example, a social planner might impose a uniform tax on debt.

Formally, suppose now that the social planner can set uniform wedges $\tau(m)$ on a subset $\hat{M} \subset M$ of shadow banking activities. As before, the social planner has a full set of controls over regulated banks. The following result characterizes optimal activity regulation.

Proposition 6. *Suppose that the planner has a set of instruments $\tau(m)$ for $m \in \hat{M} \subset M$. Then, optimal activity regulation satisfies*

$$0 = I(\hat{M}) \sum_{i \in \mathcal{S}} \mu_i H_i \left(I - \tau \nabla_{w_i} x_i \right)^{-1} \left[\left(\lambda_i I + \mathcal{M}\Xi_{\mathcal{H}-1}\Theta_{\mathcal{S}} \right) \tau + \mathcal{M}\Xi_{\mathcal{H}}^{-1}\mathcal{E} \right]. \quad (16)$$

where $I(\hat{M})$ is a diagonal matrix with $I(m, m) = 1$ for $m \in \hat{M}$ and $I(m, m) = 0$ otherwise, and where $\Theta_{\mathcal{S}} = \sum_{i \in \mathcal{S}} \mu_i \lambda_i \Xi_i$.

To understand Proposition 6, consider the terms of equation (16). The final term, $\left(\lambda_i I + \mathcal{M}\Xi_{\mathcal{H}-1}\Theta_{\mathcal{S}} \right) \tau + \mathcal{M}\Xi_{\mathcal{H}}^{-1}\mathcal{E}$, reflects the trade-off between changes in shadow bank “equity” value and pecuniary externalities. The change in bank equity is due not only to the implicit effect through the budget constraint, but also to reoptimization from price changes due to the fact that wedges imply that shadow banks are not able to achieve the

¹⁵See e.g. [Federal Reserve Bank of Minneapolis \(2017\)](#).

privately optimal allocation. Because the wedges τ are equal in equilibrium to the private marginal value to shadow banks of activities, they capture the change in private value to banks of changes in activities induced by changes in regulation. This change in activities is captured by the initial set of terms. Intuitively, H_i reflects the set of substitution effects that arise from a change in the wedge $\tau(m)$, as in Section 4. However, the substitution effects H_i have to be scaled by $\left(I - \tau \nabla_{w_i} x_i\right)^{-1}$, which reflects a set of wealth effects that arise from reoptimization. Intuitively, remitted revenue is $\tau_i x_i^*$. The wealth effect accounted for in isolating the substitution H_i is the wealth effect on $\tau_i x_i^*$ arising through a change in τ_i . However, there is also a wealth effect on x_i^* which must be accounted for. This additional term reflects this. This first-order condition is pre-multiplied by $I(\hat{M})$, which ensures that it only holds for regulatory instruments which the planner possesses.

Intuitively, equation (16) reflects a balance between welfare consequences of tax changes on different activities. A tax increase $\tau(m)$ generates a positive social welfare impact with regards to activity m' when either, averaging across the population, m' is a negative-welfare activity and it shifts activities out of m' , or m' is a positive-welfare activity and it shifts activities into m' . If a tax increase $\tau(m)$ generated a positive welfare impact across all activities, it would be unambiguously desirable to increase it further. As a result, at the optimum it must either be the case that an increase in $\tau(m)$ has no net welfare impact on any activity, or that it has positive impacts on some activities and negative impacts on others. The latter case amounts to *arbitrage across activities*, rather than arbitrage across institutions.

5.1 Welfare Benefits from Activity Regulation

We now ask the question of what the potential welfare gains from activity regulation might be. In particular, starting from an equilibrium where shadow banks are unregulated, we suppose that the planner becomes able to regulate a single activity uniformly across banks. The following result characterizes the welfare gains to first order of activity regulation.

Proposition 7. *To first order, the welfare impact of activity regulation $\tau(m)$ for previously unregulated shadow banks is*

$$\Delta \mathcal{L} \approx -\tau(m) H_S(m, \cdot) \mathcal{M} \Xi_{\mathcal{H}}^{-1} \mathcal{E} \quad (17)$$

Intuitively, Proposition 7 states that the welfare gains from activity regulation are related to the substitution effect of bank activities to that tax change. In particular, it is the average substitution effect across all shadow banks, which amounts to isolating the m -th

row of the average substitution effect matrix $H_S = \sum_{i \in \mathcal{S}} \mu_i H_i$. The welfare impact of this change in shadow bank activities is evaluated in the usual way: the regulatory arbitrage multiplier maps the initial change into equilibrium changes through price changes, which are then mapped into pecuniary externalities.

5.2 Activity Regulation versus Institution Regulation

Using previous propositions, the welfare benefit of activity regulation $\tau(m)$ relative to institution regulation τ_i is

$$-\left[\tau(m) H_S(m, \cdot) - \mu_i \tau_i' H_i \right] \mathcal{M} \Xi_{\mathcal{H}}^{-1} \mathcal{E}.$$

This tells us that to first order, the trade-off between introducing activity regulation of shadow banks and introducing institutional regulation of a specific type of shadow banks revolves around a comparison of substitution effects within and across institutions. Activity regulation is relatively valuable when regulation of a single activity induces large substitution effects across shadow banks. On the other hand, institutional regulation of a single type of shadow bank is valuable when full regulation of all activities generates large substitution effects for that institution.

6 Incomplete Instruments and Non-Pecuniary Externalities

In previous sections, we have considered the case where the social planner has a complete set of regulatory instruments (wedges) for regulated banks, and moreover considers pecuniary externalities. In this case, we consider the case where the planner possesses an incomplete set of instruments and faces non-pecuniary externalities. In particular, we consider the following form of incomplete instruments: regulation can be applied in an initial period of the model, but cannot be applied in subsequent periods. This may correspond to initial capital or liquidity requirements. For example, this form of incomplete instruments is studied in [Dávila and Korinek \(2018\)](#) in characterizing optimal regulation with pecuniary externalities and financial frictions. In this case, we show that similar results can be obtained regarding characterization of optimal regulation in terms of a regulatory arbitrage multiplier.

6.1 Model

Incorporating both incomplete instruments of the form described and broader classes of non-pecuniary externalities requires only a slight modification to the framework above. In particular, define p^* to be the vector of M^* “future prices,” given by $p^* = \Phi^* \left(\{x_i, x_h\} \right)$.¹⁶ These future prices appear in both the utility functions of agents, $U_i(a_i, p^*)$, and in the constraint sets, $\Gamma_i(a_i, z_i, x_i, p, p^*)$. A simple example would be an indirect utility function from the continuation problem. As a result, both present and future prices affect demand and supply functions. In other words, household responses are now given by $x_h(p, p^*)$ (and similarly for shadow banks).

Notably, although we refer to p^* as “future prices,” the notation is sufficiently general that it can also be used to capture non-pecuniary externalities. For example, p^* could aggregate costs to a deposit insurance scheme, or could aggregate an environmental externality arising from activities. In other words, this section can also reflect non-pecuniary externalities with complete instruments.

From here, it is helpful to define the complete vector of prices as $P = \begin{pmatrix} p \\ p^* \end{pmatrix}$. Similarly, we can define $\Phi \left(\{x_i, x_h\}, P \right) = \begin{pmatrix} X_{\mathcal{H}} - X_{\mathcal{I}} \\ \Phi^* - p^* \end{pmatrix}$, so that the equilibrium condition is $\Phi = 0$. Finally, we can write the demand functions of households as $x_h(P)$, and similarly for shadow banks.

6.2 Optimal Regulation

We can now characterize optimal regulation with and without shadow banks when there are either incomplete instruments or non-pecuniary externalities.

Proposition 8. *Under incomplete instruments or non-pecuniary externalities:*

1. *Optimal regulation without shadow banks is given by*

$$\tau_i = -\frac{1}{\lambda_i} \nabla_{x_i} \Phi \left(\nabla_P \Phi + \sum_{h \in \mathcal{H}} \nabla_P x_h \nabla_{x_h} \Phi \right)^{-1} \mathcal{E}. \quad (18)$$

¹⁶We could make p^* also depend on z and a , which would mainly complicate the derivation.

2. Optimal regulation with shadow banks is given by

$$\tau_i = -\frac{1}{\lambda_i} \nabla_{x_i} \Phi \left(\nabla_P \Phi + \sum_{h \in \mathcal{H}} \nabla_{Px_h} \nabla_{x_h} \Phi \mathcal{M}^{-1} \right)^{-1} \mathcal{E} \quad (19)$$

$$\text{where } \mathcal{M} = \left(I + \left(\sum_{h \in \mathcal{H}} \nabla_{Px_h} \nabla_{x_h} \Phi \right)^{-1} \sum_{j \in \mathcal{S}} \nabla_{Px_j} \nabla_{x_j} \Phi \right)^{-1}.$$

Optimal regulation accounts for externalities resulting from changes in future prices. In absence of shadow banks, a change in regulated bank activities induces an equilibrium change in both current and future prices. Not only is there a direct effect on prices, but there is also an indirect effect: the current behavior of households changes in response to changes in both current and future prices. This in turn induces further price changes. In other words, similar to previous sections, equilibrium changes in future prices must be such that households are willing to absorb the change in demand from banks. The total change in both current and future prices multiplies the vector \mathcal{E} of pecuniary externalities arising from all price changes.

Next introducing shadow banks, we obtain a regulatory arbitrage multiplier \mathcal{M}^* which multiplies the indirect effect. This regulatory arbitrage multiplier has a similar intuition to the previous sections, which can be uncovered with a first order approximation. Here, we have

$$\mathcal{M} \approx \underbrace{I}_{\text{Household Effect}} - \underbrace{\left(\sum_{h \in \mathcal{H}} \nabla_{Px_h} \nabla_{x_h} \Phi \right)^{-1} \sum_{j \in \mathcal{S}} \nabla_{Px_j} \nabla_{x_j} \Phi}_{\text{First Order Shadow Bank Effect}}.$$

This decomposition uncovers a similar logic to before. Changes in bank activities induce price changes, which then lead shadow banks to change activities. Changes in shadow bank activities then have further effects on prices. After accounting for the effects of shadow banks on future prices, these prices are then passed to households, who then respond and induce further price changes. Notice that the regulatory arbitrage multiplier necessarily only multiplies the indirect effect through reoptimization, and does not multiply the direct effect.

Significantly, the regulatory arbitrage multiplier \mathcal{M} is *not* generally invariant to the distribution of shadow banks and households, and so does not depend only on aggregates. This is because the equilibrium price p^* is not necessarily invariant to the cross-section of shadow bank and household positions. It is only by assuming that future prices depend

only on aggregate holdings, X_S and X_H , that we recover that these additional multipliers depend only on aggregates. As a result, knowledge of these regulatory arbitrage multipliers under incomplete instruments requires richer knowledge about the cross-section of responses of shadow banks and households.

Additional Results. Other core results of the baseline model – Propositions 5, 6, and 7 – are results that multiply changes in activities by the externality that arises through market clearing. In this case, “market clearing” simply reflects a more general set of constraints. As a result, similar characterizations are obtained under the externality definition of this section.

7 An Application to Fire Sales

In this section, we apply our theory to simple models of fire sales. We adopt notation in this section that helps ease exposition, for example labeling debt by D . However, we maintain notational consistency with the general model as much as possible.

We consider two roles for shadow banks. In the first case, both banks and shadow banks invest in a capital good, and so fire sell assets in tandem during a crisis. Nevertheless, we show that the regulatory arbitrage multiplier can be larger than 1 due to an income effect, increasing the stringency of bank regulation. In the second case, shadow banks purchase bank assets liquidated prior to maturity and so increase market depth. Nevertheless, we show that the regulatory arbitrage multiplier is less than 1, reducing the stringency of bank regulation.

7.1 Shadow Banks as Asset Sellers

In our first application, we consider shadow banks who contribute to fire sales by liquidating assets in tandem with banks. Nevertheless, we show that the regulatory arbitrage multiplier may be greater than 1.

There are three periods, $t = 0, 1, 2$, and three agents: a representative bank, a representative shadow bank, and a representative household. The representative bank and shadow bank are indexed by $i \in \{S, B\}$, to maintain notational consistency. An aggregate states $\in \{s_H, s_L\}$ is realized at date 1, with the probability of the high state being π_H .

Banks. At date 0, banks can raise debt at price 1 to finance investment. Their budget constraint at date 0 is

$$\Phi_i(I_i) \leq A_i + D_i$$

where A_i is an initial endowment of the consumption good, and D_i is debt issuance. We impose the functional form $\Phi_i(I_i) = I_i + \frac{1}{2}\kappa_i I_i^2$ in order to obtain closed-form solutions.

At date 1, bank investment experiences a quality adjustment R_i , which does not depend on the aggregate state. However, at this point banks must roll over their debt, subject to a collateral constraint, given by

$$D_i \leq p(s)L_i(s) + (1 - \theta_i(s))p(s)(R_i I_i - L_i(s))$$

where $\theta_i(s)$ is the collateral haircut in state s , and $L_i(s)$ is the amount of the asset liquidated prior to maturity. Assets liquidated prior to maturity are sold at price $p(s)$. Assets held to maturity yield 1 unit of consumption per unit of final scale.

In the high state, credit markets are deep and there is no collateral haircut, so that $\theta_i(s_H) = 0$. As a result, banks do not need to liquidate assets to roll over their debt. As a result, final bank consumption in the high state is $c_i(s_H) = R_i I_i - D_i$.

However, in the low state there are frictions in the credit market, with $\theta_i(s_L) > 0$. We assume there are sufficient frictions that the collateral constraint binds in equilibrium. As a result, bank consumption in the low state is $c_i(s_L) = R_i I_i - D_i + (p(s_L) - 1)L_i(s_L) - D_i$, where $L_i(s_L)$ is given by the binding collateral constraint.

As a result, bank optimization can be expressed as

$$\max_{I_i, D_i, L_i(s_L)} R_i I_i - D_i + (1 - \pi_H)(p(s_L) - 1)L_i(s_L)$$

subject to the budget constraint and the low-state collateral constraint. From here, we obtain the following characterization of bank behavior.

Lemma 9. *Absent regulation, optimal bank i project scale I_i and debt issuance D_i are increasing in the low state liquidation price $p(s_L)$.*

From here, the derivative of liquidations in the price is

$$\frac{\partial L_i(s_L)}{\partial p(s_L)} = \underbrace{-\frac{1}{\theta_i(s_L)p(s_L)^2}}_{\text{Income Effect}} + \underbrace{\frac{1}{\theta_i(s_L)p(s_L)} \frac{\partial D_i}{\partial p(s_L)}}_{\text{Higher Debt}} - \underbrace{\frac{1 - \theta_i(s_L)}{\theta_i(s_L)} R_i \frac{\partial I_i}{\partial p(s_L)}}_{\text{Higher Collateral}}.$$

Intellectually, there are three effects on bank liquidations from a price increase. First, there is an income effect: the higher price relaxes the collateral constraint, reducing bank liquidations. The second effect is the response of bank debt: a higher bank debt level increases risk and forces more liquidations. The final effect is the response of bank collateral: higher investment increases collateral and eases debt rollover.

Households. The representative household both lends ex ante to banks, and also is the second best user of bank capital ex post. Households are risk neutral and do not discount the future, so that the loan price is always fixed at 1. At date 1, households operate a technology $\mathcal{F}(L_h)$ to produce the consumption good, so that households have demand $p(s_L) = \mathcal{F}'(L_h(s_L))$. As such, we can write

$$\frac{\partial L_h(s_L)}{\partial p(s_L)} = \frac{1}{\mathcal{F}''(L_h)} < 0.$$

Regulatory Arbitrage Multiplier. Notice that in this model, $p(s_L)$ is the only endogenous price, and banks only have one degree of freedom in their choice set when the low state collateral constraint binds (either choosing debt, investment, or low state liquidations). We can therefore equivalently express regulation as regulating low state liquidations or initial debt. For simplicity, we focus on regulatory wedges placed directly on low state liquidations. To this end, because we have representative agents, the aggregate household supply response is $\Xi_{\mathcal{H}} = \frac{\partial L_{\mathcal{H}}(s_L)}{\partial p(s_L)}$ while the aggregate shadow bank demand response is $\Xi_{\mathcal{S}} = \frac{\partial L_{\mathcal{S}}(s_L)}{\partial p(s_L)}$. From here, we obtain the following result.

Proposition 10. *The regulatory arbitrage multiplier \mathcal{M} is given by*

$$\mathcal{M} = \left(1 + \left| \frac{1}{\mathcal{F}''(L_{\mathcal{H}}(s_L))} \right| \left| \frac{\partial L_{\mathcal{S}}(s_L)}{\partial p(s_L)} \right| \right)^{-1}.$$

The regulatory arbitrage multiplier \mathcal{M} is bigger (smaller) than 1 if $\frac{\partial L_{\mathcal{S}}}{\partial p}$ is negative (positive). A sufficient condition for the regulatory arbitrage multiplier to be larger than 1 is for the high state probability π_H to be sufficiently large.

The intuition behind Proposition 10 is straight-forward. First, when shadow banks reduce (increase) liquidations in response to a price increase, shadow bank activities amplify (dampen) the effects of bank regulation. As such, the regulatory arbitrage multiplier is larger (smaller) than 1, leading to stricter (more lenient) regulation of banks. In this environment, shadow banks reduce liquidations when the direct income effect from the higher

price relaxing the collateral constraint (reducing liquidations) outweighs the effects from higher debt issuance (increasing liquidations). The direct income effect is the dominant force when the crisis state is sufficiently low probability, and hence the responsiveness of debt issuance to the crisis state price is not too large.

7.2 Shadow Banks as Asset Buyers

In our second application, we consider shadow banks to be on the opposite side of the liquidation market: they purchase assets from banks during crises. In this case, we show that the regulatory arbitrage multiplier is in fact *less* than one.

Model. The model of regulated banks and households is the same as the previous application. On the other hand, shadow banks are unable to create their own projects, but can purchase and manage a liquidated bank project in the low state at efficiency R_S , that is turning 1 unit of the liquidated bank project purchased at date 1 into R_S units of the consumption good at date 1. However, shadow banks cannot pledge income from managing bank projects, and so much use their own financing. They have initial wealth A_S at date 0 which they can save for use at date 1 by lending to banks (or households), but can only save in non-contingent bonds. As a result, in the low state shadow banks have

$$p(s_L)L_S(s_L) = D_S$$

where $-D_S > 0$ is shadow bank savings and $-L_S > 0$ is shadow bank asset purchases. The optimization problem of shadow banks is

$$\max_{D_S} u_S(A_S + D_S) - \pi_H D_S - (1 - \pi_H) \frac{R_S}{p(s_L)} D_S$$

From here, we obtain the following result.

Proposition 11. *Shadow bank savings $-D_S$ are decreasing in the low state liquidation price ($-\frac{\partial D_S}{\partial p(s_L)} < 0$). Shadow bank low state asset purchases satisfy*

$$-p(s_L) \frac{\partial L_S(s_L)}{\partial p(s_L)} = - \underbrace{|L_S(s_L)|}_{\text{Income Effect}} - \underbrace{\frac{\partial D_S}{\partial p(s_L)}}_{\text{Lower Savings}} < 0$$

and so are decreasing in the liquidation price. The regulatory arbitrage multiplier is

$$\mathcal{M} = \left(1 + \left| \frac{1}{\mathcal{F}''(L_{\mathcal{H}}(s_L))} \right| \left| \frac{\partial L_S(s_L)}{\partial p(s_L)} \right| \right)^{-1} < 1.$$

Proposition 11 indicates that although shadow banks are *beneficial* in the sense that they increase market depth by increasing savings and asset purchases when the price falls, they result in a regulatory arbitrage multiplier *less* than one, dampening bank regulation. The intuition is straight-forward: as the price rises due to stricter bank regulation, shadow banks experience a negative income effect which reduces their asset purchases. Moreover, their value from purchasing assets also decreases, so that they also lower their savings. Both of these effects reduce asset purchases by shadow banks in the crisis state. As a result, greater regulatory stringency has a side effect of reducing loss absorbing capacity of shadow banks, undoing some of the stability gains achieved from bank regulation. As a result, the regulatory arbitrage multiplier is less than 1.

8 Conclusion

We study optimal regulation when there are pecuniary (or non-pecuniary) externalities arising from the activities of financial intermediaries, in the presence of unregulated shadow banks. The presence of shadow banks introduces a “regulatory arbitrage multiplier” into the optimal regulatory formula for banks. The multiplier is determined by the responsiveness of *aggregate* shadow banking activities to changes in prices. As a result, it provides a sufficient statistic for a regulator to determine optimal regulation with shadow banks by only looking at aggregate changes in shadow bank activities.

Although we have framed our results in terms of financial regulation and pecuniary externalities, we have also shown that our results can be extended to broader classes of non-pecuniary externalities, either in the banking context or in other externality problems. Our framework can therefore provide broader guidance to regulators who lack the ability to prevent substitution of activities into unregulated agents.

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A Proofs

A.1 Proof of Proposition 1

Because all banks are regulated, the social optimality condition for an external trade $x_i(m)$ is

$$0 = \frac{\partial \mathcal{L}_i}{\partial x_i(m)} - \lambda(m)$$

which implies that we can write the optimal wedges as

$$\tau_i = \frac{1}{\lambda_i} \lambda$$

where λ_i is the budget constraint Lagrange multiplier.

Next, by Envelope Theorem the optimal external price $p(m)$ is

$$0 = \sum_{h \in \mathcal{H}} \mu_h \frac{\partial \mathcal{L}_h}{\partial p(m)} + \sum_{i \in \mathcal{I}} \mu_i \frac{\partial \mathcal{L}_i}{\partial p(m)} + \nabla_{p(m)} X_{\mathcal{H}} \lambda$$

In vector form, this system is written as

$$\Xi_{\mathcal{H}} \lambda = -\mathcal{E}$$

so that we obtain the result.

A.2 Proof of Proposition 2

The proof follows the same steps as Proposition 1, except that we now have

$$(\Xi_{\mathcal{H}} - \Xi_{\mathcal{S}}) \lambda = -\mathcal{E}.$$

From here, we obtain

$$\tau_i = -\frac{1}{\lambda_i} (\Xi_{\mathcal{H}} - \Xi_{\mathcal{S}})^{-1} \mathcal{E} = -\frac{1}{\lambda_i} (I - \Xi_{\mathcal{H}}^{-1} \Xi_{\mathcal{S}})^{-1} \Xi_{\mathcal{H}}^{-1} \mathcal{E}$$

which gives the result.

A.3 Proof of Corollary 3

Summing budget constraints of the household sector, we have $Z_{\mathcal{H}} + \sum_{m' \in M} p(m') X_{\mathcal{H}}(m') = w_{\mathcal{H}}$. Differentiating in the price vector p , we obtain $\nabla_p Z_{\mathcal{H}} + \Xi_{\mathcal{H}} p = -X_{\mathcal{H}}$. Doing the same for shadow banks, differencing, and substituting in $p = -\Xi_{\mathcal{H}}^{-1} (X_{\mathcal{H}} + \nabla_p Z_{\mathcal{H}})$, we obtain

$$\mathcal{M} \Xi_{\mathcal{H}}^{-1} [X_{\mathcal{B}} + (\nabla_p Z_{\mathcal{H}} - \nabla_p Z_{\mathcal{S}})] = \Xi_{\mathcal{H}}^{-1} (X_{\mathcal{H}} + \nabla_p Z_{\mathcal{H}}).$$

From here, define $\nabla_p Z_{\mathcal{B}} \equiv \nabla_p Z_{\mathcal{H}} - \nabla_p Z_{\mathcal{S}}$ through market clearing. From here, we have

$$(\mathcal{M} - I) \Xi_{\mathcal{H}}^{-1} [X_{\mathcal{B}} + \nabla_p Z_{\mathcal{B}}] = \Xi_{\mathcal{H}}^{-1} (X_{\mathcal{S}} + \nabla_p Z_{\mathcal{S}}).$$

The result for the case where cross-price elasticities are zero follows immediately from here.

A.4 Proof of Proposition 4

Define the tax rate after being normalized by the wealth level as

$$\tau(\mathcal{S}) \equiv -\mathcal{M} \Xi_{\mathcal{H}}^{-1} \mathcal{E}.$$

Then, to first order we have

$$\begin{aligned} \tau(\mathcal{S}) &\approx \tau(\emptyset) - \nabla_{\mathcal{S}} \mathcal{M} \Xi_{\mathcal{H}}^{-1} \mathcal{E} |_{\mathcal{S}=\emptyset} \\ &= \tau(\emptyset) - \nabla_{\mathcal{S}} \mathcal{M} (\Xi_{\mathcal{H}}^{-1} \mathcal{E}) |_{\mathcal{S}=\emptyset} - \nabla_{\mathcal{S}} (\Xi_{\mathcal{H}}^{-1} \mathcal{E}) |_{\mathcal{S}=\emptyset}. \end{aligned}$$

From here, note that we have

$$\begin{aligned} \nabla_{\mathcal{S}} \mathcal{M} |_{\mathcal{S}=\emptyset} &= -(I - \Xi_{\mathcal{H}}^{-1} \Xi_{\mathcal{S}})^{-2} \left[\nabla_{\mathcal{S}} \Xi_{\mathcal{H}}^{-1} \Xi_{\mathcal{S}} + \Xi_{\mathcal{H}}^{-1} \nabla_{\mathcal{S}} \Xi_{\mathcal{S}} \right] |_{\mathcal{S}=\emptyset} \\ &= -\Xi_{\mathcal{H}}^{-1} \nabla_{\mathcal{S}} \Xi_{\mathcal{S}} |_{\mathcal{S}=\emptyset} \\ &= -\Xi_{\mathcal{H}}^{-1} \Xi_{\mathcal{S}} \end{aligned}$$

where $\Xi_{\mathcal{S}}$ in the final line is evaluated for the set \mathcal{S} of banks at the equilibrium allocation where $\mathcal{S} = \emptyset$. As a result, we have

$$\tau(\mathcal{S}) \approx \tau(\emptyset) + \Xi_{\mathcal{H}}^{-1} \Xi_{\mathcal{S}} (\Xi_{\mathcal{H}}^{-1} \mathcal{E}) - \nabla_{\mathcal{S}} (\Xi_{\mathcal{H}}^{-1} \mathcal{E}) |_{\mathcal{S}=\emptyset}$$

and finally, substituting for $\tau(\emptyset) = (\Xi_{\mathcal{H}}^{-1}\mathcal{E})$ yields

$$\tau(\mathcal{S}) - \tau(\emptyset) \approx \Xi_{\mathcal{H}}^{-1}\Xi_{\mathcal{S}}\tau(\emptyset) - \nabla_{\mathcal{S}}(\Xi_{\mathcal{H}}^{-1}\mathcal{E})$$

giving the result.

A.5 Proof of Proposition 5

By Envelope Theorem, we have

$$\Delta\mathcal{L} \approx \frac{\partial\mathcal{L}}{\partial\tau_i}\Delta\tau_i = -\mu_i(\nabla_{\tau_i}x_i\Delta\tau_i)'\lambda.$$

since to first order, shadow banks are at their optimum. From here, we can write the response function

$$x_i(m) = x_i(p + \tau, w_i, p, m)$$

where w_i is wealth in the budget constraint, which accounts for remissions. Denote $\frac{\partial x_i}{\partial \tilde{p}}$ to be the derivative in the budget constraint price, holding collateral prices fixed. Differentiating around $\tau_i = 0$, we have

$$\frac{\partial x_i(m')}{\partial \tau_i(m)} = \frac{\partial x_i(m')}{\partial \tilde{p}(m)} + \frac{\partial x_i(m')}{\partial w_i}x_i(m) = \frac{\partial h_i(m')}{\partial \tilde{p}(m)}$$

From here, define the matrix H_i by $H_i(m, m') = \frac{\partial h_i(m')}{\partial \tilde{p}(m)}$. Then, we have

$$\Delta\mathcal{L} \approx -\mu_i\Delta\tau_i'H_i\lambda$$

which obtains the result by substituting in for λ .

A.6 Proof of Proposition 6

Consider the social planner's Lagrangian, given as before.

$$\mathcal{L} = \sum_{h \in \mathcal{H}} \mu_h \mathcal{L}_h + \sum_{i \in \mathcal{I}} \mu_i \mathcal{L}_i + (X_{\mathcal{H}} - X_{\mathcal{I}})'\lambda.$$

Using the private shadow bank Lagrangian, accounting for activity regulation, by Envelope Theorem

$$\frac{\partial \mathcal{L}_i}{\partial \tau(m)} = \lambda_i \left(x_i^*(m) + (\nabla_{\tau(m)} x_i^*)\tau - x_i(m) \right) = \lambda_i (\nabla_{\tau(m)} x_i)\tau.$$

From here, the first order condition for activity regulation $\tau(m)$ is given by

$$0 = \sum_{i \in \mathcal{S}} \mu_i \lambda_i (\nabla_{\tau(m)} x_i) \tau - (\nabla_{\tau(m)} X_{\mathcal{S}}) \lambda.$$

Defining $x_i = x_i(p + \tau, w_i, p)$ and differentiating, we have

$$\frac{\partial x_i(m')}{\partial \tau(m)} = \frac{\partial x_i(m')}{\partial \bar{p}(m)} + \frac{\partial x_i(m')}{\partial w_i} \left(x_i(m) + (\nabla_{\tau(m)} x_i) \tau \right).$$

In matrix form, this yields

$$\nabla_{\tau(m)} x_i = \nabla_{\bar{p}(m)} x_i + x_i(m) \nabla_{w_i} x_i + \nabla_{\tau(m)} x_i \tau \nabla_{w_i} x_i.$$

Rearranging and inverting, we obtain, we have

$$\nabla_{\tau(m)} x_i = \left(\nabla_{\bar{p}(m)} x_i + x_i(m) \nabla_{w_i} x_i \right) \left(I - \tau \nabla_{w_i} x_i \right)^{-1}$$

Finally, noting that $H_i(m, \cdot) = \nabla_{\bar{p}(m)} x_i + x_i(m) \nabla_{w_i} x_i$, we have

$$\nabla_{\tau(m)} x_i = H_i(m, \cdot) \left(I - \tau \nabla_{w_i} x_i \right)^{-1}.$$

From here, we obtain the first order condition, we obtain

$$0 = \sum_{i \in \mathcal{S}} \mu_i H_i(m, \cdot) \left(I - \tau \nabla_{w_i} x_i \right)^{-1} \left(\lambda_i \tau - \lambda \right).$$

Finally, we can take the derivative in p . Note that for shadow banks, we have

$$d\mathcal{L}_i = \nabla_p \mathcal{L}_i + \lambda_i \nabla_p x_i^* \tau = \nabla_p \mathcal{L}_i + \lambda_i \nabla_p x_i \tau$$

so that we have the social FOC for p

$$0 = \mathcal{E} + \sum_{i \in \mathcal{S}} \mu_i \lambda_i \Xi_i \tau + (\Xi_{\mathcal{H}} - \Xi_{\mathcal{S}}) \lambda$$

giving

$$\lambda = -\mathcal{M} \Xi_{\mathcal{H}}^{-1} \left[\mathcal{E} + \sum_{i \in \mathcal{S}} \mu_i \lambda_i \Xi_i \tau \right]$$

giving

$$0 = \sum_{i \in \mathcal{S}} \mu_i H_i(m, \cdot) \left(I - \tau \nabla_{w_i} x_i \right)^{-1} \left(\lambda_i \tau + \mathcal{M} \Xi_{\mathcal{H}}^{-1} \left[\mathcal{E} + \sum_{i \in \mathcal{S}} \mu_i \lambda_i \Xi_i \tau \right] \right).$$

Notice that for complete instruments ($\hat{M} = M$), then given λ_i is constant across i in this case we recover $\tau = -\frac{1}{\lambda_i} \mathcal{M} \Xi_{\mathcal{H}}^{-1} \mathcal{E}$ as before. Finally, let us define $I(\hat{M})$ to be a diagonal matrix with $I(m, m) = 1$ for $m \in \hat{M}$ and $I(m, m) = 0$ otherwise. Then, notice that we can write this system of equations as

$$0 = I(\hat{M}) \sum_{i \in \mathcal{S}} \mu_i H_i \left(I - \tau \nabla_{w_i} x_i \right)^{-1} \left[\left(\lambda_i I + \mathcal{M} \Xi_{\mathcal{H}^{-1}} \Theta_{\mathcal{S}} \right) \tau + \mathcal{M} \Xi_{\mathcal{H}}^{-1} \mathcal{E} \right].$$

where we define $\Theta_{\mathcal{S}} = \sum_{i \in \mathcal{S}} \mu_i \lambda_i \Xi_i$.

A.7 Proof of Proposition 7

Using the Envelope Theorem, the first order welfare gain from activity regulation is simply the right hand side of equation (16) evaluated at $\tau = 0$, for a single activity.

A.8 Proof of Proposition 8

The Lagrangian of the social planner is given by

$$\mathcal{L} = \sum_{j \in \mathcal{I} \cup \mathcal{H}} \mu_j \mathcal{L}_j + \Phi' \lambda.$$

Both with and without shadow banks, we obtain by the usual steps

$$\tau_i = \frac{1}{\lambda_i} \nabla_{x_i} \Phi \lambda.$$

Without Shadow Banks. In absence of shadow banks, we have

$$0 = \nabla_P \sum_{j \in \mathcal{I} \cup \mathcal{H}} \mu_j \mathcal{L}_j + \left(\nabla_P \Phi + \sum_{h \in \mathcal{H}} \nabla_P x_h \nabla_{x_h} \Phi \right) \lambda.$$

Noting that $\nabla_P \Phi + \sum_{h \in \mathcal{H}} \nabla_P x_h \nabla_{x_h} \Phi$ is a square matrix, we have

$$\lambda = - \left(\nabla_P \Phi + \sum_{h \in \mathcal{H}} \nabla_P x_h \nabla_{x_h} \Phi \right)^{-1} \mathcal{E}$$

where we now have $\mathcal{E} = \nabla_P \sum_{j \in \mathcal{I} \cup \mathcal{H}} \mu_j \mathcal{L}_j$. Substituting in yields

$$\tau_i = - \frac{1}{\lambda_i} \nabla_{x_i} \Phi \left(\nabla_P \Phi + \sum_{h \in \mathcal{H}} \nabla_P x_h \nabla_{x_h} \Phi \right)^{-1} \mathcal{E}.$$

With Shadow Banks. With shadow banks, we have

$$0 = \nabla_P \sum_{j \in \mathcal{I} \cup \mathcal{H}} \mu_j \mathcal{L}_j + \left(\nabla_P \Phi + \sum_{h \in \mathcal{H}} \nabla_P x_h \nabla_{x_h} \Phi + \sum_{j \in \mathcal{S}} \nabla_P x_j \nabla_{x_j} \Phi \right) \lambda,$$

so that we have

$$\lambda = - \left(\nabla_P \Phi + \sum_{h \in \mathcal{H}} \nabla_P x_h \nabla_{x_h} \Phi \mathcal{M}^{-1} \right)^{-1} \mathcal{E},$$

where we have defined

$$\mathcal{M} = \left(I + \left(\sum_{h \in \mathcal{H}} \nabla_P x_h \nabla_{x_h} \Phi \right)^{-1} \sum_{j \in \mathcal{S}} \nabla_P x_j \nabla_{x_j} \Phi \right)^{-1}.$$

This yields the result

$$\tau_i = - \frac{1}{\lambda_i} \nabla_{x_i} \Phi \left(\nabla_P \Phi + \sum_{h \in \mathcal{H}} \nabla_P x_h \nabla_{x_h} \Phi \mathcal{M}^{-1} \right)^{-1} \mathcal{E}.$$