Multinational Banks and Financial Stability

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Abstract

We study the scope for international cooperation in macroprudential policies. Multinational banks contribute to and are affected by fire sales in countries they operate in. National governments setting quantity regulations non-cooperatively fail to achieve the globally efficient outcome, under-regulating domestic banks and over-regulating foreign banks, necessitating international cooperation. Surprisingly, non-cooperative national governments using Pigouvian taxation can achieve the global optimum. Intuitively, this occurs because applying taxes, rather than quantity regulations, leads governments to internalize the business value of foreign banks through the tax revenue collected. Our theory not only provides a unified framework to think about international bank regulations, but also yields concrete insights with the potential to improve on the current policy stance.

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1 Introduction

The banking industry is multinational in its scope: banks that are headquartered in one country lend to, borrow from, and are owned by agents across country borders. More than 30% of global bank claims are on foreign counterparties as of 2019, with more than half of foreign claims being on the non-bank private sector.[1] In the aftermath of the 2008 financial crisis, the scope of global banking has led to concerns that international bank activities can contribute to domestic financial stability risk, with foreign banks both exacerbating and being affected by domestic fire sales.[2] These financial stability concerns have motivated financial regulators to extend post-crisis macroprudential regulatory regimes – such as equity capital and liquidity requirements – to foreign banks operating domestically.[3] They have also led governments to apply capital control measures – such as residency based transaction taxes – to manage foreign capital flows. As a result of multinational banking, the effects of fire sales and macroprudential regulation also extend across country borders.

The global dimension of banking has led to concerns that uncoordinated financial regulation may not be efficient. In the words of former Bank of England Deputy Governor Paul Tucker, *"given the internationalization of finance, the problem of financial system stability is a global common-resource problem. That means that the standard of resilience needs to be shared and so agreed internationally"* (Tucker (2016)). These concerns have motivated the formation of cooperative regulatory regimes, in which country regulators coordinate macroprudential policies. Cooperative regimes in practice include agreements to common regulatory standards (Basel III) and explicit common supervisory and resolution regimes (European Banking Union, Single Point of Entry (SPOE) resolution).[4] Despite the prominence of these agreements and their attention in the policy world, there is relatively limited formal economic analysis studying the need for macroprudential cooperation in the presence of financial stability concerns from cross-border banking, or to guide policymakers in forming cooperative agreements.

We provide a simple economic framework to study the regulation of cross-border banks in the presence of fire sales.[5] Despite its simplicity, we show that its insights extend

[1] Bank for International Settlements (BIS) Consolidated Banking Statistics (CBS), among reporting countries.

[2] See e.g. Tucker (2016). See e.g. French et al. (2010) for a broader discussion of financial stability concerns and motivations for post-crisis financial regulation.

[3] For example, the Intermediate Holding Company requirement in the US applies prudential standards of The Dodd-Frank Wall Street Reform and Consumer Protection Act (Dodd-Frank Act) to foreign banks with large operations in the US.

[4] See BIS (2010) for Basel III, ECB (2018b) for an overview of the EU Single Supervisory Mechanism responsible for common supervision, and Financial Stability Board (2013) for a discussion of SPOE.

[5] We frame our model in terms of banks, but it also applies to broader classes of financial intermediaries.

to a much more general setting. Our model captures essential elements of the global banking industry and real economy. Banks engage in cross-border investment activities for a variety of reasons such as comparative advantage or diversification against domestic risk. When banks experience adverse shocks, they are forced to sell domestic assets, leading to a domestic fire sale. The model features two sources of cross-border spillovers from fire sales. First, the domestic fire sale directly spills over to foreign banks investing in the domestic economy, reducing both recovery values in liquidation and collateral values. Second, the domestic fire sale leads banks to liquidate foreign assets, exacerbating fire sales in foreign countries. Although fire sales are domestic in origin, cross-border banking leads them to propagate across countries. Domestic financial stability becomes a global concern. To speak to real world concerns about international banking and financial stability, we map the model into several important stylized empirical facts and applications involving cross-border banking, including home bias and retrenching, financial integration in the EU, capital flows between developed economies and emerging markets, and countries with large global banking presences (such as the US).

Fire sale spillovers in the model, which are not internalized by banks, motivate the consideration of macroprudential regulation. Our first contribution is to characterize the globally efficient regulatory policy, which has two important properties. First, the stringency of globally efficient regulation accounts for not only domestic fire sale spillovers, but also for international spillovers that arise through cross-border banking activities. Second, equal regulatory stringency is applied to all banks regardless of their domicile, so that banks can enjoy equally the benefits of international activities.

In practice, regulatory policies are often handled by country level governments, who may engage in cooperative agreements governing policy design. Our second contribution is to provide a theory of the non-cooperative design of macroprudential policies by independent country governments, and to ask whether international cooperation is required to achieve the globally efficient outcome. In practice, countries have regulatory jurisdiction over all activities of domestic banks, as well as the domestic activities of foreign banks operating within their borders. This implies that multiple countries have regulatory jurisdiction over the same bank. In the absence of cooperative agreements, national regulators will set macroprudential policies independently to maximize national welfare. Our framework captures and allows us to study this non-cooperative behavior.

We next provide a theory of non-cooperative design of macroprudential policies under a quantity regulation approach. Non-cooperative quantity regulation does not achieve the globally efficient outcome. Country planners regulate both domestic and foreign banks in equilibrium. However, regulation of domestic banks accounts for domestic fire sale spillovers, but not for international fire sale spillovers. By contrast, country planners ensure that foreign banks' domestic activities are sufficiently safe that they do not contribute to domestic fire sales, leading to unequal treatment of domestic and foreign banks. This is consistent with regimes that subject local subsidiaries of foreign banks to domestic regulatory requirements, often assuming no support will be provided by the parent organization.[6] These departures from efficient policy provide a theory of optimal cooperation, which both increases regulation of domestic banks and ensures equal regulatory treatment of foreign banks. This resembles and helps to understand the broad architecture and goals of existing cooperative regimes.

In practice, cooperation can be difficult to sustain when national and international interests diverge.[7] One might wonder then whether instruments other than quantity regulation could improve non-cooperative outcomes and reduce the need for cooperation. One natural candidate is Pigouvian taxation, which is a common prescription for externality problems. However, there are two reasons to expect a priori that Pigouvian taxation would yield similar outcomes to regulation: first, these instruments are typically equivalent in models lacking standard (Weitzman (1974)) price-quantity trade-offs;[8] and second, there are international spillovers associated with fire sales.

The most surprising result of our paper is that *non-cooperative* Pigouvian taxation can implement the *globally* efficient outcome. In particular, country planners set tax rates that coincide with globally optimal policy, up to a monopolistic revenue extraction distortion. When countries' monopoly power is zero due to sufficient substitutability, non-cooperative Pigouvian taxation is globally efficient. The mobility of global banking assets and the presence of large offshore financial centers suggests that low monopoly revenues at the country level is a plausible description of the world.[9]

The key feature that gives rise to the optimality of Pigouvian taxation is that taxes on foreign banks generate revenues for the domestic planner. When combined with the standard motivation to correct domestic externalities, the revenue generation motive leads to efficient outcomes. The intuition is that a country planner becomes willing to allow foreign banks to engage in socially costly domestic activities because she can collect more tax revenue as a result. This results in an alignment in preferences between country

[6]For example, US total loss absorbing capital (TLAC) requirements apply to the US intermediate holding companies (IHCs) of foreign systemically important banks, rather than to the entire banking group.

[7]One recent example is the decision by the Italian government to engage in partial bailouts of distressed Italian banks in 2017, which faced criticisms as undermining the European Banking Union. "Why Italy's €17bn bank rescue deal is making waves across Europe," *Financial Times,* June 26, 2017.

[8]For example, Erten et al. (2019) argues that "the principle of dualism...implies that every quantity-based control corresponds to an equivalent price-based control."

[9]For example, see the work by Coppola et al. (2019) on global capital flows and tax havens.

planners and foreign banks. In equilibrium, the marginal tax rates on foreign banks' domestic activities are equal to the marginal benefit to foreign banks of those activities. Moreover because domestic fire sales reduce the marginal benefit to foreign banks of domestic activities, they also reduce tax revenue collection. The motive to generate tax revenue thus not only leads country planners to internalize the marginal benefit to foreign banks of domestic activities, but also to internalize the spillovers of domestic fire sales onto foreign banks. By contrast, quantity regulation lacks the revenue generation motive, giving no reason for the domestic planner to care about the welfare of foreign banks and leading to inefficient outcomes. Revenue generation from Pigouvian taxation is what aligns incentives. If Pigouvian taxation of foreign banks were revenue neutral, it would lead to the same outcome as quantity regulation.

The efficiency of non-cooperative Pigouvian taxation has implications for both macroprudential policies and capital controls. In the macroprudential context, it implies that giving a larger role in the financial regulatory regime to Pigouvian taxation can reduce or eliminate the need for cooperative arrangements. Although in practice macroprudential regulation often takes the form of quantity regulation, rather than Pigouvian taxation, we speculate that this may have arisen in part due to a combination of perceived duality between the instruments and political obstacles to taxation. Our results contribute a new argument in favor of a Pigouvian tax approach to macroprudential policies. In the capital controls context, our results imply that price-based capital control measures set non-cooperatively can be globally efficient. This provides further motivation for use of capital control measures in managing financial stability.

Even when Pigouvian taxation does not yield exact efficiency, it has the potential to simplify cooperation in two ways: first, by restricting the need for cooperation to foreign bank activities; and second, by reducing a multilateral spillover problem to a bilateral monopolist problem. Moreover, it implies that a set of partial equilibrium elasticities can be used to evaluate the need for cooperation.

We show that our simple framework can be generalized to study a number of other salient features of global banking. Our main application is the provision of fiscal backstops such as lender of last resort (LOLR) and deposit insurance. We show that non-cooperative governments under-value fiscal backstops to both domestic and foreign banks, not internalizing the full stability benefits to foreign banks. This motivates common fiscal backstops, such as Common Deposit Insurance in the EU. Because fiscal backstops are chosen by governments rather than by banks, Pigouvian taxation does not result in efficient (non-cooperative) choices of fiscal backstops unless banks are taxed for the bailouts they expect to receive.

Our results are sufficiently general that they hold in a broader class of externality problems featuring multinational agents. We present a general model of these externality problems. We characterize two classes of externalities: *local* and *global*. Local externalities, such as local pollution, are externalities that affect domestic agents, but not foreigners. However, foreign agents can contribute to local externalities by conducting domestic activities. Global externalities, such as global pollution or climate change, on the other hand, have costs that are globally diffuse and affect foreign agents. The efficiency of non-cooperative Pigouvian taxation extends to the class of local externalities, even though foreign multinational agents contribute to them, following the same logic as in the main model. By contrast, non-cooperative Pigouvian taxation is not generally efficient for global externalities. While cooperation is not required for local externalities, it is more generally required for global externalities. We show that cooperation tends not to be required when a local externality, such as a fire sale, takes on a global dimension due to cross-border activities, because Pigouvian taxes lead the national government to internalize the externality's impact on international agents. By contrast, global externalities that spread even without cross-border activities, such as climate change, generally require cooperation.

Related Literature. First, we relate to a large empirical literature on capital flows, retrenchment, and home bias, including in the context of banks.[10] The empirical literature documents both home bias and cross-country differences in foreign investment holdings, and suggests that motivations for foreign investment extend beyond diversification. These empirical observations help motivate the assumptions underlying our baseline banking model.

Second, we relate to a smaller literature on optimal regulatory cooperation in international banking and financial markets.[11] Caballero and Simsek (2018, 2019) show fickle capital flows can be globally valuable when they provide liquidity to distressed countries. National regulators do not internalize this benefit and ban capital inflows to mitigate domestic fire sales, generating a scope for cooperation. Korinek (2017) prove a first welfare theorem in a model in which country planners control domestic agents, who interact on global markets. Their welfare theorem does not hold in our model, in which domestic

[10] For example, Avdjiev et al. (2018), Broner et al. (2013), Caballero et al. (2008) Davis and Van Wincoop (2018), Forbes and Warnock (2012), and Milesi-Ferretti and Tille (2011), Maggiori (2017), Maggiori et al. (2019), De Marco et al. (2019), Mendoza et al. (2009), Niepmann (2015), and Shen (2019). See Coeurdacier and Rey (2013) for a broader overview of the home bias literature.

[11] Additional contributions include Beck et al. (2013), Beck and Wagner (2016), Calzolari et al. (2019), and Dell'Ariccia and Marquez (2006). Niepmann and Schmidt-Eisenlohr (2013) consider cooperation over bailouts with interbank market contagion. Farhi and Werning (2017) studies international risk sharing and fiscal unions.

liquidation prices affect foreign agents and where domestic planners are able to affect these liquidation prices. Farhi and Tirole (2018) show that national regulators loosen bank supervision to dilute existing international creditors, generating a time consistency problem that motivates supranational supervision. Bolton and Oehmke (2019) study the trade-off between single- and multi-point-of-entry in bank resolution. Bengui (2014) and Kara (2016) consider regulatory cooperation when banks' operations are domestic but the asset resale market is global. Our contribution to this literature is to study a global non-cooperative regulatory problem with common agency over cross-border banks, to characterize optimal cooperative policy, and to characterize optimality of Pigouvian taxation in the global context.

Third, we relate to a large literature on macroprudential regulation and capital controls.[12] Both literatures study optimal policy in response to fire sale externalities in the domestic economy, with macroprudential policies mitigating the contribution of domestic agents to fire sales and capital controls mitigating the contribution of foreign agents to fire sales. We study optimal policy in a global economy where banks are exposed to fire sales in all countries they invest in. Domestic planners have controls over both domestic and foreign banks, so that we incorporate both macroprudential policies and capital controls.

Fourth, we relate to the literature on principal-agent problems with common agency.[13] Our model features a common agency problem between countries regulating multinational banks and externalities between banks, and we differentiate between (sub-optimal) non-cooperative regulation and (optimal) non-cooperative Pigouvian taxation.

Fifth, we relate to the literature on the distinction between quantity and price regulation.[14] The primary difference between price and quantity regulation in our paper differs from the standard one: Pigouvian taxation generates revenues for the domestic planner, whereas quantity regulation does not. This revenue generation motive is required for the optimality of non-cooperative Pigouvian taxation. If revenues from Pigouvian taxation were remitted to foreigners, rather than to domestic agents, it would be equivalent to quantity regulation in our model.

[12] For example, Bianchi (2011), Bianchi (2016), Bianchi and Mendoza (2010), Bianchi et al. (2016), Bianchi and Mendoza (2018), Caballero and Krishnamurthy (2001), Chari and Kehoe (2016), Dávila and Korinek (2018), Farhi et al. (2009), Farhi and Tirole (2012), Farhi and Werning (2016), Korinek and Simsek (2016), Lorenzoni (2008), Schmitt-Grohé and Uribe (2016), and Stein (2012). See also Erten et al. (2019) for a recent survey of the capital controls literature.

[13] See e.g. Bernheim and Whinston (1986) and Dixit et al. (1997). In common with much of this literature, we apply an equilibrium concept where all country planners take policies of other countries as given. Another solution concept involves contracting on contracts. See e.g. Szentes (2015).

[14] See e.g. Weitzman (1974). In the banking context, see Perotti and Suarez (2011).

2 Model

There are three dates, $t = 0, 1, 2$. The world economy consists of a unit continuum of countries, indexed by $i \in [0, 1]$. All countries are small and of equal measure, but are not necessarily symmetric or otherwise identical ex ante.

Each country is populated by a representative bank and a representative arbitrageur.[15] Banks raise funds from global investors to finance investment in both their home country and in foreign countries. Arbitrageurs are second-best users of bank investment projects, and purchase bank investments that are liquidated prior to maturity. Arbitrageurs and global investors exist in our model to ease solving for the general equilibrium prices that banks face. Accordingly, we make their decision problems as simple as possible.

A global state $s \in S$ is realized at date 1, at which point uncertainty resolves. The global state s is continuous with density $f(s)$. It captures all shocks in the model, including global, regional, and country-level shocks.

In the baseline model presented in this section, we adopt a leading example model of banking with simplifying assumptions on preferences, technology, financing structure, and so on. In Section 6, we extend results to a much more general environment.

2.1 Banks

Banks are risk-neutral and do not discount the future. Banks only consume at date 2, with final consumption denoted by $c_i(s)$.

2.1.1 Bank Activities

Banks engage in activities such as retail banking, lending to small and medium-sized enterprises (SMEs), and investment banking, which we refer to collectively as "projects," "investment," or "assets." Projects are illiquid, and suffer a loss when liquidated (sold) prior to maturity.

In each country, there is an investment project available to banks. We denote by I_{ij} the (date 0) investment by country i banks in the country j project. A core assumption in our model is that bank investment is *home biased*.[16]

[15]The core aspects of the model (international activities, home bias, and fire sales) apply to broader financial intermediaries, such as mutual funds. Although for expositional purposes we frame the discussion in terms of banks, the model is also applicable to this broader class of financial intermediaries.

[16]See Caballero and Simsek (2019) for a similar assumption. As highlighted in the introduction, home bias is an empirical regularity, with in the neighborhood of 66% of bank claims being on domestic counterparties (CBS).

Assumption 1 (Home Bias)**.** A bank investment portfolio is $I_i = \{I_{ij}\}_j$, where

1. Domestic investment $I_{ii} \in \mathbb{R}_+$ is a mass.
2. Foreign investment $I_{ij} : [0,1] \to \mathbb{R}_+$ is a density.

Home bias will arise in our model when domestic banks specialize in domestic activities. Fire sales will be a core focus of the model. If domestic banks did not retain a mass exposure to the domestic economy, they would only be marginally affected by domestic fire sales. Home bias ensures that domestic banks are substantially exposed to domestic fire sales. Assuming that banks retain only a marginal exposure to foreign countries is a simplifying assumption to maintain tractability.

Banks operate a technology which uses $\Phi_{ij}(I_{ij})$ units of the numeraire to produce I_{ij}, where Φ_{ij} is increasing and weakly convex. Banks' total bank investment cost is therefore $\Phi_{ii}(I_{ii}) + \int_j \Phi_{ij}(I_{ij})dj$. Examples of costs are direct lending costs and operational costs.

At date 1, projects in country j experience quality shocks $R_j(s)$. As a result, the scale of projects operated by country i banks in country j changes to $R_j(s)I_{ij}$. Projects do not yield dividends at date 1. When projects are held to maturity at date 2, banks receive $1 + r_{ij} \geq 1$ units of the consumption good per unit of project scale. Intuitively, $R_j(s)$ captures a common risk exposure from investment in a country, while r_{ij} captures different specializations (comparative advantages) in bank lending. If $r_{ii} > r_{ij}$, then banks specialize in domestic lending, providing one potential source for home bias in lending.

Projects may be liquidated prior to maturity. We denote project liquidations by L_i, defined analogously to I_i, with $0 \leq L_{ij}(s) \leq R_j(s)I_{ij}$. Projects liquidated at date 1 are sold at price $\gamma_j(s) \leq 1$ to arbitrageurs, with the discount $1 - \gamma_j(s)$ reflecting the degree of illiquidity of a project. The final return r_{ij} is lost when a project is liquidated prior to maturity.

2.1.2 Bank Budget Constraints

Banks have an initial endowment $A_i > 0$, and can also raise external debt D_i from risk-neutral global investors at price 1, where for expositional simplicity we consolidate banks' balance sheets across countries and operations. Given a fixed debt price of 1, the liquidation prices γ are the only endogenous prices in the model. The bank uses its total funds to finance its investment portfolio at date 0, so that the date 0 bank budget constraint is

$$\Phi_{ii}(I_{ii}) + \int_j \Phi_{ij}(I_{ij})dj \leq A_i + D_i. \tag{1}$$

At date 1, banks can roll over debt at a price of 1, meaning that D_i is also the amount of new debt issued at date 1. Consolidating the dates 1 and 2 budget constraints yields

$$c_i(s) \leq \mathcal{R}_{ii}(s) + \int_j \mathcal{R}_{ij}(s)dj - D_i, \tag{2}$$

where $\mathcal{R}_{ij}(s) = \gamma_j(s)L_{ij}(s) + (1 + r_{ij})(R_j(s)I_{ij} - L_{ij}(s))$ is the total return to investment in country j for country i banks from both date 1 liquidations and date 2 final payoffs.

2.1.3 Collateral Constraints

If banks faced no restrictions on rolling over debt, they would never choose to liquidate assets since liquidations always reduce bank value. To introduce a role for liquidations and fire sales, we impose a date 1 collateral constraint, which is a standard method of capturing forced deleveraging (e.g. Kiyotaki and Moore (1997)).

The date 1 collateral constraint requires banks to back debt issued at date 1 with collateral, and is given by

$$D_i \leq \underbrace{\gamma_i(s)L_{ii}(s) + \int_j \gamma_j(s)L_{ij}(s)dj}_{\text{Funds From Liquidations}} + \underbrace{(1 - h_i(s))\mathcal{C}_{ii}(s) + \int_j (1 - h_j(s))\mathcal{C}_{ij}(s)dj}_{\text{Funds From Collateral}}, \tag{3}$$

where $\mathcal{C}_{ij}(s) = \gamma_j(s)\left[R_j(s)I_{ij} - L_{ij}(s)\right]$ is the market value of collateral at date 1. The collateral haircut $h_j(s) \in [0, 1]$ reflects the extent to which investors discount a project's collateral value, and can reflect economic (e.g. uncertainty) and political (e.g. expropriation) concerns about collateral quality. Banks that cannot roll over their entire liabilities using collateral must liquidate assets to repay investors.

2.1.4 Bank Optimization

At date 0, banks choose a contract (c_i, D_i, I_i, L_i) to maximize expected utility

$$V_i^B = \int_s c_i(s)f(s)ds$$

subject to the budget constraints (1) and (2), and the collateral constraint (3). Banks take equilibrium prices γ as given.

Banks in our model choose their entire contract with commitment, including liquidations L_i. The bank liquidation decision outlined in this section and in Section 2.3 is in fact time consistent, so that the assumption is innocuous in this sense. The core advantage of allowing liquidations to be chosen with commitment is that it simplifies the regulatory problems studied in this paper.

2.2 Arbitrageurs and Liquidation Values

Country i arbitrageurs are second best users of country i projects.[17] At date 1, they purchase an amount $L_i^A(s)$ of bank projects and convert them into the consumption good using an increasing and (weakly) concave technology $\mathcal{F}_i\left(L_i^A(s), s\right)$. Arbitrageur technology is inefficient in the sense that $\frac{\partial \mathcal{F}_i(L_i^A(s),s)}{\partial L_i^A(s)} \leq 1$, so that selling projects to arbitrageurs never results in a resource gain.

Arbitrageurs obtain surplus $c_i^A(s) = \mathcal{F}_i\left(L_i^A(s), s\right) - \gamma_i(s) L_i^A(s)$ from purchasing projects. Arbitrageurs are price takers, so that the equilibrium liquidation value is

$$\gamma_i(s) = \frac{\partial \mathcal{F}_i\left(L_i^A(s), s\right)}{\partial L_i^A(s)}, \quad L_i^A(s) = L_{ii}(s) + \int_j L_{ji}(s) dj \tag{4}$$

where $L_i^A(s)$ is equal in equilibrium to total country i projects sold by *all* banks, including foreign ones. There is a *fire sale spillover* when additional liquidations reduce liquidation values, that is when the marginal product of bank projects in arbitrageur technology is strictly decreasing. The extent of the fire sale spillover reflects the ability of the economy to absorb liquidations by banks, with deeper fire sales arising when limited market depth allocates liquidated bank projects to increasingly less efficient users.

2.3 Competitive Equilibrium

In order to map our model into economically important applications, we characterize the motivations for cross-border banking in the competitive equilibrium of the model.[18] To build concrete intuition, we study the optimal choices of investment and liquidations by banks.

Lemma 2. *In the competitive equilibrium, the private optimality conditions for investment (I_{ij})*

[17]The baseline model is equivalent to a model in which there are global arbitrageurs with separable technology across countries.

[18]Formally, a competitive equilibrium of the global economy is a vector of allocations (c, D, I, L, L^A) and prices γ such that: (1) the contract (c_i, D_i, I_i, L_i) is optimal for country i banks, given prices; (2) purchases L_i^A are optimal for country i arbitrageurs, given prices; and the markets for liquidations clear.

and liquidations ($L_{ij}(s)$), respectively, are

$$0 \geq \underbrace{-\lambda_i^0 \frac{\partial \Phi_{ij}}{\partial I_{ij}} + E[\lambda_i^1] E\left[(1+r_{ij})R_j\right]}_{\text{Specialization}} + \underbrace{\text{cov}\left(\lambda_i^1, (1+r_{ij})R_j\right)}_{\text{Diversification}} + \underbrace{E\left[\overline{\xi}_{ij} R_j\right]}_{\text{Liquidity}} + \underbrace{E\left[\Lambda_i^1 (1-h_j)\gamma_j R_j\right]}_{\text{Collateral}} \tag{5}$$

$$0 = \underbrace{\lambda_i^1(s)\left(\gamma_j(s) - (1+r_{ij})\right)}_{\text{Resource Loss}} + \underbrace{\Lambda_i^1(s) h_j(s) \gamma_j(s)}_{\text{Collateral Haircut}} + \underline{\xi}_{ij}(s) - \overline{\xi}_{ij}(s) \tag{6}$$

where the (non-negative) Lagrange multipliers are λ_i^0 on the date 0 budget constraint (1), $\lambda_i^1(s)$ on the date 1 budget constraint (2), $\Lambda_i^1(s)$ on the date 1 collateral constraint (3), and $\underline{\xi}_{ij}(s), \overline{\xi}_{ij}(s)$ (respectively) on the constraints $0 \leq L_{ij}(s) \leq R_j(s) I_{ij}$ in state s.

Proof. All proofs are in the Appendix. ■

The liquidation decision in equation (6) shows that because liquidations always result in resource losses, that is $(1+r_{ij}) - \gamma_j(s) > 0$, banks only liquidate assets when the collateral constraint binds, that is $\Lambda_i^1(s) > 0$. When faced with a binding collateral constraint, banks prefer to liquidate assets with lower liquidation discounts, $(1+r_{ij}) - \gamma_j(s)$, and higher collateral haircuts, $h_j(s)$. Banks sell liquid and hard-to-collateralize assets prior to illiquid ones. A positive Lagrange multiplier $\overline{\xi}_{ij}(s)$ indicates that banks have moved to a corner solution of fully liquidating an asset, and would prefer to liquidate more of it if they were able to do so.

The investment decision in equation (5) reflects the motivations for foreign investment. "Specialization" indicates that banks invest in assets with low marginal costs, $\frac{\partial \Phi_{ij}}{\partial I_{ij}}$, and high returns, r_{ij}. For example, banks may specialize in certain lending markets or may lend in under-serviced markets. "Diversification" indicates that banks value assets that pay off in states where the value of bank wealth, $\lambda_i^1(s)$, is high. Although banks are risk neutral in their preferences, a binding collateral constraint induces a higher marginal value of wealth as banks seek to avoid forced asset sales. "Liquidity" measures the value to banks of having more of an asset available to be liquidated when faced with binding collateral constraints, and implies banks value liquid assets with a high Lagrange multiplier $\overline{\xi}_{ij}(s)$. "Collateral" measures the value of an asset as collateral for rolling over debt at date 1, and is decreasing in both the haircut, $h_{ij}(s)$, and the liquidation discount, $1 - \gamma_j(s)$.

The diverse set of motivations for cross-border investment in our model allows for different countries to have different levels and compositions of foreign investment exposures, allowing us to capture a number of important potential applications of the model.

2.4 Empirics and Applications

We now connect our model to several important stylized facts and applications.

Home Bias and Retrenchment. Bank specialization in domestic activities ($r_{ii} > r_{ij}$), for example due to information advantages or lower costs,[19] can generate two empirical regularities: home bias and retrenchment patterns.

Home bias in investment is an empirical regularity whereby banks tend to overconcentrate in domestic assets compared to their world market weight. Nearly 70% of bank claims are on domestic counterparties (BIS CBS). Equation (5) indicates that domestic specialization motivate banks to scale up domestic investment when $r_{ii} > r_{ij}$.

A second empirical regulatirty is that domestic bank retrenchment generally coincides with reductions in foreign investment into the domestic economy.[20] When $r_{ii} > r_{ij}$, the forgone final return from liquidating domestic investment is larger than from liquidating foreign investment, motivating domestic banks to retrench and liquidate foreign assets. Foreign banks likewise retrench, coinciding with the empirical pattern.

Both of these empirical regularities are meaningful to the spillover and regulatory problem studied in the baseline model. Home bias implies that domestic banks maintain a substantial exposure to domestic risk and domestic fire sales. Foreign retrenchment implies that distressed banks liquidate assets in foreign countries, contributing to foreign stability risks.

EU Financial Integration. Risk sharing and increased financial stability are two key motivations for both cross-border financial integration and organization under a common supervisory framework (the Single Supervisory Mechanism, or SSM) in the EU.[21] Our model is able to capture the European experience with a block $I \subset [0,1]$ of countries ("the EU") have both partly uncorrelated and partly correlated returns. The uncorrelated component motivates cross-border investment for diversification (equation 5). However, a

[19] For example, recent work by De Marco et al. (2019) emphasizes the role of information in determining European banks' holdings of sovereign debt.

[20] See e.g. Avdjiev et al. (2018), Broner et al. (2013), Davis and Van Wincoop (2018), and Forbes and Warnock (2012).

[21] See ECB (2018a) and ECB (2018b).

correlated EU wide shock leads all member states to fire sell assets in tandem, fueling an EU wide crisis.

Developed and Emerging Markets. A large literature has emphasized that emerging markets may wish to use capital controls to manage domestic fire sales.[22] Our model captures emerging market stability concerns through a set of developed economies $D \subset [0,1]$ and a set of emerging markets $E \subset [0,1]$. Emerging markets have high expected returns, making them a target for developed economy investment (equation (5)). Developed economies have lower collateral haircuts due to stronger legal protections and lower uncertainty, making them a target for emerging markets looking for safe and liquid assets (equation (5)).[23] Due to larger emerging market haircuts, developed economies retrench during domestic crises (equation 6)), contributing to emerging market instability.

"Systemically" Important Countries. Some countries have larger foreign banking presences, and so may be important in the sense that they can generate larger spillovers. For example, the US has a large and growing presence in global investment banking, leading to concerns about European exposure to US shocks and retrenchment.[24] Our model captures systemically important countries through a block of countries $I \subset [0,1]$ that have large endowments A_i and are efficient in foreign lending (e.g. low costs Φ_{ij}). These countries maintain a large global banking presence (equation (5)), and their retrenchment may increase foreign financial instability.

3 Globally Optimal Policy

The presence of prices in constraints suggests existence of inefficient pecuniary externalities, warranting regulatory intervention. We begin by studying the optimal policy that would be chosen by a *global* planner looking to implement a globally Pareto efficient allocation. This global planning problem provides a benchmark for globally efficient policies.[25]

The global planning problem is a constrained-efficient problem, where the global

[22] See Erten et al. (2019) for an overview and IMF (2012) for a policy perspective.

[23] See the literature on global imbalances, for example Caballero et al. (2008), Maggiori (2017), and Mendoza et al. (2009).

[24] See Goodhart and Schoenmaker (2016). See also e.g. "How US banks took over the financial world," *Financial Times*, September 16, 2018.

[25] For expositional purposes, we present results in all sections of the main text for interior solutions. See Appendix E for a generalization that allows for corner solutions.

planner maximizes a weighted sum of bank welfare

$$V^G = \int_i \omega_i \int_s c_i(s) f(s) ds di$$

subject to the same constraints (equations (1), (2), (3)) as faced by banks, but internalizing the equilibrium pricing equation (4).[26] For expositional purposes, we place welfare weights of zero on arbitrageurs, and show in the Appendix that qualitatively similar results apply with positive weights.

We characterize the solution of the global planning problem by its decentralization: the complete set of date 0 wedges $\tau = \{\tau_i^c, \tau_i^D, \tau_i^I, \tau_i^L\}$ and date-0 lump sum transfers $\mathcal{T}_i$ that implement the globally optimal allocation. The complete set of wedges placed on country i banks consists of: a wedge $\tau_i^c(s)$ on consumption in state s; a wedge τ_i^D on date 0 debt; a wedge τ_{ij}^I on investment in country j; and a wedge $\tau_{ij}^L(s)$ on liquidations of country j assets in state s.[27] Given welfare weights ω_i, the globally efficient allocation is characterized as follows.

Proposition 3. *The globally efficient allocation can be decentralized using liquidation wedges*

$$\tau_{ji}^L(s) = -\underbrace{\Omega_{i,i}(s)}_{\text{Domestic Spillovers}} - \underbrace{\int_{i'} \Omega_{i',i}(s) di'}_{\text{Foreign Spillovers}} \quad \underbrace{\forall j}_{\text{Equal Treatment}} \tag{7}$$

where $\Omega_{i,j}(s) \leq 0$ is given by

$$\Omega_{i,j}(s) = \underbrace{\frac{\partial \gamma_i(s)}{\partial L_i^A(s)}}_{\text{Price Impact}} \left[\underbrace{\frac{\lambda_i^1(s)}{\lambda_i^0} L_{ij}(s)}_{\text{Liquidation Losses}} + \underbrace{\frac{\Lambda_i^1(s)}{\lambda_i^0} \Big[L_{ij}(s) + (1 - h_j(s)) \left[R_j(s) I_{ij} - L_{ij}(s) \right] \Big]}_{\text{Collateral Constraint Spillover}} \right] \tag{8}$$

All other wedges are 0.

The globally efficient allocation corrects a fire sale spillover problem: higher liquidations reduce liquidation prices and collateral values, tightening banks' collateral constraints further and forcing further liquidations. Because both domestic and foreign banks hold domestic investment, the fire sale impacts both domestic banks ("Domestic Spillovers") and all foreign banks ("Foreign Spillovers"). The impact on any individual bank is the product

[26] We do not give the global planner controls over arbitrageurs to prevent subsidizing asset purchases, which may be considered a form of bailout. We study fiscal backstops ("bailouts") in Section 5.

[27] As usual, the decentralizing wedge is set equal to the gap between the social first order condition and the private first order condition.

of the marginal change in the liquidation price ("Price Impact") and the total impact of that price change on that bank. That total impact consists of two elements: "Liquidation Losses" and "Collateral Constraint Spillover." "Liquidation Loss' reflects that an increase in the liquidation price increases the recovery value to banks from liquidating the asset, which is weighted by the marginal value of wealth, $\lambda_i^1(s)$, in that sate. Liquidation losses are larger when banks liquidate more of that asset, that is $L_{ij}(s)$ is high, or when the marginal value of date 1 wealth is high due to a more severely binding collateral constraint. "Collateral Constraint Impact" reflects the impact of the change in price on the binding collateral constraint. An increase in the liquidation price relaxes the collateral constraint both because liquidations generate a greater recovery value to repay debt holders, and because the collateral value for debt rollover increases. As a result, foreign spillovers are particularly large when foreign banks are forced to liquidate domestic assets or face binding collateral constraints at the same time that the domestic liquidation price is particularly sensitive to additional liquidations.

Because both domestic and foreign banks can contribute to the domestic fire sale via liquidations, globally efficient policy applies wedges to both domestic and foreign banks. Moreover, globally efficient policy applies *equal treatment*: the wedge placed on liquidations of the country i asset does not depend on the domicile of the bank liquidating it. This is because both domestic and foreign banks generate the same total spillover by liquidating a domestic project. Although foreign banks can contribute to domestic instability by retrenching, they are not treated different from domestic banks under the globally efficient policy.

4 Non-Cooperative Policies

The globally efficient policy of Section 3 is predicated on a global planner setting policy. However, in practice individual countries have regulatory jurisdiction over banks within their borders. In this section, we provide a theory of how country-level governments design macroprudential regulation in the presence of fire sales, and ask whether they implement the globally efficient policy independently and non-cooperatively. We show that independent governments using *quantity regulation* are unable to achieve efficient policy, and show that the departures from efficiency help understand existing cooperative regimes. By contrast, independent governments using *Pigouvian taxation* have the potential to achieve efficient policies.

4.1 Country Planners

Each country has a designated government, or "social planner," who represents and acts in the interests of domestic agents. The social welfare function of country planner i is equal to domestic bank welfare,

$$V_i^P = \int_s c_i(s) f(s) ds.$$

The social planner has a complete set of "macroprudential" wedges on both domestic banks and domestic allocations of foreign banks. The wedges of the country i planner on country i banks are $\tau_{i,i} = (\tau_{i,i}^c, \tau_{i,i}^D, \tau_{i,i}^I, \tau_{i,i}^L)$, and are fully contingent as in Section 3. The wedges of the country i planner on country j banks are $\tau_{i,j} = (\tau_{i,ji}^I, \tau_{i,ji}^L)$, again fully contingent, reflecting that the country i planner can only directly influence the domestic activities of foreign banks.

Wedges are taxes from the perspective of banks, meaning that revenue is collected from their use. We will interpret *revenue-neutral* wedges as quantity restrictions, appealing to duality results between quantity restrictions in revenue neutral wedges in problems with single regulators.[28] We will refer to *revenue-generating* wedges as Pigouvian taxation. In this case of quantity regulation, regulators use wedges to control allocations, but not to generate revenues. In this case of Pigouvian taxation, there is also a revenue motive.[29]

The total *date 0* wedge burden borne by country i banks (excluding remissions) is $T_{i,i} + \int_j T_{j,ij} dj$, where

$$T_{i,i} = \int_s \tau_{i,i}^c(s) c_i(s) f(s) ds + \tau_{i,i}^D D_i + \tau_{i,ii}^I I_{ii} + \int_j \tau_{i,ij}^I I_{ij} dj + \int_s \left[\tau_{i,ii}^L(s) L_{ii}(s) + \int_j \tau_{i,ij}^L(s) L_{ij}(s) dj \right] f(s) ds$$

is the burden to the domestic planner, and

$$T_{j,ij} = \tau_{j,ij}^I I_{ij} + \int_s \tau_{j,ij}^L(s) L_{ij}(s) f(s) ds$$

is the burden to foreign planner j. To ease exposition, we adopt inner product notation $T_{i,i} = \tau_{i,i}^c c_i + \tau_{i,i}^D D_i + \tau_{i,i}^I I_i + \tau_{i,i} L_i$ and $T_{j,ij} = \tau_{j,ij}^I I_{ij} + \tau_{j,ij}^L L_{ij}$.

Let $T_{i,i}^*$ denote the equilibrium tax revenue collected by country planner i from all domestic banks, where in equilibrium $T_{i,i}^* = T_{i,i}$. The equilibrium tax revenue $T_{i,i}^*$ is remitted lump-sum to domestic banks, under both quantity restrictions and Pigouvian taxation.

[28] See e.g. Erten et al. (2019). In Appendix D.6, we discuss a planning problem that uses explicit quantity restrictions.

[29] Because wedges are the means of controlling allocations, we rule out explicit side payments.

Under quantity regulation, the equilibrium tax revenue collected from foreign banks is remitted globally to foreign banks. As a result, the *equilibrium* date 0 tax *burden* of country i banks is

$$T_i^{\text{Quantity}} = T_{i,i} - T_{i,i}^* + \int_j T_{j,ij} dj - T_i^G$$

where T_i^G is global remitted revenues, which country i takes as given.[30]

Under Pigouvian taxation, the equilibrium tax revenue collected from foreign banks is remitted to domestic banks. The equilibrium date 0 tax burden is

$$T_i^{\text{Pigou}} = T_{i,i} - T_{i,i}^* + \int_j T_{j,ij} dj - \int_j T_{i,ji} dj.$$

In contract to quantity regulation, the country i planner now accounts for how changes in policy affect the revenue $\int_j T_{i,ji} dj$ collected from foreign banks.

In both cases, taxes appear in the banks' date 0 budget constraint, as

$$\Phi_{ii}(I_{ii}) + \int_j \Phi_{ij}(I_{ij}) dj \leq A_i + D_i - T_i. \tag{9}$$

Banks optimally choose contracts as in Section 2, now taking into account the additional tax burden.

Equilibrium Concept. A non-cooperative equilibrium of the model is a Nash equilibrium between country planners, in which every country planner optimally chooses wedges $\tau_i = (\tau_{i,i}, \{\tau_{i,j}\})$, taking as given the wedges τ_{-i} set by all other country planners, in order to maximize domestic social welfare, accounting for how tax rates affect the demand functions of domestic and foreign banks.

Implementability. Banks are subject to wedges by planners in all countries they operate in. Because country planner i has a complete set of wedges over country i banks, we can adopt the standard approach whereby country planner i directly chooses the allocations of country i banks, and then back out the implementing wedges. Although country planner i only has controls over part of foreign banks' contracts, we derive an implementability result that shows that the same procedure can be applied for choosing the domestic allocations of foreign banks.

[30]In particular, there is the globally remitted revenue $T^G = \int_i \int_j T_{i,ji}^* dj di$ arising from the wedges, which corresponds to remitting revenue to foreigners. We assume this is remitted according to some allocation rule $\int_i T_i^G di = T^G$.

Lemma 4 (Implementability). *Under both quantity regulation and Pigouvian taxation, country planner i can directly choose the domestic allocations of foreign banks, with implementing wedges*

$$\tau^I_{i,ji} = -\tau^I_{j,ji} - \frac{\partial \Phi_{ji}}{\partial I_{ji}} + E\left[\frac{\lambda^1_j}{\lambda^0_j}(1+r_{ji})R_i\right] + \frac{1}{\lambda^0_j}E\left[\Lambda^1_j(1-h_i)\gamma_i R_i\right] \tag{10}$$

$$\tau^L_{i,ji}(s) = -\tau^L_{j,ji}(s) + \frac{\lambda^1_j(s)}{\lambda^0_j}\left(\gamma_i(s) - (1+r_{ji})\right) + \frac{1}{\lambda^0_j}\Lambda^1_j(s)h_i(s)\gamma_i(s) \tag{11}$$

where the Lagrange multipliers and the foreign wedges $\tau_{j,ji}$ are constants from the perspective of country planner i.

As a result, the optimization problem of country planner i can be written as maximizing social welfare V^P_i, directly choosing allocations $(c_i, D_i, I_i, L_i, \{I_{ji}, L_{ji}\}_j)$, subject to equations (9), (2), (3), and (4).

To choose a domestic allocation of foreign banks, country planner i unwinds the wedges placed by the foreign country planner, and then sets the wedge equal to the marginal value to foreign banks of that activity in the domestic country.[31] Because foreign banks have only a marginal investment presence in country i, allocations in country i can be influenced without filtering to the entire foreign bank contract. As a result, we can define the optimization problem of country planner i using a standard approach.

Lemma 4 allows for solving the problem using a standard approach. Importantly, notice that the implementing wedges on foreign banks *do not* appear in the objective function of country i planner under quantity regulation, because revenues are remitted to foreign banks. By contrast, they do appear in the objective function under Pigouvian taxation, through the budget constraint due to revenue collections.

4.2 Non-Cooperative Quantity Regulation

We now characterize the solution to the problem under quantity regulation, where revenue from wedges on foreign banks is remitted to foreign banks.

Proposition 5. *Under non-cooperative quantity regulation, the equilibrium has the following features.*

[31] Notice that the non-tax terms in equations (10) and (11) are terms in the first order conditions from the competitive equilibrium. Intuitively, the Lagrange multipliers $\underline{\xi}_{ij}, \overline{\xi}_{ij}$ do not appear because the planner can always ensure that the first order conditions hold with equality at corner solutions by setting tax rates appropriate. See Appendix E.2.

1. *The domestic liquidation wedges on domestic banks are*

$$\tau^L_{i,ii}(s) = -\underbrace{\Omega_{i,i}(s)}_{\text{Domestic Spillovers}} \tag{12}$$

where $\Omega_{i,i}$ is defined as in Proposition 3.

2. *The domestic liquidation wedges on foreign banks generate an allocation rule*

$$L_{ji}(s) \underbrace{\Omega_{i,i}(s)}_{\text{Domestic Spillovers}} = 0.$$

In other words, if $|\Omega_{i,i}(s)| > 0$ then $\tau^L_{i,ji}(s)$ is set high enough that foreign banks do not to liquidate domestic assets in state s.

3. *All other wedges on domestic and foreign banks are* 0*.*

Proposition 5 reflects how country planners use regulation to manage spillovers from domestic fire sales. First, country planners place wedges on domestic liquidations by domestic banks that account for the fire sale spillover cost to *domestic* banks. Because planners do not care about the welfare of foreign banks, the domestic wedges do not account for spillovers to foreign banks.

Second, country planners place wedges on liquidations by foreign banks. Because planners again do not care about foreign bank welfare, their objective is to eliminate foreign banks' contributions to domestic fire sales whenever there is an adverse domestic spillover. Country planners thus prevent foreign banks from contributing to domestic fire sales even while allowing domestic banks to take risks that lead to domestic fire sales. This effective ban on domestic liquidations by foreign banks (e.g. a ban on outflows) is too strong in practice, and arises because there is no domestic benefit to foreign investment in the baseline model. In Appendix D.1, we obtain less strong versions of this result that capture the same logic: country planners are not concerned with the benefits to foreign banks of activities in the domestic economy, and allow foreign activities and liquidations only to the extent they benefit the domestic economy.

Finally, the domestic planner does not tax *foreign* liquidations by domestic banks. This happens because the investment presence of domestic banks in any single foreign country is marginal, so that country planners do not internalize their fire sale impact in foreign countries.

Departures from Global Efficiency. Non-cooperative quantity regulation differs from globally efficient policy in two important ways.

The first departure is that unlike globally efficient policy, non-cooperative policy does not account for foreign spillovers. As a result, the globally efficient wedge $\tau^L_{ii}(s)$ is generally *higher* than the non-cooperative wedge $\tau^L_{i,ii}(s)$. Non-cooperative regulation features too little regulation of domestic banks due to the foreign spillovers from domestic fire sales. This is a *multilateral* problem, as the domestic fire sale potentially affects all foreign countries investing domestically.

The second departure is unequal treatment of foreign banks for domestic activities. Whereas globally efficient policy features equal treatment of domestic and foreign banks, under non-cooperative quantity regulation foreign banks are regulated more stringently than domestic banks. This regulatory gap $\tau^L_{i,ji}(s) - \tau^L_{i,ii}(s)$ reflects a *bilateral* problem: the marginal benefit to foreign banks of liquidating the domestic asset outweighs the marginal cost to to domestic economy. Nevertheless, foreign liquidations are banned because that positive surplus accrues to foreign banks, and not to the domestic economy.

These two departures provide a theory of optimal cooperation under quantity regulation. Optimal cooperation serves both to enforce equal regulatory treatment of foreign banks, and to increase regulation of domestic banks. Cooperation fixes a common pool problem – arbitrageurs in each country are a "common pool" for absorbing bank liquidations, but can be polluted by further liquidations. Each country planner protects its own pool by preventing foreign banks from accessing it, while also excessively polluting its own pool, which foreign banks benefit from accessing. Optimal cooperation ensures equal access for foreigners, but also internalizes the full effect of polluting the common pool.

4.3 Non-Cooperative Pigouvian Taxation

We now characterize the non-cooperative equilibrium under Pigouvian taxation, where wedge revenues from foreign banks are remitted domestically. Recall that the change in the remission rule is the only change relative to quantity regulation.

Proposition 6. *Under non-cooperative Pigouvian taxation, the equilibrium has the following features.*

1. *The domestic liquidation wedges on domestic* and *foreign banks are*

$$\tau^L_{i,ii}(s) = \tau^L_{i,ji}(s) = -\underbrace{\Omega_{i,i}(s)}_{\text{Domestic Spillovers}} - \underbrace{\int_{i'} \Omega_{i',i}(s)di'}_{\text{Foreign Spillovers}} \quad \underbrace{\forall j}_{\text{Equal Treatment}} \tag{13}$$

where $\Omega_{i,j}(s)$ are as defined in Proposition 3.

2. *The wedges on domestic investment by foreign banks are*

$$\tau^I_{i,ji} = \underbrace{\frac{\partial^2 \Phi_{ji}}{\partial I_{ji}^2} I_{ji}}_{\text{Monopolist Motive}} \geq 0$$

3. *All other wedges are* 0.

In contrast to quantity regulation, *non-cooperative* planners using Pigouvian taxation account for both the benefits to foreign banks of liquidating domestic assets, *and* for the spillover effects of domestic fire sales to foreign banks, leading to implement the efficient wedges on asset liquidations.

This difference arises from the motive to collect revenue from foreign banks. To build intuition, suppose that we start from the non-cooperative equilibrium of Proposition 5, in which foreign liquidations are banned, and perturb the equilibrium so that foreign banks are allowed to liquidate a small amount ϵ of assets. There are two immediate effects. First, the domestic planner is able to collect $\tau^L_{i,ji}(s)\epsilon$ in tax revenue from foreign banks, rather than nothing, which increases domestic welfare. The second effect is that the domestic fire sale is exacerbated, which has cost $\tau^L_{i,ii}(s)\epsilon$ to domestic banks. However, the *bilateral tension* of the non-cooperative regulatory equilibrium arises when the marginal benefit $\tau^L_{i,ji}(s)$ to foreign banks exceeds the marginal spillover cost $\tau^L_{i,ii}(s)$, so that $(\tau^L_{i,ji}(s) - \tau^L_{i,ii}(s))\epsilon > 0$. As a result, the domestic planner benefits from allowing some foreign bank liquidations, rather than none. In other words, taxation allows for the marginal surplus of foreign bank liquidations to accrue to the domestic planner, rather than foreign banks, leading the domestic planner to implement an optimal policy. This helps to understand why the wedge on liquidations by foreign banks is equal to that on domestic banks.

It is less immediate why the domestic planner also internalizes fire sale spillovers to foreign banks when setting taxes. The reason is that fire sale spillovers reduce the marginal benefit to foreign banks of domestic activities, which lowers the tax revenue collected by the domestic planner. To see the intuition, let us suppose that $h_j(s) = 1$, so that banks cannot roll over debt at date 1 and must liquidate assets to repay D_i. In this case, foreign banks value investing in and liquidating the domestic asset in part to the extent they can sell it in order to repay debt holders. A domestic fire sale lowers the value that foreign banks get from investing in and selling the domestic asset, and so also reduces their willingness to pay for domestic activities. This reduces the marginal tax rates on foreign banks, reducing tax revenue collected by the domestic planner.

Formally, when $h_j(s) = 1$ the derivative of revenue collected from country j banks in the total domestic liquidations $L_i^A(s)$ is given by

$$\begin{aligned}
\frac{1}{f(s)}\frac{\partial T^*_{i,ji}}{\partial L_i^A(s)} &= \frac{1}{f(s)}\frac{\partial \gamma_i(s)}{\partial L_i^A(s)}\frac{\partial}{\partial \gamma_i(s)}\left[\tau^I_{i,ji}I_{ji} + \int_{s'} \tau^L_{i,ji}(s')L_{ji}(s')f(s')ds'\right] \\
&= \underbrace{\frac{\partial \gamma_i(s)}{\partial L_i^A(s)}}_{\text{Price Impact}} \frac{\partial}{\partial \gamma_i(s)}\left[\underbrace{\frac{\lambda_j^1(s)}{\lambda_j^0}\gamma_i(s)L_{ji}(s)}_{\text{Liquidation Losses}} + \underbrace{\frac{\Lambda_j^1(s)}{\lambda_j^0}\gamma_i(s)L_{ji}(s)}_{\text{Collateral Constraint Spillover}}\right] \\
&= \Omega_{j,i}(s)
\end{aligned}$$

which is precisely the fire sale spillover effect. The lower liquidation value reduces the value of selling the asset (lower $\tau^L_{ji}(s)$), resulting in a "Liquidation Loss" that reduces revenue from taxes on liquidations. Similarly, the lower liquidation value also reduces the initial investment value of the asset (lower τ^I_{ji}) by reducing the value of selling it in relaxing the collateral constraint, resulting in a "Collateral Constraint Spillover" that reduces revenue from taxes on investment. This leads the domestic planner to internalize the foreign spillovers from domestic fire sales.

The revenue generation motive is critical to ensuring country planners account for effects on foreign banks when setting policy. Quantity regulation does not result in a revenue generation motive. Although the implementing wedges under quantity regulation are affected by changes in domestic liquidation prices, resulting in a need for the country planner to alter the stringency of domestic regulation required to achieve a given allocation, this higher stringency has no domestic welfare consequences. The country planner does not account for foreign banks' welfare as a result.

The revenue generation motive generates one additional force: a *monopolistic* revenue extraction motive.[32] This is similar to standard monopolistic tendencies: the country planner distorts allocations of foreign banks in order to increase their willingness to pay for domestic activities. In this model, this monopolistic motivation operates through a tax on foreign investment scale when there is curvature in the domestic investment cost function. If a country is sufficiently substitutable with other countries from an investment perspective, it will not have monopoly power and these latter terms will drop out. In

[32]Note that we have expressed this motive as a derivative of price (tax rate) in the quantity. This is equivalent to expressing it in a more familiar way of a derivative of quantity (demand) in price. In our model, it is simpler to solve for quantities, and then back out the implementing prices (taxes).

practice, the high mobility of global banking assets suggests that substitability is a pausible assumption.[33]

Efficiency of Non-Cooperative Pigouvian Taxation. When countries use Pigouvian taxation, the only difference between the non-cooperative wedges of Proposition 6 and globally efficient policy is the monopolistic distortion that gives rise to (inefficient) positive taxes on investment. As a result, if the monopolistic distortion is zero, then non-cooperative taxation implements the globally efficient outcome, and hence is efficient. We formalize this result below.

Proposition 7. *Suppose that for all i and $j \neq i$, $\frac{\partial^2 \Phi_{ij}}{\partial I_{ij}^2} = 0$. Then, the non-cooperative equilibrium under taxation is globally efficient. There is no scope for cooperation.*

Proposition 7 suggests an alternative to cooperative regulatory agreements exists in the model. If countries switch to Pigouvian taxation to manage fire sale spillovers, country planners can achieve the cooperative outcome in a non-cooperative manner. They do so even though each country maximizes domestic welfare only, even though domestic liquidation prices appear in foreign bank constraints, and even though domestic planners have market power over domestic liquidation prices.

The sufficient condition of Proposition 7 requires a notion of substitutability between countries. The condition $\frac{\partial^2 \Phi_{ij}}{\partial I_{ij}^2} = 0$ implies that the (partial equilibrium) elasticity of investment with respect to the tax rate is infinite. The infinite elasticity is a limiting case in which countries have no monopoly power over foreign banks, and so implement an efficient outcome.

Proposition 6 provides an exact efficiency result in a limiting case of an infinite elasticity. In practice, countries may not always have close investment substitutes, and so may have some monopoly power. In these cases, Pigouvian taxation nevertheless provides an advantage by restricting the need for cooperation to *foreign* bank activities. Cooperation is no longer required over domestic activities of domestic banks. Moreover, it transforms the source of inefficiency from a *multilateral* spillover problem into a *bilateral* monopolist problem. The problem of cooperation is to ensure that foreign banks, who in the non-cooperative equilibrium are taxed excessively, are taxed the same as domestic banks, who are taxed efficiently.

A final advantage is that Pigouvian taxation changes the information required to determine the need for and terms of a cooperative agreement. The multilateral financial

[33] See e.g. the work of Coppola et al. (2019) on global capital flows and tax havens.

stability spillovers that must be corrected under regulation are general equilibrium effects that arise in the future, and there may be substantial disagreements between countries as to their magnitudes and cross-country correlations.

By contrast, the information required to enforce a cooperative agreement under taxation in the baseline model is the elasticity of investment with respect to the tax rate (the cost of investment). Moreover, the elasticities needed are *partial equilibrium* elasticities, not general equilibrium ones. The tax formula of Proposition 6 implies that cooperation is required when the elasticity of investment in the tax rate (cost of investment) is low, and not required when it is high.

Relation to Side Payments. Our model rules out explicit side payments between planners and banks when designing quantity regulation or Pigouvian taxation. The revenue generated by Pigouvian taxation acts as an indirect side payment from foreign banks to domestic planners. In this sense, Pigouvian taxation allows foreign banks to conduct activities in exchange for a side payment commensurate with the externality generated. Because the fire sale reduces the side payment a foreign bank is willing to pay, the planner internalizes the impact of the domestic fire sale on foreign bank welfare through the side payment received. This highlights the importance of a implicit or explicit side payments in achieving global efficiency, and suggests that a quantity regulation approach with explicit side payments could also achieve the efficient outcome.

4.4 Discussion

Our theory has both positive implications that help to understand the design of existing bank regulatory regimes and cooperative agreements, and normative implications that help to inform the optimal design of regulatory policy.

4.4.1 Real World Quantity Regulation and Cooperation

Home and Host Country Regulation. A common regulatory arrangement in practice is that the domestic planner regulates activities of the domestic subsidiary of a foreign bank holding company, while the foreign planner regulates the overall balance sheet of the bank holding company.[34] The former is host country regulation, while the latter is home country regulation. Non-cooperative regulation in our model resembles this combination, with the

[34] For example, the US applies TLAC requirements to both systemically important US banks and to the US IHCs of systemically important foreign banks. The objective of these requirements is to “help to reduce risks to financial stability.” 82 FR 8266.

domestic planner managing risk specific to the domestic economy, and the foreign planner managing how risks translate back into the foreign economy.Alternatively, it can also be interpreted as a quantity based capital control, such as a restriction on outflows.

Basel III and the European Banking Union. The model suggests non-cooperative regulation of domestic banks is overly lax while at the same time there is unequal treatment of foreign banks. Both the Basel III accords and the European Banking Union aim to enhance bank regulatory standards to address cross-border stability risks, for example by strengthening bank capital and liquidity requirements.[35] Moreover, equal treatment is recognized as an important aspect of cooperative regulatory policy in practice.[36]

Ring Fencing, SPOE, and MPOE. Non-cooperative regulation in the model prohibits domestic liquidations by foreign banks. This resembles equity capital and liquidity *ring fencing*, where countries require foreign banks to adhere to domestic regulatory standards at the level of their domestic operations, potentially independent of their parent company.[37]

For capital ring fencing, this connects to the debate between single-point-of-entry (SPOE), in which an international bank holds loss-absorbing capital at an international level and so shares losses and spillovers across jurisdictions, and multiple-point-of-entry (MPOE) resolution, in which an international bank holds loss-absorbing capital at a country level to ensure domestic stability.[38] Our model suggests that non-cooperative regulators prefer to adopt MPOE regimes, while cooperative policy more closely resembles SPOE.[39]

For liquidity ring fencing, this relates to a recent proposal by the US Federal Reserve

[35]Basel III serves to "rais[e] the resilience of the banking sector by strengthening the regulatory capital framework," with a particular mind to cross-border interconnectedness and global risks (BIS (2010)). The SSM of the European Banking Union seeks to ensure "the stability of the financial system by ensuring that banking supervision across the euro area is of a high standard" (ECB (2018b)).

[36]The ECB lists a goal of the Single Supervisory Mechanism as "ensuring a level playing field and equal treatment of all supervised institutions" (ECB (2018b)). In the capital control context, the IMF states that it "is generally preferable that [capital flow management measures] not discriminate between residents and non-residents" (IMF (2012)).

[37]In applying US TLAC requirements to US IHCs of foreign systemically important banks, the rules were"tailored to the potential risks presented by the U.S. operations of foreign GSIBs to the U.S. financial system." Commenters on the rules suggested considering allowing guarantees from the parent to qualify towards TLAC requirements to reduce "ring fencing and "misallocation risk,"" the final TLAC rule did not allow parent guarantees to qualify towards TLAC requirements because they "would not be pre-positioned in the United States and available for use during a period of stress without additional actions by the foreign GSIB parent." 82 FR 8266.

[38]See Bolton and Oehmke (2019) for a formal analysis of SPOE versus MPOE. See Financial Stability Board (2013) and Tucker (2014) for additional discussions.

[39]This coincides with this notion that SPOE is an efficient strategy for banks that "operate in a highly integrated manner (through, for example, centralised liquidity, trading, hedging and risk management)" (Financial Stability Board (2013)).

Board to "impose standardized liquidity requirements on the U.S. branch and agency network of a foreign banking organization" (Federal Reserve System (2019b)). This has raised concerns that the policy would lead other countries to adopt similar proposals.[40] Consistent with these concerns, our model suggests that countries would engage in excessive liquidity ring fencing.

Branches versus Subsidiaries and Reciprocity. In the model, the host country regulator regulates domestic activities of foreign banks. In practice, this corresponds to cross-border bank expansion using subsidiaries, which are typically regulated by the host country, rather than branches, which are typically regulated by the home country.[41] This raises a natural question of whether cooperative policy could be implemented by bilateral treaties or reciprocity agreements that allow expansions using branches (or otherwise enforce equal treatment).[42] The answer is no - although such agreements may alleviate unequal treatment, they do not fix the under-regulation of domestic banks.

4.4.2 Design of Bank Regulation and Capital Controls

The contrast between the inefficiency of non-cooperative quantity regulation and the potential efficiency of non-cooperative Pigouvian taxation has normative implications regarding the optimal design of macroprudential policies and capital control measures.

Emerging Markets and Capital Controls. Emerging markets in practice use both quantity- and priced-based capital control measures to manage capital flows.[43] Provided emerging markets have low monopoly power, our model suggests that they set *price-based* capital controls in a globally efficient manner even when they act non-cooperatively. By contrast,

[40]The Federal Reserve stated that it "intends to further evaluate" concerns from commenters that such a requirement "could lead other jurisdictions to implement similar requirements" and "lead to market fragmentation," so that such rules should be "discussed...at the global level by international regulatory groups" (Federal Reserve System (2019a)).

[41]This distinction is not always true in practice. First, host country regulators can require certain activities to occur in subsidiaries. For example, the US generally requires insured deposits to be held in subsidiaries (see Nolle (2012)). Second, host regulators in theory have the ability to regulate domestic branches of foreign banks, for example by imposing liquidity requirements (Federal Reserve System (2019a) and Federal Reserve System (2019b)).

[42]Indeed, reciprocity is recognized as important in practice. For example, Federal Reserve Chair Jerome Powell stated that "U.S. regulators have a long-standing policy of treating foreign banks the same as we treat domestic banks...a level playing field at home helps to level the playing field for U.S. banks when they compete abroad." Federal Reserve Press Release, "Federal Reserve Board finalizes rules that tailor its regulations for domestic and foreign banks to more closely match their risk profiles," October 10, 2019.

[43]See e.g. IMF (2012).

quantity controls are not set efficiently and are overly restrictive. Our model provides an argument in favor of use of price-based capital controls, rather than quantity-based ones.

Macroprudential Regulation. Common macroprudential regulatory requirements are minimum equity capital and liquidity requirements, which are quantity restrictions. However, there have been proposals for and discussions of Pigouvian taxes, such as a tax on debt, as an alternative to quantity regulations.[44] Our results suggest that the Pigouvian tax approach may lead to efficient outcomes if countries have low monopoly power, whereas quantity regulation does not.

Relationship to the Quantity Regulation Paradigm. In practice, macroprudential policies commonly take the form of quantity restrictions, rather than taxes. We highlight two complementary reasons this may have arisen. First, revenue-neutral Pigouvian taxation may be perceived as roughly equivalent to quantity regulation. Even in academic debates, duality is a common assumption.[45] Moreover, quantity regulation can incorporate Pigouvian-like features, for example using risk weights and capital surcharges, which tie quantity requirements to the banks' risk profile.[46] In this context, it is not particularly surprising to see a perception of duality. Second, Pigouvian taxes may be politically more difficult to implement than quantity restrictions. Taxes can be politically unpopular, particularly when the impact is perceived as falling on the general public.[47] In a world of perceived duality, quantity regulation may simply be politically easier to implement.

One of our core contributions is to argue that even when quantity and price regulation are equivalent in a purely domestic context, they are not equivalent in the international context. Quantity regulation does not generate revenue for country planners. As we have shown, this revenue generation is critical to ensuring non-cooperative efficiency. Our results provide a new argument in favor of the Pigouvian tax approach.

Asymmetric Countries and Difficulties of Cooperation. Cooperation is often perceived to be difficult when countries are sufficiently asymmetric.[48] Asymmetric agreements may require explicit international transfers, which may be politically difficult to implement.

[44] See for example Cochrane (2014), De Nicoló et al. (2014), Kocherlakota (2010), and Tucker (2016).

[45] See e.g. Erten et al. (2019). Indeed in our model, quantity regulation in Section 4 could also be considered a revenue-neutral Pigouvian tax.

[46] See e.g. Greenwood et al. (2017).

[47] Although largely outside the banking context, see for example Mankiw (2009) and Masur and Posner (2015). Relatedly, Baker III et al. (2017) argue that revenue neutrality is important to ensuring political support for a carbon tax.

[48] See for example Bolton and Oehmke (2019) and Dell'Ariccia and Marquez (2006).

Pigouvian taxation functions by implementing the required transfers in a decentralized manner, and may help facilitate efficient outcomes when countries are relatively asymmetric (e.g. developed economies and emerging markets).

5 Bailouts and Fiscal Backstops

In this section, we study the main extension of the banking model: fiscal backstops, or "bailouts." Fiscal backstops – such as deposit insurance, lender of last resort (LOLR), asset purchases, and debt guarantees – are an important consideration for financial stability and may be complementary to an effective regulatory regime.[49] International fire sale spillovers motivate studying cooperation over fiscal backstop measures, since bailouts that enhance domestic stability have positive spillovers to foreign banks. In practice, common fiscal backstops include the ECB as a EU-wide LOLR, and porposals for an EU Common Deposit Insurance scheme.[50]

We show that the results on macroprudential policies in the baseline model also apply to bailouts. Globally efficient bailout policies account for the total set of domestic and foreign spillovers, and feature a form of equal treatment of domestic and foreign banks. Non-cooperative bailouts do not achieve the efficient allocation if governments use quantity regulations. However, non-cooperative bailouts and Pigouvian taxation can achieve the global optimum if banks are charged a levy for the bailouts they expect to receive.

5.1 Incorporating Bailouts

Incorporating bailouts into the model requires a notion of how a bailout can be used to affect banks' liquidations at the country level – for example, bailing out the local subsidiary of a foreign bank. We extend the baseline model by incorporating country-specific debt claims D_{ij}, which is the amount owed to (or from) investors from bank operations in a specific country, rather than an overall debt contract D_i. These claims reflect the ability of the bank to reallocate funds across operations. We model bailouts as ex ante lump sum transfer commitments $T^1_{ij}(s) \geq 0$, which provides a tractable way of representing the

[49] See e.g. Bianchi (2016), Clayton and Schaab (2020), Jeanne and Korinek (2020), and Keister (2016). For a policy perspective, see Geithner (2016).

[50] See Acharya et al. (2016) for discussion and analysis of the ECB as a LOLR. See European Commision (2015) for a proposal for Common Deposit Insurance.

various possible bailout instruments.[51] The date 1 country-level liquid net worth of bank i is $A^1_{ij}(s) = -D_{ij} + T^1_{ij}(s)$, which may be negative. The date 1 budget constraint accounts for the bailout transfers,

$$c_i(s) \leq A^1_i(s) + (\gamma_i(s) - 1) L_{ii}(s) + R_i(s) I_i + \int_j \left((\gamma_j(s) - 1) L_{ij}(s) + R_j(s) I_{ij} \right) dj, \quad (14)$$

where $A^1_i(s) = A^1_{ii}(s) + \int_j A^1_{ij}(s) dj$ is the total liquid net worth of the bank, and where for simplicity we set $r_{ii} = r_{ij} = 0$. The date 0 bank budget constraint is unchanged relative to before, except that total date 0 bank debt is $D_i = D_{ii} + \int_j D_{ij} dj$.

Banks now face country-level collateral constraints that force country-level liquidations, given by

$$L_{ij}(s) \geq -\frac{1}{h_j(s)\gamma_j(s)} A^1_{ij}(s) - \frac{(1 - h_j(s))}{h_j(s)} R_j(s) I_{ij} \quad \forall j. \quad (15)$$

In addition, an incomplete markets constraint limits the ability of the bank to reallocate funds across operations

$$\Gamma_i \left(\phi_{ii}(D_{ii}) + \int_j \phi_{ij}(D_{ij}) dj \right) \geq 0, \quad (16)$$

and so forces liquidations in specific countries. Applying collateral constraints at the country level allows us to consider how bailouts of operations in a specific country affect the riskiness of banks' activities in that country.

We treat D_{ij}, rather than $L_{ij}(s)$, as the choice variable of banks for this section that determines liquidations. In doing so, we internalize the liquidation rule that arises from equation (15) into the date 1 budget constraint.

Macroprudential Regulation versus Bailouts. Country planners can reduce liquidations either with bailouts or macroprudential regulation. Because bailouts are state contingent, the two policies are not perfect substitutes. Both may be desirable to mitigate fire sales.

Financing Bailouts. Bailouts are financed by domestic taxpayers, with a utility cost $V^T_i(\mathcal{T}_i)$ of tax revenue collections.[52] Country planners trade state-contingent claims on

[51] See Appendix C.6 for a discussion of time inconsistent bailouts. Although in theory fiscal backstops such as deposit insurance and LOLR rule out bad equilibria without being used on the equilibrium path, in practice these measures often are associated with undesirable transfers and moral hazard.

[52] See Appendix C.1 for a foundation.

taxpayer revenue, yielding a tax-bailout budget constraint

$$\int_s \left[T^1_{i,ii}(s) + \int_j T^1_{i,ji}(s) + \int_j T^1_{i,ij}(s) \right] f(s)ds \leq G_i + \mathcal{T}_i \tag{17}$$

where $T^1_{i,ii}(s) + \int_j T^1_{i,ji}(s) + \int_j T^1_{i,ij}(s)$ is required revenue for bailouts in state s. G_i is an existing inter-country tax revenue claim, with $\int_i G_i di = 0$, which we use in decentralizing the cooperative outcome.

5.2 Globally Efficient and Non-Cooperative Policies

We now characterize the globally efficient bailout policies, and discuss the non-cooperative bailout rules. We leave formal characterizations of regulatory policies and of the non-cooperative results to Appendix C.

Globally Efficient Bailouts. We begin by considering the globally efficient allocation, which includes the choice of bailouts. The global planner places welfare weights ω_i^T on taxpayers, so that the global objective function is

$$V^G = \int_i \left[\omega_i \left[\int_s c_i(s) f(s) ds + \omega_i^T V_i^T(\mathcal{T}_i) \right] \right] di.$$

The global planning problem includes a set of unrestricted lump-sump transfers $\int_i G_i di \leq 0$ that reallocate claims on tax revenues. The following proposition characterizes the globally efficient bailout rule.[53]

Proposition 8. *The globally efficient bailout rule for $T^1_{ij}(s)$ is*

$$\underbrace{\frac{\omega_i \omega_i^T}{\lambda_i^0} \left| \frac{\partial V_i^T}{\partial \mathcal{T}_i} \right|}_{\text{Taxpayer Cost}} \geq \underbrace{B^1_{ij}(s)}_{\text{Bank Benefit}} + \underbrace{\Omega^B_{j,j}(s) \frac{\partial L_{ij}(s)}{\partial A^1_{ij}(s)}}_{\text{Domestic Spillovers}} + \underbrace{\int_{i'} \Omega^B_{i',j}(s) di' \frac{\partial L_{ij}(s)}{\partial A^1_{ij}(s)}}_{\text{Foreign Spillovers}} \tag{18}$$

where the terms B^1_{ij}, $\Omega^B_{j,j}$, and $\Omega_{i',j}$ are defined in the proof.

The globally efficient bailout rules trade off the marginal cost of the bailout to taxpayers against both the direct benefit to the bank receiving the bailout, and the spillover benefits

[53] The Appendix also characterizes cooperative regulation, optimal tax collection, and an irrelevance result for bailout sharing rules.

from reduced liquidations and fire sales. As in the baseline regulatory problem, globally efficient policy considers the complete set of spillovers when designing bailouts. There is equal bailout treatment in the sense that domestic and foreign banks that have the same benefit $B^1_{ij}(s)$ and the liquidation responsiveness $\frac{\partial L_{ij}(s)}{\partial A^1_{ij}(s)}$ from the bailout face the same marginal bailout rule.

Non-Cooperative Quantity Regulation and Bailouts. The non-cooperative bailout rules differ from efficient rules in manners similar to the regulatory model. For bailouts of the domestic activities of domestic banks, the domestic planner neglects foreign spillovers and provides too weak of a backstop. At the same time, when deciding whether to bail out the domestic activities of a foreign bank, the domestic planner additionally neglects the benefit that foreign bank receives from the bailout, resulting in unequal treatment. Optimal cooperation increases backstops of all banks, including domestic ones.

Non-Cooperative Pigouvian Taxation and Bailouts. Under Pigouvian taxation, bailout rules are still not efficient. The reason is that bailouts are chosen by governments, not by banks, so that there is not an equilibrium tax rate associated with them. This problem can be fixed if there is a mechanism in place to charge banks for the bailouts they expect to receive. Such a mechanism, which is effectively a Pigouvian tax on bailouts, restores efficiency, including over bailout rules. One example of such a mechanism would be a deposit insurance levy.

5.3 Discussion

We now discuss the relationship between the results of the bailout model and bailout policy and cooperation in practice.

Bailout Home Bias. Our model predicts that country planners provide stronger backstops for domestic banks and domestic operations. There are several examples of home bias in deposit insurance, including: US deposit insurance not applying to foreign branches of US banks; Iceland's decision not to honor deposit guarantee obligations to UK depositors after its despoit guarantee scheme was breached; and EU policies against deposit insurance discrimination by nationality.[54]

[54] See 78 FR 56583; "Iceland Triumphs in Icesave court battle," *Financial Times*, January 28, 2013; and European Commision (2015)

Common LOLR and Common Deposit Insurance in the EU. Our model predicts overly weak fiscal backstops. This coincides with the EU motivation for Common Deposit Insurance, whose purpose is to "increase the resilience of the Banking Union against future crises" (European Commision (2015)). It further coincides the ECB acting as a common LOLR to the European Union.

Asymmetric Contributions to Backstops. Concerns may arise about sharing a fiscal backstop if countries benefit asymmetrically from it, for example if some countries are net contributors while others are net recipients One manifestation of this is countries with large banking sectors may have difficulty providing their own backstop (e.g. the case of Iceland). Proposition 8 implies bailouts by foreign countries may be optimal if they mitigate the domestic fire sale and promote cross-border investment. In this sense, asymmetric bailout cooperation requires financial integration.

6 A General Framework

In this section, we study whether the insights of the baseline model extend to other externality problems, such as environmental externalities. In addition to generalizing the results of the baseline model, we characterize classes of externalities for which the results of the baseline model, in particular non-cooperative Pigouvian tax efficiency, apply.[55]

6.1 Model

We frame the general problem as follows. In each country i, there is a representative multinational agent. The representative multinational agent has a vector $a_{ij} = \{a_{ij}(m)\}_{m \in M}$ of continuous and real-valued actions available in country j, where M is an indexing set and where $a_{ij}(m) \geq 0$.[56] The action $a_{ij}(m) = 0$ indicates not conducting activity m in country j. Multinational agents are home biased, so that domestic actions are a mass while foreign actions are a density.

We use country-level aggregates to capture spillover effects in the model. In particular, define $a_j^A(m) = a_{jj}(m) + \int_{i'} a_{i'j}(m) dj$ to be the aggregate action m in country j, with $a_j^A = \{a_j^A(m)\}$. In the baseline model, the relevant aggregate for spillovers was aggregate

[55] Appendix E contains additional extensions to this section.

[56] An example of an indexing set is $M = \{0\} \cup \{\{1\} \times S\}$, which denotes an action $a_{ij}(0)$ at date 0 and an action $a_{ij}(1, s)$ at date 1 in state s. We can impose that there are only actions $M' \subset M$ in country j by making actions $m \notin M'$ valueless.

liquidations $L_j^A(s)$, which affected multinational banks by determining the liquidation price. In the example of multinational firms generating pollution, the relevant aggregate would be total pollution (or the activity that generates it).

Country i multinational agents have a utility function $U_i\left(u_i(a_i), u_i^A(a_i, a^A)\right)$, where $u_i(a_i) = u_{ii}(a_{ii}) + \int_j u_{ij}(a_{ij})dj$ and $u_i^A(a_i, a^A) = u_{ii}^A(a_{ii}, a_i^A) + \int_j u_{ij}^A(a_{ij}, a_j^A)dj$.[57] This preference structure provides a flexible way to add up the utility impact of activities in difference countries – for example, a consumption good in each country – while ensuring sufficient continuity so that a change in foreign activities generates a utility impact proportional to the measure of those activities. Multinational agents face constraint sets $\Gamma_i(W_i, \phi_i(a_i), \phi_i^A(a_i, a^A)) \geq 0$, where $W_i = A_i - T_i$ is the wealth of the multinational agents (accounting for taxes), and where ϕ_i, ϕ_i^A are defined analogously to u_i, u_i^A and have the same motivation. Taken together, the optimization problem of country i multinational agents is

$$\max_{a_i} U_i\left(u_i(a_i), u_i^A(a_i, a^A))\right) \quad \text{s.t.} \quad \Gamma_i\left(W_i, \phi_i(a_i), \phi_i^A(a_i, a^A)\right) \geq 0, \tag{19}$$

where all multinational agents take the vector a^A of aggregates as given.

6.2 Globally Efficient and Non-Cooperative Policies

We first characterize the globally efficient allocation, which is virtually identical to Proposition 3 except for the definition of spillovers.

Proposition 9. *The globally efficient allocation can be decentralized by wedges*

$$\tau_{ji}(m) = -\underbrace{\Omega_{i,i}(m)}_{\text{Domestic Spillovers}} - \underbrace{\int_{i'} \Omega_{i',i}(m)di'}_{\text{Foreign Spillovers}} \quad \underbrace{\forall j}_{\text{Equal Treatment}}$$

where $\Omega_{i,j}(m)$ is given by

$$\Omega_{i,j}(m) = \underbrace{\frac{\omega_i}{\lambda_i^0}\frac{\partial U_i}{\partial u_i^A}\frac{\partial u_{ij}^A}{\partial a_j^A(m)}}_{\text{Utility Spillovers}} + \underbrace{\frac{1}{\lambda_i^0}\Lambda_i\frac{\partial \Gamma_i}{\partial \phi_i^A}\frac{\partial \phi_{ij}^A}{\partial a_j^A(m)}}_{\text{Constraint Set Spillovers}}$$

where Λ_i is the Lagrange multiplier on the constraint set, and where $\lambda_i^0 \equiv \Lambda_i\frac{\partial \Gamma_i}{\partial W_i}$ is the marginal

[57]Note that u_i, u_i^A can be functions in a generalized sense – for example, a vector of real numbers or a vector of functions defined over an underlying state space.

value of wealth to country i multinational agents.

Globally optimal policy in the general model features the same two core features as the baseline model. First, globally optimal policy enforces equal treatment of foreign agents, so that they are able to enjoy equally the benefits of cross-border activities. Second, globally optimal policy accounts for both domestic and foreign spillovers. There are two forms of spillovers in the general model that are reflected in $\Omega_{i,j}(m)$. The first set of spillovers are direct utility spillovers, a leading example of which is pollution externalities. The second second of spillovers are constraint set spillovers, a leading example of which is fire sale externalities.

Non-Cooperative Quantity Regulation. We leave the formal characterization to the appendix, but the inefficiencies of quantity regulation are effectively identical to those highlighted in Proposition 5 for each activity m. Non-cooperative quantity regulation neglects international spillovers and results in unequal treatment. Rather than necessarily being banned, foreign agent activities are allowed only to the extent they benefit the domestic economy. Indeed, we show that non-cooperative quantity regulation is generically inefficient in settings with externalities arising from cross-border activities.

Non-Cooperative Pigouvian Taxation. The efficiency of non-cooperative Pigouvian taxation highlighted in Proposition 7 applies in the general model under two conditions. The first is a similar notion of no monopoly rents, which carries the same intuition and is formalized in the Appendix.[58]

The second is the following on the manner in which foreign aggregates can affect a domestic agent.

Assumption 10. For all i and $j \neq i$, u^A_{ij} and ϕ^A_{ij} are homogeneous of degree 1 in a_{ij}, holding a^A_j fixed. That is, $u^A_{ij}(\beta a_{ij}, a^A_j) = \beta u^A_{ij}(a_{ij}, a^A_j)$ and $\phi^A_{ij}(\beta a_{ij}, a^A_j) = \beta \phi^A_{ij}(a_{ij}, a^A_j)$.

Assumption 10 states that domestic agents' exposure to aggregates in a foreign country scales with their activities in that foreign country.[59] For example, in the case where action m

[58]Note that the requirement of no monopoly rents implies that our theory does not provide a solution to terms of trade manipulation, given that the monopolist distortion is similar to terms of trade manipulation. More subtly, it implies that Pigouvian taxation may have trouble addressing problems of domestic market power of foreign multinational agents. This is because when a multinational agent earns monopoly rents in a country, the country planner may in turn gain some monopoly power over it.

[59]Assumption 10 requires linear scaling due to the fact that the Pigouvian tax is linear.

has a local price $\gamma_j(a_j^A)$ attached to it, we obtain a linear form $\gamma_j(a_j^A)a_{ij}(m)$, which satisfies Assumption 10. This is the case in the baseline model, where banks are affected by foreign aggregate liquidations through equilibrium prices.[60] Assumption 10 therefore restricts the form that cross-border externalities can take. Notice, however, that Assumption 10 places no restrictions on the form that *domestic* externalities that affect domestic agents can take.

To understand the role of Assumption 10 more clearly, we can use the results of Appendix A and the proof of Proposition 14 to decompose the gap between the globally efficient wedge and the non-cooperative Pigouvian tax. For expositional simplicity, we illustrate the decomposition for utility spillovers alone.[61] Assuming that monopoly rents are 0 and there are no constraint set spillovers, the gap between the globally efficient and non-cooperative wedges, evaluated at the globally efficient outcome, is given by

$$\tau_{ji}(m) - \tau_{i,ji}(m) = \int_{i'} \frac{1}{\lambda_{i'}^0} \omega_{i'} \frac{\partial U_{i'}}{\partial u_{i'}^A} \Bigg(\underbrace{\frac{\partial}{\partial a_i^A(m)} \left[\frac{\partial u_{i'i}^A}{\partial a_{i'i}} a_{i'i} \right]}_{\text{Tax Revenue Derivative}} - \underbrace{\frac{\partial u_{i'i}^A}{\partial a_i^A(m)}}_{\text{Externality}} \Bigg) di' \tag{20}$$

This gap is determined by the gap between the true externality effect on foreign agents, $\frac{\partial u_{i'i}^A}{\partial a_i^A(m)}$, and the change in tax revenue collected from foreign agents, $\frac{\partial}{\partial a_i^A(m)} \left[\frac{\partial u_{i'i}^A}{\partial a_{i'i}} a_{i'i} \right]$, that arises because changes in domestic aggregates affect the willingness-to-pay of foreign agents for domestic activities. Under Assumption 10, this tax revenue derivative is precisely equal to the externality effect because homogeneity of degree 1 implies that $\frac{\partial u_{i'i}^A}{\partial a_{i'i}} a_{i'i} = u_{i'i}^A$. This leads to the efficiency result.

6.3 Local versus Global Externalities

We now discuss the implications of this section for broader externality problems. We discuss the efficiency of non-cooperative Pigouvian taxation in terms of two broad classes of externality problems: local externalities and global externalities. These classes of externality problems allow for the model to be translated into a number of alternative settings of potential interesting.

Local Externalities. Local externalities are a class of purely domestic externalities, where the aggregates a_i^A only appear in the utility functions and constraint sets of country i

[60] Notice that Assumption 10 allows for multiple externalities, for example fire sales in multiple assets with cross-asset spillovers, or a combination of fire sales and spillovers to the real economy.

[61] Including constraint set spillovers simply adds a second and analogous term to the decomposition.

multinational agents.[62] For example, local externalities can include local pollution and local economic costs of bank failures. Local externalities result in unequal treatment of foreign agents under quantity regulation, by the same logic as in the baseline model.

Consider the decomposition of equation (20). Because local externalities do not result in foreign spillovers, we have $\frac{\partial u^A_{i'i}}{\partial a^A_i(m)} = 0$. However, for the same reason, domestic aggregates also do not affect tax revenue collection, that is $\frac{\partial}{\partial a^A_i(m)}\left[\frac{\partial u^A_{i'i}}{\partial a_{i'i}} a_{i'i}\right] = 0$. As a result, there is no gap between the globally efficient and non-cooperative Pigouvian tax outcomes, and non-cooperative Pigouvian taxation implements the global optimum. A Pigouvian tax approach to externality problems can result in the globally efficient outcome, provided monopoly rents are zero, for the entire class of local externalities that affect domestic agents, even though foreign agents can contribute to these externalities through their activities. Cooperation is not required.

Global Externalities. Global externalities are externalities that affect foreign agents, so that aggregates a^A_i appear in the utility functions and constraint sets of foreign agents. In addition to the example of fire sales in the baseline model, other examples of interest include global pollution and climate change.

Assumption 10 and equation (20) indicate that the way in which a domestic aggregate (or externality) affects foreign agents is crucial to determining the efficiency of non-cooperative Pigouvian taxation. Assumption 10 requires that the domestic externality affect foreign agents proportional to their domestic activities so that the tax revenue impact coincides with the externality. This is true in the case of local pecuniary externalities, such as fire sales, where an equilibrium local price is the source of the externality.

However, Assumption 10 is not generally satisfied for externality problems arising from global pollution or climate change. To see this, consider a simple model where there is a single domestic activity, "production" a_{ij}. Production generates carbon emissions, so that social welfare of country i agents is $u_{ii}(a_{ii}) + \int_j u_{ij}(a_{ij})dj + W_i - \int_j a^A_{jj} dj$. It is immediate that this utility function does not satisfy Assumption 10, and so non-cooperative Pigouvian taxation does not generate the globally efficient outcome.

To see the failure of non-cooperative Pigouvian taxation, consider the decomposition in equation (20). Climate change generates an adverse utility spillover, $\frac{\partial u^A_{i'i}}{\partial a^A_i(m)} = -1$. However, this utility spillover is separable in utility from the objective function $u_i(a_{ii}) + \int_j u_{ij}(a_{ij})dj + W_i$ that determines optimal production. As a result, the domestic aggregate

[62] Of course, the *actions* a_{ji} can still appear in the utility functions and constraint sets of country j multinational agents.

affects the welfare of domestic and foreign agents, but does not affect their willingness to pay for activities. As a result, we have a tax revenue derivative $\frac{\partial}{\partial a_i^A(m)}\left[\frac{\partial u_{i'i}^A}{\partial a_{i'i}}a_{i'i}\right]=0$. The non-cooperative equilibrium under Pigouvian taxation does not account at all for climate change.

This implies that careful attention must be paid to the form of cross-border externalities. When an externality spreads endogenously because cross-border activities expose multinational agents to foreign spillovers, such as under fire sales, non-cooperative Pigouvian taxation may be well-equipped to handle the problem. However, when an externality diffuses even without cross-border interactions, such as with climate change, cooperation is likely to be required.

7 Extensions

Appendix D provides extensions to the model of multinational banking, while Appendix E provides extensions to the general model of Section 6.

Appendix D.1 allows for positive welfare weights on arbitrageurs and for an extended set of stakeholders in bank activities, such as SMEs who benefit from bank lending. The qualitative results of the model are unaffected, except that planners become more accepting of foreign liquidations when they are associated with positive spillovers (either arbitrageur surplus or by allowing greater domestic activity by foreign banks). Fire sale spillovers to foreign banks are still not internalized under quantity restrictions, while Pigouvian taxes are efficient.

Appendix D.2 allows for domestic multinational banks to be partly foreign-owned. Foreign ownership leads non-cooperative planners to over-weight spillovers to the domestic economy. Neither quantity regulation nor Pigouvian taxation achieves the efficient outcome when there is foreign ownership, and when both fire sales are real economy spillovers are important to determination of optimal regulation.

Appendix D.3 considers the possibility that country planners might use wedges in a protectionist manner to shield domestic banks from competition by introducing endogenous local capital goods and prices. Non-cooperative quantity regulation leads to protectionism, whereas Pigouvian taxation remains efficient. This suggests that price-based barriers to capital flows can provide a means of achieving stability goals while reducing perverse protectionist tendencies.

Appendix D.4 allows for both local and global arbitrageurs. Global arbitrageurs generate an additional global spillover and channel of contagion unless they can be subjected to the Pigouvian tax. This channel is important to the extent that the marginal purchaser of additional bank liquidations is global, rather than locals. We briefly discuss in the context of empirical retrenchment patterns, where declines in foreign inflows tend to accompany domestic retrenchment, suggesting that local arbitrageurs may be important in equilibrium price determination.

Appendix D.5 considers two possible types of regulatory arbitrage. We first consider arbitrage between a country planner and the cooperative agreement, which disrupts cooperation but does not affect the optimality of Pigouvian taxation, where full autonomy is retained. We second consider arbitrage in the form of an unregulated "shadow banking" sector. Wholly domestic shadow banks do not affect our qualitative results, whereas a partly international shadow banking sector generates uninternalized global spillovers that warrant cooperation even under Pigouvian taxation. However, if country planners are able to regulate shadow banks, the efficiency of non-cooperative Pigouvian taxation is restored.

In Appendix D.6, we consider a quantity regulation game with quantity ceilings rather than revenue-neutral wedges. This problem yields the same outcome as the revenue neutral wedges.

In Appendix E provides several extensions to the general model studied in Section 6 and show that the qualitative results still hold. Appendix E.1 extends the model to include global goods with endogenous global prices (e.g. Arrow securities). Appendix E.2 allows for the possibility of (linear) local constraints on foreign bank allocations, thus allowing for corner solutions. Appendix E.3 allows for the possibility of heterogeneous agents. Appendix E.4 considers the case where the activities of multinational banks aggregate non-linearly. Appendix E.5 allows for a general set of government actions, such as bailouts. Appendix E.6 considers the case where country planners have different preferences than agents, for example due to paternalism or special interest groups. It shows that Pigouvian taxation is efficient from the perspective of *country planner preferences* provided that foreign spillovers are limited to constraint set spillovers, such as fire sales. Appendix E.7 considers a finite country game where all countries are large, and shows that Pigouvian taxation account for foreign spillovers. However, it also results in a monopolist distortion that arises because changes in a country result in changes in the entire contracts of foreign agents. Provided these distortions are small, for example due to a large number of countries as in the baseline model, optimality of Pigouvian taxation is restored.

8 Conclusion

We study a model of cross border banking, in which endogenous cross-border propagation of fire sales generates international financial stability spillovers. We characterize globally efficient banking activities, and compare it to the outcome achieved by non-cooperative national governments using quantity regulations. Absent cooperation, countries under-regulate domestic banks and over-regulate foreign banks, not accounting for welfare impacts of domestic regulation on foreign banks. This provides a theory of optimal cooperation, which both enforces equal treatment of foreign banks and increases regulation of domestic banks, and helps to understand the architecture of existing cooperative regimes.

Our most surprising normative contribution is to show that non-cooperative Pigouvian taxation can also implement the globally efficient allocation, eliminating the need for cooperation. From a policy perspective, this suggests that giving a more prominent role to Pigouvian policies in the macroprudential regime may be desirable. By doing so, policymakers may be able to reduce the need for cooperative regulatory agreements, and so avoid the inherent difficulties of cooperation.

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Online Appendix

A General Framework: Non-Cooperative

This Appendix presents the results of the non-cooperative problem in the general framework of Section 6.

A.1 Non-Cooperative Setup

The setup of the non-cooperative problem is analogous to the setup in the baseline model. Country planner i maximizes the welfare of domestic agents using a complete set of wedges $\tau_{i,i} = \{\tau_{i,ij}(m)\}_{j,m}$ on the actions of domestic agents, and wedges $\tau_{i,ji} = \{\tau_{i,ji}(m)\}_m$ on domestic actions of foreign agents. The total tax burden faced by country i agents from the domestic planner (excluding remissions) is therefore $T_{i,i} = \tau_{i,ii}a_{i,ii} + \int_j \tau_{i,ij}a_{i,ij}dj$, while the total tax burden from foreign planner j is given by $T_{j,ij} = \tau_{j,ij}a_{i,ij}$. These taxes appear in the wealth of the multinational agent.

As in the baseline model, under *quantity regulation* wedges are revenue-neutral, while under *Pigouvian taxation* wedges generate revenues from foreign banks.

Implementability. As in the baseline model, the approach to implementability is standard for domestic agents. Moreover, an implementability result analogous to Lemma 4 holds in the general environment, allowing us to apply the standard approach for domestic actions of foreign agents.

Lemma 11. *The domestic actions of foreign agents can be chosen by the domestic planner, with implementing wedges*

$$\tau_{i,ji}(m) = -\tau_{j,ji}(m) + \frac{1}{\lambda_j^0}\left[\omega_j \frac{\partial U_j}{\partial u_j}\frac{\partial u_{ji}}{\partial a_{ji}(m)} + \omega_j \frac{\partial U_j}{\partial u_j^A}\frac{\partial u_{ji}^A}{\partial a_{ji}(m)} + \Lambda_j \frac{\partial \Gamma_j}{\partial \phi_j}\frac{\partial \phi_{ji}}{\partial a_{ji}(m)} + \Lambda_j \frac{\partial \Gamma_j}{\partial \phi_j^A}\frac{\partial \phi_{ji}^A}{\partial a_{ji}(m)}\right]$$

where $\tau_{j,ji}$, λ_j^0, Λ_j, $\frac{\partial U_j}{\partial u_j}$, $\frac{\partial U_j}{\partial u_j^A}$, $\frac{\partial \Gamma_j}{\partial \phi_j}$, *and* $\frac{\partial \Gamma_j}{\partial \phi_j^A}$ *are constants from the perspective of country planner i.*

The intuition behind these implementability conditions is analogous to the baseline model: the planner first unwinds the wedge placed by the foreign planner, and then sets the residual wedge equal to the benefit to foreign agents of conducting that activity.

A.2 Non-Cooperative Quantity Regulation

We now characterize the non-cooperative equilibrium under quantity regulation, where wedges are revenue neutral. We obtain the following characterization of the non-cooperative equilibrium.

Proposition 12. *Under non-cooperative quantity regulation, the equilibrium has the following features.*

1. *The domestic wedges on domestic activities of domestic agents are*

$$\tau_{i,ii}(m) = -\Omega_{i,i}(m)$$

while the domestic wedges on foreign activities of domestic agents are 0*.*

2. *The domestic wedges on foreign banks generate an allocation rule*

$$\Omega_{i,i}(m)a_{ji}(m) = 0$$

so that foreign activities are allowed only up to the point they increase domestic welfare.

Proposition 12 reflects logic closely related to the baseline model. On the one hand, regulatory policies applied to domestic agents account for spillovers to domestic agents, but not to foreign agents. On the other hand, regulatory policies applied to foreign agents' domestic activities do not account for benefits to foreign agents of domestic activities. Foreign agents are allowed to conduct domestic activities only to the extent the *domestic* benefits of those activities outweigh *domestic* costs.

This characterization leads to a generic inefficiency result in the presence of cross-border activities. We say that there are *cross border activities* if $\exists M' \subset M$ and $I, J \subset [0,1]$ such that $a_{ii}(m), a_{ji}(m) > 0 \; \forall m \in M', i \in I, j \in J$.

Proposition 13. *Suppose that a globally efficient allocation features cross border activities over a triple (M', I, J). The non-cooperative equilibrium under quantity regulation generates this globally efficient allocation* only if *the globally efficient allocation features $\Omega_{i,i}(m) = \int_{i'} \Omega_{i',i}(m) di' = 0$ $\forall m \in M', i \in I$.*

Proposition 13 provides a strong and generic result that quantity regulation does not generate an efficient allocation when there are regulated cross-border activities. In particular, cross-border activities must generate no *net* domestic externality to avoid the problem of unequal treatment, and cross-border activities must generate no *net* foreign externalities to avoid the problem of uninternalized foreign spillovers. Notice that efficient under Proposition 13 requires no regulation of cross-border activities in the globally efficient policy.

A.3 Non-Cooperative Pigouvian Taxation

Finally, we characterize non-cooperative Pigouvian taxation and its optimality.

Proposition 14. *Suppose Assumption 10 holds. The equilibrium under non-cooperative Pigouvian taxation has the following features.*

1. *The domestic wedges on domestic activities of domestic agents are*

$$\tau_{i,ii}(m) = -\Omega_{i,i}(m) - \int_j \Omega_{j,i}(m)dj$$

   while domestic wedges on foreign activities of domestic agents are 0*.*

2. *The domestic wedges on domestic activities of foreign agents are*

$$\tau_{i,ji}(m) = \tau_{i,ii}(m) - \frac{\partial \tau_{i,ji}}{\partial a_{ji}(m)} a_{ji}$$

As in the baseline model, the derivatives of foreign tax revenue in domestic liquidation prices yield the foreign spillovers, so that planners account for these effects in designing policy. However, revenue collection generates a monopolistic distortion. The generalized problem therefore reflects the same logic as Proposition 6, with the only difference being the nature of the spillovers and of the monopolistic distortion. As in the baseline model, when this monopolist distortion is zero, non-cooperative Pigouvian taxation results in a globally efficient allocation.

Proposition 15. *Suppose Assumption 10 holds, and suppose that $u_{ij}, \phi_{ij}, u^A_{ij}, \phi^A_{ij}$ are linear in a_{ij} (given a^A_j) for all i and $j \neq i$. Then the non-cooperative equilibrium under taxation is globally efficient, and there is no scope for cooperation.*

Proof. Observe that when $u_{ij}, \phi_{ij}, u^A_{ij}, \phi^A_{ij}$ are linear in a_{ij} (given a^A_j) for all i and $j \neq i$, then the non-cooperative tax rates align with the cooperative ones, resulting in an efficient allocation. ■

Non-cooperative taxation is globally efficient if Assumption 10 holds, and if the monopolistic distortions are zero. The assumption of linearity on $u_{ij}, \phi_{ij}, u_{ij}^A, \phi_{ij}^A$ ensures that (partial equilibrium) elasticities of foreign activities with respect to tax rates are infinite, so that monopolistic distortions are zero. This reflects the same notion of sufficient substitutability as in the baseline model, and generalizes Proposition 7 to a broader class of problems.

As in the baseline model, Proposition 15 provides a limiting case of exactly efficiency. Comparing Propositions 14 and 9 reveals that even without exact efficiency, there are three appealing properties of Pigouvian taxation. The first is that the need for cooperation is restricted to foreign activities of multinational agents. The second is that cooperation is needed only to correct bilateral monopolist problems. The third is that the information needed to determine the magnitude of these problems is a set of partial equilibrium elasticities. This provides a potential method to evaluate the need for cooperation in practice.

B Proofs

Proofs from the baseline model (Sections 2-4) and the general model (Section 6) are contained in this appendix. Proofs from the bailouts model (Section 5) are contained in Appendix C.

B.1 Proofs of Baseline Model

Propositions 3, 5, 6, and 7 as well as Lemma 4, are applications of the corresponding propositions in Section 6.[63]

The proof of Lemma 2 is given below.

[63]For the proof of Proposition 6, one might be concerned that the planner's objective function becomes linear in foreign investment when $\partial^2\Phi_{ji}/\partial I_{ji} = 0$. This problem is easily addressed by introducing curvature elsewhere into the problem, for example by incorporating a local capital price (Appendix D.3) or by incorporating a small utility spillover to the domestic economy from total investment scale. Such spillovers would satisfy Assumption 10.

B.1.1 Proof of Lemma 2

The bank Lagrangian is

$$
\begin{aligned}
\mathcal{L}_i = & \int_s c_i(s) f(s) ds + \lambda_i^0 \left[A_i + D_i - \Phi_{ii}(I_{ii}) - \int_j \Phi_{ij}(I_{ij}) dj \right] \\
& + \int_s \lambda_i^1(s) \Bigg[\gamma_i(s) L_{ii}(s) + (1 + r_{ii})(R_i(s) I_{ii} - L_{ii}(s)) \\
& \qquad + \int_j \left[\gamma_j(s) L_{ij}(s) + (1 + r_{ij})(R_j(s) I_{ij} - L_{ij}(s)) \right] dj - c_i(s) - D_i \Bigg] f(s) ds \\
& + \int_s \Lambda_i^1(s) \left[-D_i + \gamma_i(s) L_{ii}(s) + \int_j \gamma_j(s) L_{ij}(s) dj + (1 - h_i(s)) \mathcal{C}_{ii}(s) + \int_j (1 - h_j(s)) \mathcal{C}_{ij}(s) dj \right] f(s) ds \\
& + \int_s \left[\underline{\xi}_{ii}(s) L_{ij}(s) + \overline{\xi}_{ii}(s)(R_i I_{ii} - L_{ii}(s)) + \int_j \left(\underline{\xi}_{ij}(s) L_{ij}(s) + \overline{\xi}_{ij}(s)(R_j I_{ij} - L_{ij}(s)) \right) \right] f(s) ds
\end{aligned}
$$

where we recall that $\mathcal{C}_{ij}(s) = \gamma_j(s) \left[R_j(s) I_{ij} - L_{ij}(s) \right]$.

FOC for I_{ij}. Taking the first order condition in I_{ij}, we obtain

$$
0 \geq -\lambda_i^0 \frac{\partial \Phi_{ij}}{\partial I_{ij}} + E\left[\lambda_i^1 (1 + r_{ij}) R_j \right] + E\left[\Lambda_i^1 (1 - h_j) \gamma_j R_j \right] + E\left[\overline{\xi}_{ij} R_j \right].
$$

Expanding the first expectation, we obtain the result.

FOC for $L_{ij}(s)$. Taking the first order condition in $L_{ij}(s)$, we obtain

$$
0 = \lambda_i^1(s)(\gamma_j(s) - (1 + r_{ij})) f(s) + \Lambda_i^1(s)(\gamma_j(s) - (1 - h_j(s)) \gamma_j(s)) f(s) + \underline{\xi}_{ij}(s) f(s) - \overline{\xi}_{ij}(s) f(s)
$$

which simplifies to the result.

B.2 Section 6 Proofs

B.2.1 Proof of Proposition 9

The Lagrangian of the global planner is

$$
\mathcal{L}^G = \int_i \left[\omega_i U_i \left(u_i(a_i), u_i^A(a_i, a^A) \right) + \Lambda_i \Gamma_i \left(A_i + \mathcal{T}_i, \phi_i(a_i), \phi_i^A(a_i, a^A) \right) \right] di - \lambda^0 \int_i \mathcal{T}_i di \Bigg].
$$

From here, we have

$$\frac{d\mathcal{L}^G}{da_{ij}(m)} = \frac{\partial \mathcal{L}_i}{\partial a_{ij}(m)} + \frac{\partial \mathcal{L}_j}{\partial a_j^A(m)} + \int_{i'} \frac{\partial \mathcal{L}_{i'}}{\partial a_j^A(m)} di'$$

so that we obtain the required wedge

$$\tau_{ij}(m) = -\frac{1}{\lambda_i^0}\left[\frac{\partial \mathcal{L}_j}{\partial a_j^A(m)} + \int_{i'} \frac{\partial \mathcal{L}_{i'}}{\partial a_j^A(m)} di'\right]$$

where we define $\lambda_i^0 \equiv \Lambda_i \frac{\partial \Gamma_i}{\partial W_i}$. Next, by the Envelope Theorem we can characterize the derivative

$$\frac{\partial \mathcal{L}_i}{\partial a_j^A(m)} = \omega_i \frac{\partial U_i}{\partial u_i^A} \frac{\partial u_{ij}^A}{\partial a_j^A(m)} + \Lambda_i \frac{\partial \Gamma_i}{\partial \phi_i^A} \frac{\partial \phi_{ij}^A}{\partial a_j^A(m)}.$$

Finally, defining $\Omega_{i,j}(m) = \frac{1}{\lambda_i^0} \frac{\partial \mathcal{L}_i}{\partial a_j^A(m)}$ and using that $\lambda^0 = \lambda_i^0$ (from the FOC for $\mathcal{T}_i$), we obtain

$$\tau_{ij}(m) = -\Omega_{j,j}(m) - \int_{i'} \Omega_{i',j}(m) di'$$

giving the result.

B.2.2 Proof of Lemma 11

Taking the Lagrangian of bank i

$$\mathcal{L}_i = \omega_i U_i \left(u_i(a_i), u_i^A(a_i, a^A))\right) + \Lambda_i \Gamma_i \left(A_i - T_i, \phi_i(a_i), \phi_i^A(a_i, a^A)\right)$$

and taking the first order condition in $a_{ij}(m)$, we obtain

$$0 = \omega_i \frac{\partial U_i}{\partial u_i} \frac{\partial u_{ij}}{\partial a_{ij}(m)} + \omega_i \frac{\partial U_i}{\partial u_i^A} \frac{\partial u_{ij}^A}{\partial a_{ij}(m)} + \Lambda_i \frac{\partial \Gamma_i}{\partial W_i}\left(-\tau_{i,ij}(m) - \tau_{j,ij}(m)\right) + \Lambda_i \frac{\partial \Gamma_i}{\partial \phi_i} \frac{\partial \phi_{ij}}{\partial a_{ij}(m)} + \Lambda_i \frac{\partial \Gamma_i}{\partial \phi_i^A} \frac{\partial \phi_{ij}^A}{\partial a_{ij}(m)}.$$

Defining $\lambda_i^0 = \Lambda_i \frac{\partial \Gamma_i}{\partial W_i}$ and rearranging, we obtain

$$\tau_{j,ij}(m) = -\tau_{i,ij}(m) + \frac{1}{\lambda_i^0}\left[\omega_i \frac{\partial U_i}{\partial u_i} \frac{\partial u_{ij}}{\partial a_{ij}(m)} + \omega_i \frac{\partial U_i}{\partial u_i^A} \frac{\partial u_{ij}^A}{\partial a_{ij}(m)} + \Lambda_i \frac{\partial \Gamma_i}{\partial \phi_i} \frac{\partial \phi_{ij}}{\partial a_{ij}(m)} + \Lambda_i \frac{\partial \Gamma_i}{\partial \phi_i^A} \frac{\partial \phi_{ij}^A}{\partial a_{ij}(m)}\right]$$

giving the relevant equation. From here, notice that the allocations a_{ij} and aggregates a_j^A appear to first order only in the tax rate equations in country j. As a result, considering any candidate equilibrium, the first order conditions for optimality for allocations by country i banks outside of country j are not affected (to first order) by policies in country j, and so continue to hold independent of a_{ij} and a_j^A. As a result, any allocation a_{ij} can be implemented with the above tax rates. The implementability result follows.

B.2.3 Proof of Proposition 12

Substituting in the equilibrium tax revenue, the optimization problem of the country i social planner is

$$\max_{a_i, \{a_{ji}\}} \omega_i U_i \left(u_i(a_i), u_i^A(a_i, a^A)) \right) \quad \text{s.t.} \quad \Gamma_i \left(A_i - \int_j \tau_{j,ij} a_{ij} dj, \phi_i(a_i), \phi_i^A(a_i, a^A) \right) \geq 0$$

and subject to the implementability conditions of Lemma 11. Note that the wedges rates $\tau_{i,ji}$ do not appear except in the implementability conditions, meaning that they are set to clear implementability but do not contribute to welfare. As a result, the Lagrange multipliers on implementability are 0, and the Lagrangian of planner i is given by

$$\mathcal{L}_i^{SP} = \omega_i U_i \left(u_i(a_i), u_i^A(a_i, a^A)) \right) + \Lambda_i \Gamma_i \left(A_i - \int_j \tau_{j,ij} a_{ij} dj, \phi_i(a_i), \phi_i^A(a_i, a^A) \right).$$

First of all, note that the social planner does not internalize impacts on foreign aggregates. As a result, $\frac{d\mathcal{L}_i^{SP}}{da_{ij}(m)} = \frac{\partial \mathcal{L}_i^{SP}}{\partial a_{ij}(m)}$. Social and private preferences align, and therefore we have $\tau_{i,ij}(m) = 0$.

Next, consider a domestic policy $a_{ii}(m)$. Here, we have $\frac{d\mathcal{L}_i^{SP}}{da_{ii}(m)} = \frac{\partial \mathcal{L}_i^{SP}}{\partial a_{ii}(m)} + \frac{\partial \mathcal{L}_i^{SP}}{\partial a_i^A(m)}$. To align preferences, the domestic planner therefore sets

$$\tau_{i,ii}(m) = -\frac{1}{\lambda_i^0} \frac{\partial \mathcal{L}_i^{SP}}{\partial a_i^A(m)} = -\Omega_{i,i}(m)$$

where the final equality follows as in the proof of Proposition 9.

Finally, consider $a_{ji}(m)$. Here, we have $\frac{d\mathcal{L}_i^{SP}}{da_{ji}(m)} = \frac{\partial \mathcal{L}_i^{SP}}{\partial a_i^A(m)} = \lambda_i^0 \Omega_{i,i}(m)$, giving the allocation rule.

B.2.4 Proof of Proposition 13

Given a globally efficient allocation with cross border activites over (M', I, J), suppose that the non-cooperative equilibrium under quantity regulation generates this allocation. From Proposition 12, $a_{ji}(m) > 0$ implies that $\Omega_{i,i}(m) = 0$ over (M, I, J). Using Propositions 9 and 12, $\tau_{i,ii}(m) = \tau_{ii}(m)$ and $\Omega_{i,i}(m) = 0$ implies that $\int_{i'} \Omega_{i',i}(m) di' = 0$ over (M, I, J), completing the proof.

B.2.5 Proof of Proposition 14

It is helpful to begin by characterizing the derivative of revenue from foreign agents in the domestic aggregate. Using the implementability conditions of Lemma 11, the revenue collected by planner j from country i agents is

$$T^*_{j,ij} = \tau_{j,ij}a_{ij} = -\tau_{i,ij}a_{ij} + \frac{1}{\lambda^0_i}\left[\omega_i \frac{\partial U_i}{\partial u_i}\frac{\partial u_{ij}}{\partial a_{ij}}a_{ij} + \omega_i \frac{\partial U_i}{\partial u^A_i}\frac{\partial u^A_{ij}}{\partial a_{ij}}a_{ij} + \Lambda_i \frac{\partial \Gamma_i}{\partial \phi_i}\frac{\partial \phi_{ij}}{\partial a_{ij}}a_{ij} + \Lambda_i \frac{\partial \Gamma_i}{\partial \phi^A_i}\frac{\partial \phi^A_{ij}}{\partial a_{ij}}a_{ij}\right].$$

Applying Assumption 10, $\frac{\partial u^A_{ij}}{\partial a_{ij}}a_{ij} = u^A_{ij}$ and $\frac{\partial \phi^A_{ij}}{\partial a_{ij}}a_{ij} = \phi^A_{ij}$, so that we obtain

$$T^*_{j,ij} = -\tau_{i,ij}a_{ij} + \frac{1}{\lambda^0_i}\left[\omega_i \frac{\partial U_i}{\partial u_i}\frac{\partial u_{ij}}{\partial a_{ij}}a_{ij} + \omega_i \frac{\partial U_i}{\partial u^A_i}u^A_{ij} + \Lambda_i \frac{\partial \Gamma_i}{\partial \phi_i}\frac{\partial \phi_{ij}}{\partial a_{ij}}a_{ij} + \Lambda_i \frac{\partial \Gamma_i}{\partial \phi^A_i}\phi^A_{ij}\right].$$

Finally, differentiating in $a^A_j(m)$, we obtain

$$\frac{\partial T^*_{j,ij}}{\partial a^A_j(m)} = \frac{1}{\lambda^0_i}\left[\omega_i \frac{\partial U_i}{\partial u^A_i}\frac{\partial u^A_{ij}}{\partial a^A_j(m)} + \Lambda_i \frac{\partial \Gamma_i}{\partial \phi^A_i}\frac{\partial \phi^A_{ij}}{\partial a^A_j(m)}\right] = \Omega_{i,j}(m)$$

which is the spillover effect. From here, the country i social planner's Lagrangian is given by

$$\mathcal{L}^{SP}_i = \omega_i U_i\left(u_i(a_i), u^A_i(a_i, a^A))\right) + \Lambda_i \Gamma_i\left(A_i - \int_j \tau_{j,ij}a_{ij}dj + \int_j T^*_{i,ji}dj, \phi_i(a_i), \phi^A_i(a_i, a^A)\right).$$

From here, derivation follows as in the proof of Proposition 12, except for the additional derivative in revenue. For $a_{ij}(m)$, there is no additional revenue derivative, and so $\tau_{i,ij}(m) = 0$ as before. For $a_{ii}(m)$, we have following the steps of Proposition 12

$$\tau_{i,ii}(m) = -\frac{1}{\lambda^0_i}\frac{\partial \mathcal{L}^{SP}_i}{\partial a^A_i(m)} - \frac{1}{\lambda^0_i}\Lambda_i \frac{\partial \Gamma_i}{\partial W_i}\int_j \frac{\partial T^*_{i,ji}}{\partial a^A_i(m)}dj = -\Omega_{i,i}(m) - \int_j \Omega_{j,i}(m)dj$$

giving the first result.

Finally, considering a foreign allocation a_{ji}, we have

$$0 = \frac{d\mathcal{L}^{SP}_i}{da_{ji}(m)} = \frac{\partial \mathcal{L}^{SP}_i}{\partial a^A_i(m)} + \lambda^0_i\left[\frac{dT^*_{i,ji}}{da_{ji}(m)} + \int_j \frac{\partial T^*_{i,ji}}{\partial a^A_i(m)}dj\right].$$

From here, noting that we have $\frac{dT^*_{i,ji}}{da_{ji}(m)} = \tau_{i,ji}(m) + \frac{\partial \tau_{i,ji}}{\partial a_{ji}(m)} a_{ji}$, we obtain

$$0 = -\tau_{i,ii}(m) + \tau_{i,ji}(m) + \frac{\partial \tau_{i,ji}}{\partial a_{ji}(m)} a_{ji}$$

which rearranges to the result.

C Bailouts Model: Additional Results and Derivations

In this appendix, we provide the additional results and derivations from the bailouts section, including the characterization of taxpayers, the relevant implementability conditions, characterization of regulatory policy, and characterization of non-cooperative taxation.

C.1 Taxpayers

We provide a foundation for the reduced-form indirect utility function $V_i(\mathcal{T}_i)$ of tax revenue collections from taxpayers, and show a tax smoothing result.

A unit continuum of domestic taxpayers are born at date 1 with an endowment $\overline{T}^1_i(s)$ of the consumption good. Given tax collections $T^1_i(s) \leq \overline{T}^1_i(s)$, taxpayers enjoy consumption utility $u^T_i\left(\overline{T}^1_i(s) - T^1_i(s), s\right)$.[64] These tax collections generate total bailout revenue $\mathcal{T}_i = \int_s q(s) T^1_i(s) f(s) ds$.[65] We characterize the optimal tax collection problem of country planner i, who has decided to collect a total $\mathcal{T}_i$ in tax revenue for use in bailouts.

Lemma 16. *Taxpayer utility can be represented by the indirect utility function*

$$V^T_i(\mathcal{T}_i) = \int_s u_i\left(\overline{T}^1_i(s) - T^1_i\left(\mathcal{T}_i, s\right), s\right) f(s) ds \tag{21}$$

where $\mathcal{T}^1_i(\mathcal{T}_i, s)$ is given by the tax smoothing condition

$$\frac{1}{q(s)} u'_i\left(\overline{T}^1_i(s) - T^1_i(s), s\right) = \frac{1}{q(s')} u'_i\left(\overline{T}^1_i(s') - T^1_i(s'), s'\right) \quad \forall s, s'.$$

Lemma 16 allows us to directly incorporate the indirect utility function $V^T_i(\mathcal{T}_i)$ into planner i preferences, and to use total revenue collected $\mathcal{T}_i$ as the choice variable. It implies that

[64] We impose $u^{T\prime}_i(0, s) = +\infty$. We think of u^T_i as incorporating both consumption preferences and distortionary effects of taxation.

[65] We could assume that the government pays a different price vector $\overline{q}$ for bailout claims, with derivations largely unchanged.

countries engage in tax smoothing without cooperation, but does not guarantee that they engage in the globally efficient level of bailouts.

C.1.1 Proof of Lemma 16

The optimization problem is

$$\max \omega_i^T \int_s u_i \left(\overline{T}_i^1(s) - T_i^1(s), s\right) f(s) ds \quad \text{s.t.} \quad \int_s q(s) T_i^1(s) f(s) ds \geq \mathcal{T}_i$$

The FOCs are

$$\omega_i^T \frac{\partial u_i \left(\overline{T}_i^1(s) - T_i^1(s), s\right)}{\partial c_i^T(s)} f(s) - \mu q(s) f(s) = 0$$

Combining the FOCs across states, we obtain the result.

C.2 Implementability Conditions

We characterize the implementability conditions for domestic allocations of foreign banks, in a manner analogous to the characterization in Lemma 4. Note that now, the domestic choice variables of foreign banks are (D_{ij}, I_{ij}).

Lemma 17. *Country planner j can directly choose all domestic allocations of foreign banks, with implementing wedges*

$$\tau_{j,ij}^I = -\tau_{i,ij}^I - \frac{\partial \Phi_{ij}}{\partial I_{ij}} + \frac{1}{\lambda_i^0} E\left[\lambda_i^1(s) \left(\left(\gamma_j(s) - 1\right) \frac{\partial L_{ij}(s)}{\partial I_{ij}} + R_{ij}(s)\right)\right] \tag{22}$$

$$\tau_{j,ij}^D = -\tau_{i,ij}^D - E\left[\frac{\lambda_i^1(s)}{\lambda_i^0} \left(1 + \left(\gamma_j(s) - 1\right) \frac{\partial L_{ij}(s)}{\partial A_{ij}^1(s)}\right)\right] + \frac{\Lambda_i^0}{\lambda_i^0} \frac{\partial \Gamma_i}{\partial \phi_i} \frac{\partial \phi_{ij}}{\partial D_{ij}} \tag{23}$$

Using Lemma 17, we can characterize the non-cooperative equilibrium in the same manner as the baseline model. In particular, we isolate the decision problem of the country i planner, who optimizes domestic bank welfare choosing domestic and foreign allocations, subject to domestic bank constraints and to the implementability conditions of Lemma 17, taking as given foreign planner wedges and foreign bailouts.

C.3 Non-Cooperative and Cooperative Regulation

We now characterize optimal non-cooperative regulation.

In the non-cooperative equilibrium, country planners choose both the wedges and the bailouts $T^1_{i,ij}(s)$, taking as given the wedges and bailouts of other countries, to maximize domestic social welfare

$$V_i^P = \omega_i \left[\int_s c_i(s) f(s) ds + \omega_i^T V_i^T(\mathcal{T}_i) \right].$$

The following proposition describes optimal bailout policy under regulation.

Proposition 18. *The bailout rules in the non-cooperative equilibrium under quantity regulation are as follows.*

1. *The optimal bailout rule for the domestic operations of a domestic bank is*

$$\underbrace{\frac{\omega_i \omega_i^T}{\lambda_i^0} \left| \frac{\partial V_i^T}{\partial \mathcal{T}_i} \right|}_{\text{Taxpayer Cost}} \geq \underbrace{B^1_{ii}(s)}_{\text{Bank Benefit}} + \underbrace{\Omega^B_{i,i}(s) \frac{\partial L_{ii}(s)}{\partial A^1_{ii}(s)}}_{\text{Domestic Spillovers}} \tag{24}$$

2. *The optimal bailout rule for the foreign operations of a domestic bank is*

$$\underbrace{\frac{\omega_i \omega_i^T}{\lambda_i^0} \left| \frac{\partial V_i^T}{\partial \mathcal{T}_i} \right|}_{\text{Taxpayer Cost}} \geq \underbrace{B^1_{ij}(s)}_{\text{Bank Benefit}} \tag{25}$$

3. *The optimal bailout rule for the domestic operations of a foreign bank is*

$$\underbrace{\frac{\omega_i \omega_i^T}{\lambda_i^0} \left| \frac{\partial V_i^T}{\partial \mathcal{T}_i} \right|}_{\text{Taxpayer Cost}} \geq \underbrace{\Omega^B_{i,i}(s) \frac{\partial L_{ji}(s)}{\partial A^1_{ji}(s)}}_{\text{Domestic Spillovers}} \tag{26}$$

Three factors govern the non-cooperative bailout rules: the social cost of taxes, the direct benefit of bailouts to banks, and the domestic fire sale spillover. When choosing bailouts of domestic activities of domestic banks, the domestic planner considers all three factors, but neglects spillover costs to foreign banks. Moreover, the domestic planner neglects the benefits of alleviating foreign fire sales when choosing bailouts of foreign activities of domestic banks, and neglects the benefits of the bailout transfer when choosing bailouts of domestic activities of foreign banks. Country planners are *home biased* in their bailout decisions, generally preferring to bail out domestic activities of domestic banks.

Relative to the globally efficient bailout rule, non-cooperative planners under-value all bailout activities, including bailouts of domestic activities of domestic banks, not accounting for either benefits or spillovers to foreign banks. The cooperative agreement increases bailouts of both domestic and foreign banks. Multilateral fire sale spillovers imply the need for multilateral bailout cooperation.

Proposition 19. *Optimal non-cooperative regulation is given as follows.*

1. *Domestic taxes on domestic banks' domestic activities are given by*

$$\tau^D_{i,ii} = E\left[\Omega^B_{i,i}(s)\frac{\partial L_{ii}(s)}{\partial A^1_{ii}(s)}\right] \tag{27}$$

$$\tau^I_{i,ii} = -E\left[\Omega^B_{i,i}(s)\frac{\partial L_{ii}(s)}{\partial I_{ii}}\right] \tag{28}$$

while other domestic taxes on domestic banks are zero. $\Omega^B_{i,i}(s)$ *is defined in the proof.*

2. *If there is an adverse price spillover* $-\Omega^B_{i,i}(s) > 0$*, then regulation of foreign banks is equivalent to a ban on foreign liquidations.*

To understand Proposition 19, the fact that liquidations are now determined indirectly, rather than directly, implies that the spillovers $\Omega^B_{i,i}(s)$ now form a basis to price the cost of policies that increase liquidations. This is reflected in the optimal tax rates.

At the same time, the domestic planner prefers sufficiently stringent regulation to prevent foreign banks from contributing to domestic fire sales. This is equivalent to requiring foreign banks to maintain domestic allocations that set $L^L_{ji} = 0$.

Next, we can characterize regulatory policy under the optimal cooperative agreement (global planning).

Proposition 20. *Optimal cooperative policy consists of taxes on investment scale and debt, given by*

$$\tau^D_{ij} = E\left[\left[\Omega^B_{j,j}(s) + \int_{i'} \Omega^B_{i',j}(s)di'\right]\frac{\partial L_{ij}(s)}{\partial A^1_{ij}(s)}\right] \tag{29}$$

$$\tau^I_{ij} = -E\left[\left[\Omega^B_{j,j}(s) + \int_{i'} \Omega^B_{i',j}(s)\right]\frac{\partial L_{ij}(s)}{\partial I_{ij}}\right] \tag{30}$$

The intuition of Proposition 20 is analogous to the intuition of Proposition 3. Globally optimal policy accounts for the full set of spillovers. Note that cooperative policy no longer features equal treatment in tax rates, to the extent that the responses of different banks' liquidation rules are different on the margin. There is equal treatment in the sense that the basis of spillover effects $\Omega_{i,j}(s)$ are the same, independent of which country generates the spillover.

Finally, we can characterize the optimal tax collection and bailout sharing rules of the cooperative agreement.

Proposition 21. *Globally optimal tax collection and bailout sharing are as follows.*

1. *Optimal cross-country bailout sharing is given by*

$$\omega_i \omega_i^T \frac{\partial V_i^T(\mathcal{T}_i)}{\partial \mathcal{T}_i} = \omega_j \omega_j^T \frac{\partial V_j^T(\mathcal{T}_j)}{\partial \mathcal{T}_j} \quad \forall i, j \tag{31}$$

2. *Any bailout sharing rule* $\left(T^1_{i,ij}(s), T^1_{j,ij}(s)\right)$ *satisfying* $T^1_{ij}(s) = T^1_{i,ij}(s) + T^1_{j,ij}(s)$ *can be used to implement the globally optimal allocation. Different bailout sharing rules differ in the initial distribution of tax revenue claims* G_i*. We can set* $T^1_{i,i'j}(s) = 0$ *whenever* $i \notin \{i', j\}$ *without loss of generality.*

The bailout sharing rule (31) implies that tax burdens of bailouts are smoothed across countries in an average sense but not state-by-state at date 1, so that some countries may be net contributors or recepients of bailouts in any given state s.[66] Bailout sharing rule irrelevance describes equivalent set of bailout sharing rules, and implies that in principle bailout obligations can be delegated entirely to one country (or to one international organization).[67]

C.4 Non-Cooperative Taxation

We next consider non-cooperative taxation in the model.

Proposition 22. *Suppose that the monopolist distortion is* 0*. Then, non-cooperative optimal taxation is as follows.*

[66]For example, if countries have the same indirect utility functions and are equally weighted globally, then expected tax burdens are the same across countries.

[67]For example, the responsibility for deposit insurance can be entirely vested in a single entity (the host country, the home country, or an international deposit guarantee scheme). Once the bailout authority has been delegated to a single entity, the goal of the global planner will be to ensure that that entity chooses bailouts optimally. In practice, imperfectly controllable political economy distortions may lead to bailout funds being misused. See Foarta (2018).

1. *Domestic taxes on domestic banks' domestic activities are*

$$\tau^D_{i,ii} = E\left[\left[\Omega^B_{i,i}(s) + \int_{i'} \Omega^B_{i',i}(s)di' + \int_{i'} \Delta^T_{i'i}(s)T^1_{i'i}(s)di'\right] \frac{\partial L^(_{ii}s)}{\partial A^1_{ii}(s)}\right] \tag{32}$$

$$\tau^I_{i,ii} = -E\left[\left[\Omega^B_{i,i}(s) + \int_{i'} \Omega^B_{i',i}(s)di' + \int_{i'} \Delta_{i'i}(s)T^1_{i'i}(s)di'\right] \frac{\partial L^(_{ii}s)}{\partial I_{ii}}\right] \tag{33}$$

where $\Delta^T_{ij}(s)$ is defined in the proof.

2. *Domestic taxes on foreign banks' domestic activities are*

$$\tau^D_{i,ji} = E\left[\left[\Omega^B_{i,i}(s) + \int_{i'} \Omega^B_{i',i}(s)di' + \int_{i'} \Delta^T_{i'i}(s)T^1_{i'i}(s)di'\right] \frac{\partial I^L_{ji}(s)}{\partial A^1_{ji}(s)}\right] \tag{34}$$

$$\tau^I_{i,ji} = -E\left[\left[\Omega^B_{i,i}(s) + \int_{i'} \Omega^B_{i',i}(s)di' + \int_{i'} \Delta^T_{i'i}(s)T^1_{i'i}(s)di'\right] \frac{\partial I^L_{ji}(s)}{\partial I_{ji}}\right] \tag{35}$$

Although the result here appears largely as in the baseline model, there is one substantive difference: the additional terms $\Delta_{i'i}(s)T^1_{i'i}(s)$ that arise in the revenue derivatives. These terms arise whenever there are bailouts by *some country* (not necessarily i) of domestic activities of foreign banks. This effect arises because bailout revenue is not a choice variable of private agents, but rather is an untaxed and unpriced action of governments. In absence of bailouts ($T^1_{ji}(s) = 0$), this term disappears and we revert to the effective characterizations in the first half of this paper. In other words, bailouts lead to a violation of Assumption 10.[68]

Finally, we could consider the bailout rule for banks. The bailout rule for domestic banks is of the same form as in Proposition 18, except for the change in the spillover. Importantly, however, it is immediate to observe that the bailout rule for foreign banks does not consider the direct revenue benefit to banks from bailout revenue, because there is no tax on bailouts (i.e. it is not a private choice variable). However, there are effects on tax revenue.

Proposition 23. *The optimal bailout rule for domestic activities of a foreign bank is as in Proposition 18, but with the spillover effect in Proposition 22.*

[68]Note that the bailouts model features the nonlinear aggregates property of Appendix E.4, but that Assumption 10 is still the relevant assumption in that section.

Proposition 23 indicates that in addition to imperfect internalization of spillovers, planners are not able to account for direct benefits to foreign banks of receiving bailouts. As a result, cooperation is likely to be required over bailouts of cross-border banks even if non-cooperative Pigouvian taxation is able to achieve close-to-optimal internalization of spillovers. However, it is worthwhile to note that if the terms Δ_{ij}^{T} are close to zero, then Pigouvian taxation transforms the bailout problem to a *bilateral* problem, where the domestic planner simply neglects the benefit to foreign banks of receiving a bailout. Transforming the problem into a bilateral surplus problem, rather than a multilateral problem, may simplify cooperation over bailouts. For example, it may allow for simple agreements such as reciprocity on provision of deposit insurance and access to LOLR facilities.

C.5 Restoring Non-Cooperative Optimality With Bailout Levies

The above results imply that the existence of bailouts limits the ability for non-cooperative Pigouvian taxation to generate efficient policies. This failure arises because bailouts are not priced or otherwise optimally chosen by private banks. This implies that if bailouts *were* chosen by private banks, either explicitly or implicitly, we could restore efficiency.

Suppose that banks can in fact purchase bailout claims from the government, or alternatively that banks are charged ex ante for the bailout claims they will receive. In partciular, banks can purchase claims $T_{ij}^{1}(s) \geq 0$ at date 0, at a cost $\overline{q} > 1$, so that bailout claims are more expensive than state-contingent securities. The first-order condition for bailout claim purchases in state s is

$$\tau_{j,ij}^{T}(s) = -\tau_{i,ij}^{T}(s) - \overline{q} + \frac{\lambda_i^1(s)}{\lambda_i^0} \left(1 + \left(\gamma_j(s) - 1\right) \frac{\partial L_{ij}(s)}{\partial A_{ij}^1(s)}\right). \tag{36}$$

Following the logic of previous sections, we have $\tau_{i,ij}^{T} = 0$, since banks now purchase bail out claims and since country i does not internalize impacts on foreign fire sales. As a result, domestic planners never force banks to increase their backstop for foreign activities. On the other hand, the revenue that country planner j raises at date 0 from taxing the bailout purchases is of country i banks is $\tau_{j,ij}^{T}(s) T_{ij}^{1}(s)$. From here, it is easy to see that the efficiency results of the baseline model are restored. Planner j accounts for the direct benefits of bailouts, and also for the spillover costs.

The results of this section imply that *bailout* cooperation is also not necessary if it can be given a "market mechanism" and taxed. In practice, we could think about these taxes as corresponding to levies on banks for deposit insurance or access to lender of last resort, with the levies calibrated based on how much the bank expects to receive from them. Such levies are consistent with the fact that the Single Resolution Fund in the EU is funded by bank levies, and the Orderly Liquidity Fund in the US is designed to recoup expenditures from either the resolved bank or from other large financial institutions.[69]

The framework suggests that bailout policies are most naturally delegated to the *host* country, who can internalize the benefits and spillovers to foreign banks when using Pigouvian regulation combined with a market mechanism for bailouts. For example, this could correspond to a host country insuring the deposits of the local subsidiary of a foreign bank. This synergizes with other possible considerations, such as benefits to domestic depositors of deposit insurance, that might help to ensure that bailout policies are time consistent.

Time Consistency and Bailout Sharing. The results of this section assume that bailouts and bailout sharing rules are chosen ex ante with commitment. In practice, a key concern may be time consistency problems, where countries that ex post are obliged to send bailout funds to foreign countries renege on their international claims. If there are time consistency problems that prevent non-cooperative sharing of taxpayer funds, there may be a role for cooperation to enforce risk sharing agreements. However, the results of this section imply that the role of cooperation would be limited to enforcement of risk sharing, and would not need to specify the level of risk sharing.

C.6 Time Inconsistent Bailouts

Suppose that we had time inconsistent bailouts. As a result, the liquidation rule becomes

$$L_{ij}(s) \geq -\frac{1}{h_j(s)\gamma_j(s)}A^1_{ij}(s) - \frac{(1-h_j(s))}{h_j(s)}R_j(s)I_{ij}$$

where the bailout transfer is now chosen ex post, and for simplicity we assume it is targeted. Suppose for simplicity that the government is utilitarian ex post and the cost of taxpayer funds is linear with marginal cost of 1, and suppose that there is no fire sale. An ex post utilitarian government always fully bails out its own banks, regardless of investment location. As a result, the bailout rule satisfies

$$0 \geq -\frac{1}{h_j(s)\gamma_j(s)}A^1_{ij}(s) - \frac{(1-h_j(s))}{h_j(s)}R_j(s)I_{ij}.$$

[69]See https://srb.europa.eu/en/content/single-resolution-fund for the former, and US Department of Treasury (2018) for the latter.

Given that banks know they will be bailed out, they internalize the effects of debt and investment on bailout transfers, given by the above rule. At the same time, banks are always bailed out by their domestic planner.

The result is a moral hazard problem: higher debt issuance, for example, increases the bailout subsidy. The bailout cost is not internalized by banks, creating a role for regulation. However, domestic regulators only regulate domestic banks, because they are not tempted to bail out foreign banks. Nevertheless, notice that there is equal treatment in the sense that if both domestic and foreign banks expect to be bailed out, simply by different planners, then both are regulated for the externality cost of that bailout. As a result, the equilibrium achieved is in fact efficient.

Now, suppose instead that bailouts have a cost greater than the resource loss, so that planners never bail out banks to save resources, but that bank liquidations also have an externality cost κ. When κ is large, domestic planners bail out any failing bank, regardless of domicile, ex post. This reintroduces a motivation to regulate foreign banks, and results in the bans on foreign liquidations to avoid the cost of taxpayer bailouts. By contrast, domestic banks are not banned but are restricted to account for the bailout cost. This restores unequal treatment and inefficiency.

This suggests that the moral hazard view of bailouts may affect who regulates and bails out what banks, but does not fundamentally alter the intuitions of the regulatory model. When moral hazard is concentrated in domestic banks, the regulatory equilibrium features efficient outcomes even though domestic regulators only regulate domestic banks. When moral hazard also arises from foreign banks, the regulatory equilibrium features inefficient outcomes and over-regulation of foreign banks, for the same reason as in the baseline model.

C.7 Bailout Proofs

C.7.1 Proof of Lemma 17

The Lagrangian of the country i bank problem is given by

$$\mathcal{L}_i = \int_s c_i(s) f(s) ds + \lambda_i^0 \left[A_i + D_i - T_i - \Phi_{ii}(I_{ii}) - \int_j \Phi_{ij}(I_{ij}) dj \right]$$

$$+ \int_s \lambda_i^1(s) \left[A_i^1(s) + (\gamma_i(s) - 1) L_{ii}(s) + R_i(s) I_i + \int_j \left((\gamma_j(s) - 1) L_{ij}(s) + R_j(s) I_j \right) dj - c_i(s) \right] f(s) ds$$

$$+ \Lambda_i^0 \Gamma_i^x \left(\phi_{ii}(D_{ii}) + \int_j \phi_{ij}(D_{ij}) dj \right)$$

where we have implicitly internalized the demand liquidation function $L_{ij}(s) = \max\{0, -\frac{1}{h_j(s)\gamma_j(s)} A^1_{ij}(s) - \frac{(1-h_j(s))}{h_j(s)} R_j(s) I_{ij}(s)\}$. Taking the FOC is I_{ij} and rearranging, we obtain

$$\tau^I_{j,ij} = -\tau^I_{i,ij} - \frac{\partial \Phi_{ij}}{\partial I_{ij}} + \frac{1}{\lambda^0_i} E\left[\lambda^1_i(s)\left(\left(\gamma_j(s) - 1\right)\frac{\partial L_{ij}(s)}{\partial I_{ij}} + R_{ij}(s)\right)\right].$$

Similarly, taking the FOC for $x_{ij}(s)$ and rearranging, we obtain

$$\tau^D_{j,ij} = -\tau^D_{i,ij} - E\left[\frac{1}{\lambda^0_i}\lambda^1_i(s)\left(1 + \left(\gamma_j(s) - 1\right)\frac{\partial L_{ij}(s)}{\partial A^1_{ij}(s)}\right)\right] + \frac{1}{\lambda^0_i}\Lambda_i \frac{\partial \Gamma_i}{\partial \phi_i}\frac{\partial \phi_{ij}}{\partial D_{ij}}.$$

C.7.2 Proof of Propositions 18 and 19

As in the baseline model, the implementing tax rates of Lemma 17 do not otherwise appear in the country i planning problem. These constraints simply determine these tax rates, for the chosen allocation.

Now, consider the decision problem of the country i planner. The only twist is that the liquidation discount is now given by the equation

$$\gamma_i(s) = \gamma_i\left(L_{ii}(s) + \int_j L_{ji}(s) dj, s\right),$$

where we have adopted the shorthand $\gamma_i = \frac{\partial \mathcal{F}_i}{\partial I^A_i}$. From here, we characterize the response of the liquidation price to an increase ϵ in total liquidations. Totally differentiating the above equation in total liquidations, we have

$$\frac{\partial \gamma_i(s)}{\partial \epsilon} = \frac{\partial \gamma_i(s)}{\partial L^A_i(s)}\left[1 + \frac{\partial [L_{ii}(s) + \int_j L_{ji}(s) dj]}{\partial \gamma_i(s)}\frac{\partial \gamma_i(s)}{\partial \epsilon}\right],$$

where $L_{ii}(s)$ and $L_{ij}(s)$ depend on $\gamma_i(s)$ due to the collateral constraint. Rearranging from here, we obtain the equilibrium country i price response

$$\frac{\partial \gamma_i(s)}{\partial \epsilon} = \frac{1}{1 - \frac{\partial \gamma_i(s)}{\partial L^A_i(s)}\frac{\partial L^A_i(s)}{\partial \gamma_i(s)}}\frac{\partial \gamma_i(s)}{\partial L^A_i(s)}.$$

This characterization is useful, since externalities in this proof arise from changes in total liquidations.

Now, consider the Lagrangian of the country i planner. The Lagrangian of the planner can be written as

$$\mathcal{L}_i^{SP} = \mathcal{L}_i + \omega_i^T V_i^T(\mathcal{T}_i) + \lambda_i^T\left[G_i + \mathcal{T}_i - \int_s \left[T^1_{i,ii}(s) + \int_j T^1_{i,ji}(s) + \int_j T^1_{i,ij}(s)\right] f(s)ds\right.$$

where $\mathcal{L}_i$ internalizes the liquidation response and liquidation price relationships.

We first characterize the regulatory policies (Proposition 19), and then characterize the bailout policies (Proposition 18).

Regulatory Policies. Consider first the domestic allocations of domestic banks. For foreign allocations and consumption of the bank, the planner and bank derivatives coincide, and no wedges are applied, that is $\tau^D_{i,ij} = \tau^I_{i,ij} = 0$ for all $j \neq i$.

For domestic investment, the planner's derivative is

$$\frac{\partial \mathcal{L}_i^{SP}}{\partial I_{ii}} = \frac{\partial \mathcal{L}_i}{\partial I_{ii}} + \int_s \underbrace{\frac{\partial \mathcal{L}_i}{\partial \gamma_i(s)} \frac{\partial \gamma_i(s)}{\partial \epsilon}}_{\equiv \lambda_i^0 \Omega^B_{i,i}(s) f(s)} \frac{\partial L_{ii}(s)}{\partial I_{ii}} ds$$

so that the domestic tax on domestic investment scale is given by

$$\tau^I_{i,ii} = -E\left[\Omega^B_{i,i}(s) \frac{\partial L_{ii}(s)}{\partial I_{ii}}\right],$$

which is simply the expected spillover effect. Next, we can apply the same argument to taxes on domestic state-contingent securities D_{ii}. We have

$$\frac{\partial \mathcal{L}_i^{SP}}{\partial D_{ii}} = \frac{\partial \mathcal{L}_i}{\partial D_{ii}} + \int_s \frac{\partial \mathcal{L}_i}{\partial \gamma_i(s)} \frac{\partial \gamma_i(s)}{\partial \epsilon} \frac{\partial L_{ii}(s)}{\partial D_{ii}}$$

so that the required tax rate is

$$\tau^D_{i,ii} = E\left[\Omega^B_{i,i}(s) \frac{\partial L_{ii}(s)}{\partial A^1_{ii}(s)}\right].$$

Finally, considering domestic allocations of foreign banks, we only have the price spillover effect. This implies that there is a liquidation ban whenever there is an adverse price spillover, $-\Omega^B_{i,i}(s) > 0$.

Note that we can formally characterize the spillover effect $\Omega^B_{i,j}(s)$ by evaluating

$$\frac{\partial \mathcal{L}_i}{\partial \gamma_j(s)} = \lambda^1_i(s)\left[L_{ij}(s) + (\gamma_j(s)-1)\frac{\partial L_{ij}(s)}{\partial \gamma_j(s)}\right] f(s)$$

so that we have

$$\Omega^B_{i,j}(s) = \frac{\frac{\partial \mathcal{L}_i}{\partial \gamma_j(s)}\frac{\partial \gamma_j(s)}{\partial \epsilon}}{\lambda^0_i f(s)} = \frac{\lambda^1_i(s)}{\lambda^0_i}\left[L_{ij}(s) + (\gamma_j(s)-1)\frac{\partial L_{ij}(s)}{\partial \gamma_j(s)}\right]\frac{\partial \gamma_j(s)}{\partial \epsilon}.$$

Bailout Policies. We next characterize the optimal bailout policies. Consider first the bailout rule for domestic activities of domestic banks, where we have

$$\frac{\partial \mathcal{L}^{SP}_i}{\partial T^1_{i,ii}(s)} = \underbrace{\frac{\partial \mathcal{L}_i}{\partial A^1_{ii}(s)}}_{\equiv \lambda^0_i B^1_{ii}(s) f(s)} + \frac{\partial \mathcal{L}_i}{\partial \gamma_i(s)}\frac{\partial \gamma_i(s)}{\partial \epsilon}\frac{\partial L_{ii}(s)}{\partial A^1_{ii}(s)} - \lambda^T_i f(s).$$

Now, the FOC for tax collection tells us that $\lambda^T_i = -\omega^T_i \frac{\partial V^T_i}{\partial \mathcal{T}_i}$. Noting that $\frac{\partial V^T_i}{\partial \mathcal{T}_i} < 0$, we rearrange and obtain the bailout rule

$$\frac{\omega^T_i}{\lambda^0_i}\left|\frac{\partial V^T_i}{\partial \mathcal{T}_i}\right| \geq B^1_{ii}(s) + \Omega^B_{i,i}(s)\frac{\partial L_{ii}(s)}{\partial A^1_{ii}(s)}.$$

The remaining two equations follow simply by noting that the spillover term does not appear in the FOC for bailouts of foreign activities of domestic banks, while the bank benefit term does not appear in the FOC for bailouts of domestic activities of foreign banks.

C.7.3 Proof of Propositions 20 and 21

The Lagrangian of the global planner is given by

$$\mathcal{L}^G_i = \int_i \left[\mathcal{L}_i + \omega^T_i V^T_i(\mathcal{T}_i) + \lambda^T_i \left[G_i + \mathcal{T}_i - \int_s \left[T^1_{i,ii}(s) + \int_j T^1_{i,ji}(s) + \int_j T^1_{i,ij}(s)\right] f(s) ds\right] + \int_i \left[\lambda^0 T_i + \lambda^T G_i\right] di$$

where the last terms reflect the set of lump sum transfers. The FOC for G_i implies $\lambda^T = \lambda^T_i$ while the FOC for T_i implies $\lambda^0 = \lambda^0_i$. From here, the regulation and bailout rules follow by the same steps as in the non-cooperative equilibrium, except that now the full set of spillovers appear, and the benefits to banks of bailouts are always accounted for.

Next, the relationship $\lambda^T = \lambda^T_i$ gives the tax sharing rule. Bailout irrelevance arises by setting $G_i = \int_s \left[T^1_{i,ii}(s) + \int_j T^1_{i,ji}(s)dj + \int_j T^1_{i,ij}(s)dj \right] dj - \mathcal{T}_i$, for the desired bailout rule.

C.7.4 Proof of Propositions 22 and 23

The country planner Lagrangian is the same as under regulation, except that there is now also tax revenue collected from foreign banks.

The tax revenue collected by country j from country i banks is given by

$$T^*_{j,ij} = \tau_{j,ij} I_{ij} + \tau^D_{j,ij} D_{ij}$$

so that differentiating in total liquidations in state s, we have

$$\frac{\partial T^*_{j,ij}}{\partial \epsilon} = \frac{\partial}{\partial \gamma_j(s)} \left[\frac{\lambda^1_i(s)}{\lambda^0_i} \left(\gamma_j(s) - 1\right) \left[\frac{\partial L_{ij}(s)}{\partial I_{ij}} I_{ij} - \frac{\partial L_{ij}(s)}{\partial A^1_{ij}(s)} D_{ij} \right] \right] \frac{\partial \gamma_j(s)}{\partial \epsilon} f(s)$$

from here, we note that $L_{ij}(s)$ is homogeneous of degree 1 in $(I_{ij}, A^1_{ij}(s))$, given γ_j, so that we can write

$$\begin{aligned} \frac{\partial T^*_{j,ij}}{\partial \gamma_j(s)} &= \frac{\partial}{\partial \gamma_j(s)} \left[\frac{\lambda^1_i(s)}{\lambda^0_i} \left(\gamma_j(s) - 1\right) \left[L_{ij}(s) - \frac{\partial L_{ij}(s)}{\partial A^1_{ij}(s)} T^1_{ij}(s) \right] \right] \frac{\partial \gamma_j(s)}{\partial \epsilon} f(s) \\ &= \Omega^B_{j,i}(s) f(s) + \Delta^T_{ij}(s) T^1_{ij}(s) f(s) \end{aligned}$$

where we have defined $\Delta^T_{ij}(s) = -\frac{\partial}{\partial \gamma_j(s)} \left[\frac{\lambda^1_i(s)}{\lambda^0_i} \left(\gamma_j(s) - 1\right) \frac{\partial L_{ij}(s)}{\partial A^1_{ij}(s)} T^1_{ij}(s) \right] \frac{\partial \gamma_j(s)}{\partial \epsilon}$.

From here, results on regulation follow by the usual steps. Moreover, results on bailouts also follow the usual steps, noting the bailout has indirect effects on tax rates through the liquidation price, but does not have direct effects due to the linear nature of $L_{ij}(s)$.

D Extensions of the Banking Model

In this Appendix, we present a number of extensions to and discussions of the model, as applied to the banking context. To ease exposition, we express all results in this appendix for interior solutions, except for foreign allocations under non-cooperative regulation.

D.1 Extended Stakeholders, Real Economy Spillovers, and Arbitrageurs

Bank activities and failures can affect a wide range of economic agents, leading to spillovers from the financial sector to the real sector of the economy. In the baseline model, there were no benefits from foreign banking, and fire sales affected only the consumption profiles of agents in the "financial sector" (banks and arbitrageurs); moreover, arbitrageurs were given zero welfare weight. In practice, the spillover to the real economy's "extended stakeholders" in the financial sector are an important consideration for macroprudential policy.[70] Examples of extended stakeholders include: (1) SMEs, who rely on banks for financing; (2) bank/SME employees, whose employment changes based on bank activities; (3) retail customers, who benefit from retail banking (e.g. deposit) activities; and (4) other local banks, who hold related assets and are affected by fire sales.

For simplicity, we model these spillovers as a reduced form utility spillover onto "extended stakeholders." The indirect utility function of extended stakeholders $V_i^E\left(I_i^A, L_i^A\right)$, where $I_i^A = I_{ii} + \int_j I_{ji} dj$ is total investment in country i at date 0. This additional indirect utility function appears in social welfare, so that country i social welfare is now

$$V_i^P = \int_s c_i(s) f(s) ds + \omega_i^E V_i^E \left(I_i^A, L_i^A\right) \tag{37}$$

where ω_i^E is the aggregate welfare weight on extended stakeholders. The model setup is otherwise the same. Notice that positive welfare weights on arbitrageurs is a special case of this setup, where the indirect utility function V_i^E is the expected consumption surplus of arbitrageurs.

This model is a straight-forward application of Section 6 when we interpret V_i^P as representative agent welfare with total utility spillovers $\omega_i^E V_i^E$. Note that it is irrelevant that V_i^E does not satisfy homogeneity of degree 1, since it is for country i allocations. As a result, all of the core properties of Section 6 go through: quantity restrictions are inefficient and features unequal treatment + uninternalized foreign spillovers, whereas Pigouvian taxation is efficient in absence of monopoly rents.

Nevertheless, in contrast to the baseline model, there are "real economy" domestic spillover from banking activities $\Omega_{i,i}^{I,E} = \omega_i^E \frac{\partial V_i^E}{\partial I_i^A}$ and $\Omega_{i,i}^{L,E} = \omega_i^E \frac{\partial V_i^E}{\partial L_i^A(s)}$ which may make the domestic government more accepting of international banking activities.

In particular, suppose that there is a positive domestic liquidation spillover perfectly

[70]"The objective of the [Basel III] reforms is to improve the banking sector's ability to absorb shocks... thus reducing the risk of spillover from the financial sector to the real economy...."[B]anks provide critical services to customers, small and medium-sized enterprised, large corporate firms and governments who rely on them to conduct their daily business." BIS (2010)

offsets the domestic fire sale cost at the global optimum.[71] Section 6 implies that the domestic planner would on the margin regulate foreign banks correctly. However, the domestic planner would nevertheless still under-regulate domestic banks. A cooperative agreement would still be required to achieve global efficiency, and would increase regulation of domestic banks.

Positive Arbitrageur Welfare Weights. With positive welfare weights on arbitrageurs, we have $V_i^E = E\left[\mathcal{F}_i^A(L_i^A(s), s) - \gamma_i^A(s)L_i^A(s)\right]$. This delivers a positive utility spillover from liquidations,

$$\frac{\partial c_i^A(s)}{\partial L_i^A(s)} = -\frac{\partial \gamma_i^A(s)}{\partial L_i^A(s)} L_i^A(s) \geq 0$$

and so positive welfare weights on arbitrageurs tend to make domestic planners more accepting of liquidations, both by domestic and foreign banks.

Extended Stakeholders Without Fire Sales. What if instead, there were no fire sales but there were spillovers to extended stakeholders? Optimal regulation would account for the domestic spillover, but there is no longer any international spillover. As a result, optimal cooperation only is needed to enforce equal treatment, but does not require an increase in regulation of domestic banks.

D.2 Dispersed Bank Ownership

Banks in practice are multinational not only in their activities, but also in ownership: even though a bank is headquartered in one country, part of its equity can be owned by foreigners. This invites a natural question: do regulatory incentives change when part of the value of banks accrues to non-domestic agents, and if so does it cause inefficiencies?

[71] Although positive spillovers from bank liquidations may seem counter intuitive, they can be understood in two ways. The first way to understand why there might be a positive liquidation spillover is that it is a reduced-form representation of the idea that banks may not be able to provide valuable services if they are subject to too strict regulatory stringency. For example, they may choose not to lend to certain types of borrowers entirely. This may lead domestic regulators to be more accepting of foreign bank liquidations, to avoid crowding out these valuable lending services. The second way to understand a positive spillover is that optimal regulation in the model may imply *investment subsidies*, despite the fact that we are using "regulation." In this sense, it may be natural to impose an instrument restriction $\tau_{i,ji}^I \geq 0$, which we could also reflect as a utility cost for choosing negative wedges under regulation. When banning liquidations reduces the value of domestic investment, this may make the value of domestic investment negative, making the allocation unfeasible. As a result, domestic regulators need to allow some foreign liquidations in order to motivate them to invest domestically.

Formally, we extend the model as follows. $c_i(s)$ is the total payoff to the (domestic and foreign) equity holders of country i banks, whose objective is as in the baseline model.[72] Country i equity holders are risk-neutral agents who own a portfolio of bank equity shares, with exogenous portfolio shares $\left(\alpha_i, \{1-\alpha_j\}\right)$. α_i is the level of domestic ownership of country i banks. For simplicity, we assume that foreign equity holdings of domestic banks is equally dispersed across all countries. The consumption profile of country i equity holders is $c_i^{\text{Equity}}(s) = \alpha_i c_i(s) + \int_{i'} (1-\alpha_{i'}) c_{i'}(s) di'$.[73]

As we will show, non-cooperative and cooperative policy are invariant to foreign ownership if there are only spillovers to multinational banks. To this end, we allow for a second set of spillovers onto purely domestic agents, which we reflect by a reduced-form social cost of bank liquidations. For example, this may correspond to fire sale spillovers to wholly domestic local banks, or to real economy spillovers (as in Appendix D.1). Incorporating the value of equity holders and the reduced-form spillover cost, country i social welfare is now

$$V_i^P = \alpha_i \int_s c_i(s) f(s) ds + \int_{i'} (1-\alpha_{i'}) \int_s c_{i'}(s) f(s) ds di' - \omega_i^E \int_s L_i^A(s) f(s) ds,$$

where ω_i^E is the welfare weight on the additional spillover cost.

Note that this falls into the framework of Section 6, except that the "aggregate" $c_{i'}(s)$ now appears in the utility of foreign multinational banks. It does not satisfy the form of Assumption 10, and so will not generate efficient results under either quantity restrictions or taxes.

Expressing the inefficiency in the form of consumption payout is unintuitive. Instead, let us represent it in form of more intuitive arguments. Suppose for simplicity that the global planner is utilitarian, so that the global welfare function is

$$V_i^G = \int_i \left[\int_s c_i(s) f(s) ds - \omega_i^E \int_s L_i^A(s) f(s) ds \right].$$

Consider instead the optimization function of country planner i. Their objective function

[72] Even if banks only care about domestic equity holders, α_i is a constant scaling of the objective function that does not affect optimization.

[73] We treat the ownership structure of banks as given, implying that regulation (or taxation) is either chosen *after* ownership is determined or is chosen simultaneously but is not easily verifiable or contractible. This is related to the time consistency problem of Farhi and Tirole (2018), which revolves around debt holdings by foreigners.

is a monotone transformation of the optimization problem

$$V_i^{P,*} = \int_s c_i(s) f(s) ds - \frac{1}{\alpha_i} \omega_i^E \int_s L_i^A(s) f(s) ds.$$

From here, we see that the spillover is equivalent to an upweighting of domestic real economy spillovers. As a result, even under Pigouvian taxation, the efficient outcome is not achieved because of foreign ownership of domestic banks. Foreign ownership leads to an inefficient upweighting of other domestic interests.

In this case, the "global spillover" that must be corrected is equivalent to a *positive* utility spillover $(\frac{1}{\alpha_i} - 1) L_i^A(s)$. It is positive in this case because the spillover to extended stakeholders is negative, leading to over-regulation of domestic banks.

As a result, if there are no net spillovers to domestic non-bank agents (or those spillovers are not valued), the equilibrium is efficient even under foreign ownership. This implies that the goal of macroprudential regulation matters substantially for determining the efficiency properties under foreign ownership. When policy orients principally around mitigating fire sale impacts on multinational banks ($\omega_i^E = 0$), there is no (or only a small) distortion, as foreign ownership only results in a constant scaling of the social objective function.

A second implication is that the composition of the banking sector matters. When the banking system consists only of multinational banks (no other spillovers), foreign ownership is irrelevant to optimal policy design. By contrast when there is a mixture of multinational and local banks, increasing foreign ownership of multinational banks up-weights the relative weight that domestic planners place on spillovers to local banks, leading to excessively stringent regulation of multinational banks.

D.3 Local Capital Goods and Protectionism

Although financial stability and fire sales have been highlighted as justifications for post-crisis regulation, cooperative agreements predate the crisis, including the previous Basel accords. In this context, regulators may care about additional considerations such as domestic spillovers (Appendix D.1). Additionally, regulators may care about controlling local costs of investment, for example wishing to ensure that (less strictly regulated) foreign banks are not at a competitive advantage over domestic banks.[74]

[74] The mechanism in this extension is closely related to Dell'Ariccia and Marquez (2006), who use a similar model to motivate a coordinated increase in domestic regulation. Our main contribution relative to their paper is to allow for common agency, to study the impacts of Pigouvian taxation, and to relate this mechanism to fire sales.

We consider such a motivation by extending the model to include a common domestic investment price. In particular, we augment the model with local capital goods, which are used to produce domestic projects. For simplicity, we rule out all spillovers besides capital good prices in this sections. As a result, there are no fire sales and no extended stakeholder spillovers.

Banks can produce projects using both initial endowment, and with units of a local capital good. Bank i purchases a vector k_i of local capital goods, with k_{ij} being the capital good of country j, at prices p_j. When using k_i of the capital good, it costs an additional $\Phi_{ii}(I_{ii}, k_{ii}) + \int_j \Phi_{ij}(I_{ij}, k_{ij})dj$ to produce the vector I_i of projects. The date 0 budget constraint of bank i is

$$p_i k_{ii} + \int_j p_j k_{ij} dj + \Phi_{ii}(I_{ii}, k_{ii}) + \int_j \Phi_{ij}(I_{ij}, k_{ij})dj \leq A_i + D_i.$$

The optimization problem of banks is otherwise unchanged, except that k_i is now a choice variable of banks.

In each country, there is a representative capital producing firm. The capital producing firm produces the capital good out of the consumption good with an increasing and weakly convex cost function $\mathcal{K}_i(K_i)$, and so has an optimization problem

$$\max_{K_i} p_i K_i - \mathcal{K}_i(K_i).$$

The resulting equilibrium capital good price in country i is

$$p_i = \frac{\partial \mathcal{K}_i(K_i)}{\partial K_i}, \quad K_i = k_{ii} + \int_j k_{ji} dj. \tag{38}$$

The local capital producing firm cannot be controlled by country planners, so that equation (38) is an implementability condition of the model. Note that $\frac{\partial p_i}{\partial K_i} \geq 0$.[75]

Finally, the social planner places a welfare weight ω_i^K on the capital producing firm, so that the social welfare function is

$$V_i^P = \int_s c_i(s) f(s) ds + \omega_i^K \Big[p_i(K_i) K_i - \mathcal{K}_i(K_i) \Big].$$

From here, note that the model is in the form of Section 6 when we interpret profits of the capital producing firm as a utility spillover to the domestic representative bank.

[75]In order to ensure that firm profits are bounded above, we will assume that $\frac{\partial p_i}{\partial K_i} = 0$ above some point K^*, which amounts to assuming that $\mathcal{K}_i(K_i)$ becomes linear on the margin above K^*.

From here, we see that there are spillovers to both domestic and foreign agents from changes in capital purchases, given by

$$\Omega^K_{i,i} = -\frac{\partial p_i}{\partial K_i} k_{ii} + \frac{\omega^K_i}{\lambda^0_i} \frac{\partial p_i}{\partial K_i} K_i$$

$$\Omega^K_{j,i} = -\frac{\partial p_i}{\partial K_i} k_{ji}$$

The spillover from the capital price increase is the additional resource cost to the bank of purchasing their existing level of the capital good. This is closely related to the direct price spillover under fire sales.

Let us suppose that we are in an environment where the domestic planner wishes to subsidize domestic banks by keeping capital cheap. We represent this by the limiting case $\omega^K_i = 0$. In this case, there is a *negative* spillover from increases in the capital price to both domestic and foreign banks, which make capital more expensive.

The globally efficient policy subsidizes capital by limiting capital purchases of all banks. By contrast, non-cooperative quantity regulation is protectionist and bans foreign banks from purchasing the domestic capital. In effect, it shields domestic banks from foreign competition.

Nevertheless, the "pecuniary externality" here falls within the class of problems under Assumption 10. As a result, assuming no monopoly power, the non-cooperative equilibrium under Pigouvian taxation is globally efficient.

Relationship to the Pre-Crisis World. In addition to understanding the Basel accords, this result also helps contextualize the historical aversion to capital control measures or other barriers to capital flows. In a purely non-cooperative environment, countries are tempted to engage in inefficient protectionism to shield domestic banks from foreign competition. Protectionism is inefficient because all countries do so, and so countries benefit from agreements against protectionist policies. For example, agreements might allow expansion via branches, rather than subsidiaries, in addition to lifting other barriers to capital flows. Our results suggest that although quantity-based measures lead to inefficient protectionist policies, priced-based measures (taxes) do not. This provides another advantage of tax-based policies in the international context.

Differences from Fire Sales. Although the general characterizations in this extension are closely related in a general sense to the characterizations of the main paper under fire sales, there are two important differences.

The first important difference is the form of restrictions on foreign banks. Under fire sales, non-cooperative policies were meant to restrict premature liquidations. This corresponded most naturally to either ring fencing type policies, or to restrictions on capital outflows. By contrast, with local capital prices, non-cooperative policies are meant to restrict investment in the first place, and so more closely resemble either greater regulation on domestic activities of foreign banks, or bans on capital inflows. The motivation under the former is to enhance domestic financial stability, while the motivation under the latter is more protectionist in nature.

The second important distinction is in the implications for cooperation. Under fire sales, cooperation was required among countries who invest across borders *and* who share common crisis states. By contrast under local capital goods, cross border investment alone determines the need for cooperation.

D.4 Global Resale Markets

In the baseline model, investment resale markets are local: local arbitrageurs always buy liquidated projects. We extend the model to allow for global arbitrageurs, so that resale markets are partly global. For simplicity, we will assume that local and global arbitrageurs are not valued by any country planner. At the end of the section, we provide a brief discussion of relation to the literature on empirical capital flows.

In addition to local arbitrageurs, there is also a representative global arbitrageur, who has a production technology $\mathcal{F}^G\left(\int_i F_i^G\left(L_i^G(s),s\right)di,s\right)$, where $F_i^G(s):\mathbb{R}\to\mathbb{R}_+^M$ and $\mathcal{F}^G:\mathbb{R}_+^M\to\mathbb{R}_+$. Global markets may be fully integrated, may be partially segmented, or may be fully segmented.[76]

The optimality conditions of the global arbitrageur are

$$\frac{\partial\mathcal{F}^G(s)}{\partial F^G(s)}\frac{\partial F_i^G(s)}{\partial L_i^G(s)}=\gamma_i(s)$$

while the optimality conditions of local arbitrageurs are $\frac{\partial\mathcal{F}_i(s)}{\partial L_i^A(s)}=\gamma_i(s)$, as before. Inverting these equilibrium conditions, we obtain the demand functions of global and local arbitrageurs, respectively, as $L_i^G(\gamma_i(s),F^G(s),s)$ and $L_i^A(\gamma_i(s),s)$. Note that the demand functions of global arbitrageurs also depend on $F^G(s)$ through the derivative $\frac{\partial\mathcal{F}^G}{\partial F^G}$. As a result, we also have $\gamma_i(s)$ being a function of $F^G(s)$.

[76] For example, partial segmentation could be two regions, $I_1\cup I_2=[0,1]$, with production $\mathcal{F}^G=\mathcal{F}_1^G(\int_{i\in I_1}F_i^G di)+\mathcal{F}_2^G(\int_{i\in I_2}F_i^G di)$. Full segmentation would be $\mathcal{F}^G=\int_{i\in I_i}F_i^G di$.

From the perspective of individual country planners, this problem is no different from that in the baseline model, except for the change in the equilibrium price determination. In particular, the sensitivity of the equilibrium price to total country i liquidations, from the perspective of country planner i, is given by

$$1 = \frac{\partial L_i^A(s)}{\partial \gamma_i(s)} \frac{\partial \gamma_i(s)}{\partial L_i^{\text{Tot}}} + \frac{\partial L_i^G(s)}{\partial \gamma_i(s)} \frac{\partial \gamma_i(s)}{\partial L_i^{\text{Tot}}}$$

which gives us the equilibrium movement of the country i price from the perspective of country planner i. Defining $\alpha_i^A(s) = L_i^A(s)/L_i^{\text{Tot}}(s)$, we have

$$\xi_{\gamma_i, L_i^{\text{Tot}}}(s) = \frac{1}{\alpha_i^A(s)\xi_{L_i^A,\gamma_i}(s) + (1 - \alpha_i^A(s))\xi_{L_i^G,\gamma_i}}$$

which relates the elasticity of the liquidation price in total liquidations to a weighted sum of the demand elasticities of local and global arbitrageurs.

Suppose first that $\partial \mathcal{F}^G(s)/\partial F^G(s)$ is constant, so that I_i^G is not a function of F^G. Then, this problem proceeds as in the baseline model, with the domestic liquidation price being a function only of domestic liquidations. All results are unchanged.

By contrast, suppose that $\partial \mathcal{F}^G(s)/\partial F^G(s)$ is not constant. In this case, suppose that an increase in foreign liquidations (in countries outside of i) induces an increase in a single direction $F_0^G(s)$. From here, we can characterize the resulting equilibrium price change.

Lemma 24. *Holding country i allocations fixed, the equilibrium country i price change from an increase in foreign liquidations $F_0^G(s)$ is*

$$\frac{\partial \gamma_i}{\partial F_0^G} = \frac{\left(1 - \alpha_i^A(s)\right) \xi_{L_i^G,\gamma_i}(s)}{\left(1 - \alpha_i^A(s)\right) \xi_{L_i^G,\gamma_i}(s) + \alpha_i^A(s)\xi_{L_i^A,\gamma_i}(s)} \frac{\partial^2 \mathcal{F}^G}{\partial F^G \partial F_0^G} \frac{\partial F_i^G}{\partial L_i^G}$$

where $\alpha_i^A(s)$ is the share of liquidations purchased by local arbitrageurs, and $\xi_{L_i^G,\gamma_i}$ ($\xi_{L_i^A,\gamma_i}$) is the demand elasticity of global (local) arbitrageurs in the local liquidation price.

Lemma 24 illustrates the change in equilibrium liquidation price in country i as a direct result of changes in foreign country liquidations filtering through global resale markets. This change is not internalized by any individual planner, and so reflects an uninternalized global spillover.

This uninternalized global resale market is governed by two forces. The first is a measure of the marginal importance of global arbitrageurs in the country i resale market.

When either global arbitrageur demand (on the margin) for local liquidated assets is relatively inelastic, or when global arbitrageurs account for a relatively small share of domestic purchases, spillovers from global resale markets are muted. Similarly, if global resale markets have relatively low impacts on each other, then likewise global resale market spillovers are muted. From here, we obtain the following result.

Proposition 25. *Suppose that the global resale market spillover is* 0*. Then, the non-cooperative equilibrium under Pigouvian taxation is globally efficient under the conditions of Proposition 9.*

Proof. When $\partial\gamma_i/\partial F_0^G = 0$, additional global spillover is 0, and so the proof follows as before. ∎

There are two cases under which global resale market spillovers are 0. The first is that local arbitrageurs are the relevant marginal pricing agent, that is $(1-\alpha_i^A(s))\xi_{L_i^G,\gamma_i}(s) = 0$. The second that that global resale markets are not interconnected, that is $\partial^2 F^G/\partial F^G \partial F_0^G = 0$.

The derivative $\frac{\partial^2 \mathcal{F}^G(s)}{\partial F^G(s)\partial F_0^G}$ indicates that the global asset market spillover depends on a notion of asset similarity between countries. In particular, suppose that there are two blocks of countries, I and $I' = [0,1]\backslash I$, so that asset resale markets are integrated within each block, but not across blocks. This can be represented by a global technology $\mathcal{F}^G = \mathcal{F}_I^G\left(\int_{i\in I} F_i^G(L_i^G(s),s),s\right) + \mathcal{F}_{I'}^G\left(\int_{i\in I'} F_i^G(L_i^G(s),s),s\right)$. Here, we have $\frac{\partial^2 \mathcal{F}^G(s)}{\partial F_I^G(s)\partial F_{I'}^G(s)} = 0$, and there are no spillovers on global resale markets between the two blocks.

D.4.1 Relation to Empirics

In practice, the empirical literature has highlighted that international capital flows tend to co-move, with declines in foreign inflows at the same time as declines in domestic outflows.[77] If in the model global arbitrageurs were the principal pricing agents, this would suggest that there should be foreign inflows at the same time as retrenchment. This is suggestive of a role for local arbitrageurs in determining prices.[78]

D.4.2 Proof of Lemma 24

For exposition, we suppress the s notation on equilibrium objects. The domestic liquidation price is a function $\gamma_i(L_i^{\text{Tot}}, F^G, s)$ of domestic total liquidations and of foreign

[77] See e.g. Broner et al. (2013).

[78] Of course, these foreign inflows might be offset by outflows by foreign banks, generating the empirical patterns.

liquidations F^G, while the equilibrium demand functions are $L_i^A(\gamma_i, s)$ and $L_i^G(\gamma_i, F^G, s)$. Market clearing implies that

$$L_i^{\text{Tot}} = L_i^A\left(\gamma_i(L_i^{\text{Tot}}, F^G, s), s\right) + L_i^G\left(\gamma_i(L_i^{\text{Tot}}, F^G, s), F^G, s\right)$$

so that differentiating in a single direction F_0^G, holding L_i^{Tot} fixed, we have

$$0 = \left[\frac{\partial L_i^A}{\partial \gamma_i} + \frac{\partial L_i^G}{\partial \gamma_i}\right]\frac{\partial \gamma_i}{\partial F_0^G} + \frac{\partial L_i^G}{\partial F_0^G}.$$

Next, differentiating the optimality condition of the global arbitrageur in F_0^G, we have

$$\frac{\partial^2 \mathcal{F}^G}{\partial F^G \partial F_0^G}\frac{\partial F_i^G}{\partial L_i^G} + \frac{\partial \mathcal{F}^G}{\partial F^G}\frac{\partial^2 F_i^G}{\partial \left(L_i^G\right)^2}\left(\frac{\partial L_i^G}{\partial \gamma_i}\frac{\partial \gamma_i}{\partial F_0^G} + \frac{\partial L_i^G}{\partial F_0^G}\right) = \frac{\partial \gamma_i}{\partial F_0^G}.$$

Finally, note that from the optimality condition of the global arbitrageur, we also obtain the price response $\frac{\partial \mathcal{F}^G}{\partial F^G}\frac{\partial^2 F_i^G}{\partial \left(L_i^G\right)^2}\frac{\partial L_i^G}{\partial \gamma_i} = 1$. Substituting into the above equation and rearranging, we obtain

$$\frac{\partial \gamma_i}{\partial F_0^G} = \frac{\left(1 - \alpha_i^A(s)\right)\xi_{L_i^G, \gamma_i}(s)}{\left(1 - \alpha_i^A(s)\right)\xi_{L_i^G, \gamma_i}(s) + \alpha_i^A(s)\xi_{L_i^A, \gamma_i}(s)}\frac{\partial^2 \mathcal{F}^G}{\partial F^G \partial F_0^G}\frac{\partial F_i^G}{\partial L_i^G}$$

where ξ are the elasticities, and α_i^A is the share of liquidations purchased by local arbitrageurs.

D.5 Regulatory Arbitrage

An important consideration in macroprudential regulation is regulatory arbitrage. In our model, there are two potential sources of regulatory arbitrage. First, country planners may be unwilling or unable to form cooperative agreements around all activities, creating possibilities for arbitrage around the cooperative agreement. Second, there may be a group of agents ("shadow banks") beyond the regulatory control of country planners.

D.5.1 Arbitrage Around Cooperative Agreements

The possibility for arbitrage around cooperative agreements may arise if planners retain autonomy over a set of instruments, which the global planner cannot control. In the baseline model, this could be captured by restricting the set of wedges the global planner could place on country planners.

It is obvious that such arbitrage has the potential to disrupt cooperative agreements. However, this form of arbitrage has no bearing on the efficiency of non-cooperative Pigouvian taxation, where all authority is vested in country planners and efficiency is nevertheless obtained.

Regulatory arbitrage of this form can therefore be seen as an additional difficulty of cooperation. It provides another argument for advantages of adopting Pigouvian taxation.

D.5.2 Unregulated Sectors ("Shadow Banks")

A second form of regulatory arbitrage arises if there is a "shadow banking" sector, which conducts similar activities to banks but cannot be regulated.

Suppose for simplicity that country planners are not concerned with the welfare of the shadow banking sector. Given this, the welfare-relevant aspect of the shadow banking sector is their contribution to financial stability. The only welfare-relevant aspect of the shadow banking sector, therefore, is their asset liquidations $L_i^{SB}(s)$, which lead total domestic liquidations in country i to be $L_i^A(s) = L_{ii}(s) + \int_j L_{ji}(s)dj + L_i^{SB}(s)$.

Suppose first that the shadow banking sector is wholly domestic. In this case, we represent the liquidations of the shadow banking sector as a function $L_i^{SB}(\gamma_i(s), s)$ of the domestic liquidation price in state s. This function is taken as exogenous by country planners. The equilibrium liquidation price is given as before by arbitrageur optimization, that is

$$\gamma_i(s) = \frac{\partial \mathcal{F}_i\left(L_i^A(s), s\right)}{\partial L_i^A(s)}, \quad L_i^A(s) = L_{ii}(s) + \int_j L_{ji}(s)dj + L_i^{SB}(\gamma_i(s), s)$$

where we now have a fixed-point relationship. Inverting this fixed point relationship and rearranging, we obtain

$$(\mathcal{F}_i')^{-1}(\gamma_i(s), s) - L_i^{SB}(\gamma_i(s), s) = L_{ii}(s) + \int_j L_{ji}(s)dj$$

so that the equilibrium liquidation price determination takes the same generic form as the baseline model - the equilibrium response of the shadow banking sector simply feeds into the ability of the arbitrageur sector to absorb losses. Under natural assumptions, $L_i^{SB}(\gamma_i(s), s)$ increases in $\gamma_i(s)$, so that the effects of reducing bank liquidations are partially offset by increasing risks in the shadow banking sector. However, because the problem remains of the same form as the baseline model, all results of the baseline model continue to apply, including the efficiency of non-cooperative Pigouvian taxation.

Consider next the case where the shadow banking sector is international, but home-biased. In this case, we can instead represent shadow banking sector liquidations by a function $L_i^{SB}(\gamma_i(s), \gamma_{-i}(s), s)$. The arbitrageur pricing relationship takes the same form

$$(\mathcal{F}_i')^{-1}(\gamma_i(s), s) - L_i^{SB}(\gamma_i(s), \gamma_{-i}(s), s) = L_{ii}(s) + \int_j L_{ji}(s) dj$$

except that now, the liquidation price in country i depends on liquidation prices abroad. This generates an uninternalized spillover of the same generic nature as the model of global arbitrageurs in Appendix D.4. This implies that global shadow banking activities may provide an alternate need for regulatory cooperation. Moreover, as in the case of global resale markets, this implies that the "liquidation demand elasticity" of the shadow banking sector is relevant for determining the need for cooperation. When the shadow banking sector's response is highly elastic, the need for cooperation will be greater. When the shadow banking sector's response is relatively inelastic, the need for cooperation is smaller.

Finally, notice that the inefficiency arises because shadow banks are not regulated. Were shadow banks regulated, they would fall under the framework of Section 6 and the efficiency of Pigouvian taxation would be restored. Surprisingly, this implies that by solving the problem of unregulated shadow banks, country planners can also solve the problem of international cooperation due to resale market spillovers.

D.6 Quantity Restrictions in the Form of Ceilings

In the baseline model, we adopt a revenue-neutral wedge approach to quantity restrictions. Here, we consider a variant of the problem where planners are allowed to place ceiling restrictions, and make use of the fact that all wedges in the non-cooperative equilibrium were non-negative in the baseline model. For expositional simplicity, we will restrict attention to quantity ceilings in liquidations, as no other ceilings would be used in equilibrium.[79]

Suppose that each planner can place quantity ceilings $\overline{L}_{i,ij}(s)$ and $\overline{L}_{i,ji}(s)$ on liquidations by banks. Banks facing multiple quantity ceilings must respect the more stringent ceiling, that is $L_{ij}(s) \leq \overline{L}_{ij}(s) = \min\{\overline{L}_{i,ij}(s), \overline{L}_{j,ij}(s)\}$. From here, the demand function of

[79]For this section, we assume the bank optimization problem is convex.

banks for liquidations can be written as

$$L_{ji}(s) = \begin{cases} 0, & \frac{\lambda_j^1(s)}{\lambda_j^0}\left(\gamma_i(s) - (1+r_{ji})\right) + \frac{1}{\lambda_j^0}\Lambda_j^1(s)h_i(s)\gamma_i(s) < 0 \\ \in [0, \overline{L}_{ji}(s)], & \frac{\lambda_j^1(s)}{\lambda_j^0}\left(\gamma_i(s) - (1+r_{ji})\right) + \frac{1}{\lambda_j^0}\Lambda_j^1(s)h_i(s)\gamma_i(s) = 0 \\ \overline{L}_{ji}(s), & \frac{\lambda_j^1(s)}{\lambda_j^0}\left(\gamma_i(s) - (1+r_{ji})\right) + \frac{1}{\lambda_j^0}\Lambda_j^1(s)h_i(s)\gamma_i(s) > 0 \end{cases}$$

Consider first the global planning problem. We implement the global constrained efficient allocation with quantity ceilings $\overline{L}_{ij}(s) = L^*_{ij}(s)$, where $L^*_{ij}(s)$ is the constrained efficient liquidation rule. Notice that the Lagrange multipliers $\tau^L_{ij}(s)$ on the quantity restriction constraints are given by the wedge formulas in Proposition 3.

From here, let us consider the regulatory game between country planners. Conjecture an equilibrium where all foreign planners impose quantity ceilings that correspond with the optimal foreign liquidations in the equilibrium under Proposition 5, and consider the optimal policy of country i. Suppose that country planner i wishes to impose the equilibrium of Proposition 5. It does so by banning foreign liquidations in the associated states of Proposition 5, and by restricting domestic liquidations to their equilibrium level under Proposition 5. This enforces the equilibrium under Proposition 5. But note that the equilibrium under Proposition 5 was an optimal response for country i when country i had unrestricted control over allocations. Finally, note that the Lagrange multipliers on the binding liquidation constraints are the same as the wedges in Proposition 5.

E Extensions of the General Model

In this Appendix, we provide extensions of the general model presented in Section 6.

E.1 World Prices

We now extend the model to incorporate world prices, for example allowing for state contingent securities prices at date 0 to be endogenous. We show that provided that global prices only enter constraints through the wealth level, the problem is unaffected. This result is in line with Korinek (2017) and follows similarly.

Let $x_i = \{x_i(n)\}_{n \in N}$ be a vector of global goods held by country i, so that market clearing implies $\int_i x_i(n)di = 0$. Global goods trade at prices q, so that the wealth level of country i multinational agents is

$$W_i = A_i - T_i - \sum_n q(n)x_i(n).$$

Global goods enter into $u_{ii}, u_{ii}^A, \phi_{ii}, \phi_{ii}^A$, but prices do not enter except through the wealth level. Note that because global goods enter into domestic functions, they do not influence Assumption 10. From here, we obtain the following result.

Proposition 26. *The optimal cooperative wedges are of the same form as Proposition 9, with no wedges on x_i. Pigouvian taxation is efficient under the same conditions as Proposition 15.*

Proposition 26 may apply, for example, to a global market for liabilities at date 0.

E.1.1 Proof of Proposition 26

The global planning problem has a Lagrangian

$$\mathcal{L}^G = \int_i \left[\omega_i U_i \left(u_i(a_i, x_i), u_i^A(a_i, x_i, a_i^A))\right) + \Lambda_i \Gamma_i \left(A_i + \mathcal{T}_i, \phi_i(a_i, x_i), \phi_i^A(a_i, x_i, a_i^A)\right) - \lambda^0 \mathcal{T}_i - \lambda^0 Q x_i\right] di$$

where we have suggestively denoted $Q(n)$ to be the Lagrange multiplier on the global goods market clearing for good n. Differentiating in $x_i(n)$, we obtain

$$0 = \frac{\partial \mathcal{L}_i}{\partial x_i(n)} - \lambda^0 Q(n)$$

so that world prices $q(n) = Q(n)$ form an equilibrium (recall that $\lambda_i^0 = \lambda^0$). Globally efficient policy is as in Proposition 9, with no wedges placed on x_i.

E.2 Local Constraints on Allocations

We extend Section 6 to incorporate local constraints on allocations. Note that such constraints are already available through Γ_i for domestic allocations, but that such constraints are not available in countries $j \neq i$. The extension captures, for example, the constraints $0 \leq L_{ij}(s) \leq R_j(s) I_{ij}$ and $I_{ij} \geq 0$ imposed in the main paper.

Suppose that in country j, there is a vector of linear constraints $\chi_{ij}(a_j^A) a_{ij} \leq b_{ij}$ on allocations, where $\chi_{ij}(a_j^A)$ potentially depends on aggregates in country j and where $b_{ij} \geq 0$.[80] We impose linearity in the spirit of the required conditions for optimality of

[80] We impose $b_{ij} \geq 0$ to ensure that non-participation ($a_{ij} = 0$) is always feasible.

Pigouvian taxation in Proposition 15. We obtain the following revised implementability result for foreign allocations, which mirrors Lemma 11

Lemma 27. *Any domestic allocation of foreign agents satisfying constraints $\chi_{ij}(a_j^A)a_{ij} \leq b_{ij}$ is optimally implemented with the wedges in Lemma 11.*

Lemma 27 implies that implementability constraints are the same as in Section 6. The only difference is that now the constraint set on local allocations is a constraint of the local planner. Note that this implies that the local planner directly internalizes spillovers of domestic aggregates onto the constraint set $\chi_{ij}(a_j^A)a_{ij} \leq b_{ij}$, so that such spillovers are not an issue.

From here, all results proceed as in Section 6. Intuitively, the only adjustment we need to make is that $\chi_{ij}(a_j^A)a_{ij} \leq b_{ij}$ is now a constraint set of planner j. Without loss of generality, scale the Lagrange multiplier ν_{ij} by λ_i^0, and define the "local constraint spillover" of a change in aggregates by

$$\Omega_{j,ij}^{LC}(m) = -\nu_{ij} \frac{\partial \chi_{ij}}{\partial a_j^A(m)} a_{ij}$$

so that we can define the total domestic local constraint set spillover as

$$\Omega_j^{LC}(m) = -\int_i \Omega_{j,ij}^{LC}(m) di$$

From here, it follows that the results of Section 6 apply, treating the total domestic spillover as $\Omega_{j,j}(m) + \Omega_j^{LC}(m)$.[81]

Note that if the local constraints were non-linear, this would not generally hold, as we would not be able to recovery the complementary slackness condition precisely in the above proof. As a result, the domestic planner may have an incentive to manipulate the tax rates that implement corner solutions in order to increase revenue. This would amount to another form of "monopolistic" revenue distortion in the model.

E.2.1 Proof of Lemma 27

For expositional ease, we suppress the notation $\chi_{ij}(a_j^A)$ and simply write χ_{ij}. Let $\nu_{ij} \geq 0$ be the Lagrange multipliers on the local feasibility constraints $b_{ij} - \chi_{ij}a_{ij} \geq 0$. The first order

[81] To see that $\tau_{i,ij}(m) = 0$ constitutes an equilibrium policy for $j \neq i$, suppose that $\tau_{j,ij}(m)$ is set to clear the first-order condition. Then, the first order condition of country planner i for $a_{ij}(m)$ is satisfied with equality, and so we must have $\nu_{ij} = 0$, so that there is no value to country planner i of relaxing the local constraints in country j at the equilibrium. As a result, the preferences of country planner i align with country i agents over actions in country j, and we have $\tau_{i,ij}(m) = 0$.

condition for an action m is

$$0 = \omega_i \frac{\partial U_i}{\partial u_i}\frac{\partial u_{ij}}{\partial a_{ij}(m)} + \omega_i \frac{\partial U_i}{\partial u_i^A}\frac{\partial u_i^A}{\partial a_{ij}(m)} + \Lambda_i \frac{\partial \Gamma_i}{\partial W_i}\left(-\tau_{i,ij}(m) - \tau_{j,ij}(m)\right) + \Lambda_i \frac{\partial \Gamma_i}{\partial \phi_i}\frac{\partial \phi_{ij}}{\partial a_{ij}(m)} + \Lambda_i \frac{\partial \Gamma_i}{\partial \phi_i^A}\frac{\partial \phi_i^A}{\partial a_{ij}(m)} - \nu_{ij}\chi_{ij}(m)$$

So that rearranging, we obtain

$$\lambda_i^0 \tau_{j,ij}(m) + \nu_{ij}\chi_{ij}(m) = -\lambda_i^0 \tau_{i,ij}(m) + \omega_i \frac{\partial U_i}{\partial u_i}\frac{\partial u_{ij}}{\partial a_{ij}(m)} + \omega_i \frac{\partial U_i}{\partial u_i^A}\frac{\partial u_i^A}{\partial a_{ij}(m)} + \Lambda_i \frac{\partial \Gamma_i}{\partial \phi_i}\frac{\partial \phi_{ij}}{\partial a_{ij}(m)} + \Lambda_i \frac{\partial \Gamma_i}{\partial \phi_i^A}\frac{\partial \phi_i^A}{\partial a_{ij}(m)}.$$

Notice that the left-hand side is constant for a given allocation, and is the same formula as in Lemma 11. Denote it to be $\lambda_i^0 \tau_{j,ij}^*(m)$, so that we have $\tau_{j,ij}(m) = \tau_{j,ij}^*(m)$ if $\nu_{ij} = 0$. Given corner solutions, there may be multiple vectors of tax rates that implement this allocation. We can express the problem of country planner j therefore maximizing tax revenue collected while implementing the same allocation, that is

$$\max_{\nu_{ij}, \tau_{j,ij}} \tau_{j,ij} a_{ij}^* \quad \text{s.t.} \quad \lambda_i^0 \tau_{j,ij}(m) + \nu_{ij}\chi_{ij}(m) = \lambda_i^0 \tau_{j,ij}^*(m), \quad \nu_{ij}\left(b_{ij} - \chi_{ij} a_{ij}^*\right) = 0$$

where the second constraint is complementary slackness. Substituting in for $\tau_{j,ij}$ and substituting in the complementary slackness condition, we obtain

$$\max_{\nu_{ij} \geq 0} \tau_{j,ij}^* a_{ij}^* - \frac{1}{\lambda_i^0}\nu_{ij} b_{ij}$$

Because $b_{ij} \geq 0$, revenue collection is maximized at $\nu_{ij} = 0$, so that we have $\tau_{j,ij} = \tau_{j,ij}^*$. As a result, the implementability conditions of Lemma 11 hold.

E.3 Heterogeneous Agents

We extend the model of Section 6 by allowing for agents agents within a country. Suppose that in each country, there are $\mathcal{K} = \{1, ..., K\}$ agents, who differ in their utility functions and constraint sets, whom we index i_k. Some agents may not be able to conduct cross-border activities, in which case foreign actions would not appear in their utility function or constraint sets. Agents of type i_k have relative mass μ_{i_k} and are assigned a social welfare weight ω_{i_k}.

It is easy to see that we can treat the problem as if there were a single representative agent in country i. In particular, define $a_i = \{a_{i_k}\}_{k\in K}$, $U_i = \sum_k \mu_{i_k}\omega_{i_k}U_{i_k}$, and $\Gamma_i = (\Gamma_{i_1}, ..., \Gamma_{i_K})$. The problem is as-if we have a single representative agent who solves

$$\max_{a_i} U_i \quad \text{s.t.} \quad \Gamma_i \geq 0,$$

since this decision problem is fully separable in a_{i_k} and yields the optimality conditions of each agent type. The only difference relative to Section 6 is that there are K different measures of wealth, W_{i_k}. Domestic lump sum transfers imply that $\lambda^0_{i_k} = \lambda^0_i$ is independent of k, and the characterization of optimal policy follows as in Section 6.

E.4 Nonlinear Aggregates

In Section 6, we assumed that aggregates are linear, that is $a^A_i(m) = a_{ii}(m) + \int_j a_{ij}(m)dj$. The welfare-relevant aggregates may not necessarily be linear. We can represent this by $a^A_i(m) = z_{ii}(a_{ii}, m) + \int_j z_{ji}(a_{ji}, m)dj$ for some functions z. The key change in the model is that we now have spillover effects that depend on the identity of the country investing, as in the bailouts model. The optimality of non-cooperative Pigouvian taxation follows from the same steps and logic as the baseline model. This clarifies once again that the homogeneity property of Assumption 10 applies to allocations, not to aggregates.

The possibility for non-linear aggregation helps to generalize the results to settings where regulation is set at an initial date, but the economy is not regulated thereafter. This is the case in the bailouts model when we assume away bailouts, where planners set policies at date 0 but then do not intervene at date 1.

E.5 General Government Actions

We extend the model to feature more general government actions, for example bailouts as in Section 5. In particular, country planner i can take actions $g_{i,jk}(m) \geq 0$ (for either $i = j$ or $i = k$), which affect country j agents in the same way as action m in country k. As such, we can define the total domestic action of agent i as

$$\bar{a}_{ii}(m) = a_{ii}(m) + g_{i,ii}(m)$$

and the total foreign action of agent i as

$$\bar{a}_{ij}(m) = a_{ij}(m) + g_{i,ij}(m) + g_{j,ij}(m).$$

This classification allows for a rich set of both agent and government actions. For example, a domestic action m that can only be taken by the government, such as government debt issuance or a bailout, could feature a feasibility constraint $a_{ii}(m) = 0$. From here, the domestic aggregates are given by

$$a_i^A(m) = \bar{a}_{ii}(m) + \int_j \bar{a}_{ji}(m) dj.$$

The flow utility of the country i representative agent is now given by

$$\max_{a_i} U_i\left(u_i(a_i, g_i, \bar{a}_i), u_i^A(a_i, g_i, \bar{a}_i, a_i^A))\right) \quad \text{s.t.} \quad \Gamma_i\left(W_i, \phi_i(a_i, g_i, \bar{a}_i), \phi_i^A(a_i, g_i, \bar{a}_i, a^A)\right) \geq 0,$$

where we have $u_i(a_i, g_i, \bar{a}_i) = u_{ii}(a_{ii}, u_{ii}^g(g_{i,ii}) + \int_j u_{ij}^g(g_{i,ji})dj, \bar{a}_{ii}) + \int_j u_{ij}(a_{ij}, g_{i,ij}, \bar{a}_{ij})dj$ and so on. It simplifies exposition to include in Γ_i any government feasibility constraints, for example government budget constraints. Observe that such constraints would be assigned Lagrange multipliers of 0 by the representative agent, but not by the social planner.

From here, we begin by characterizing the globally efficient allocation. Observe first that the optimal wedges for private actions are still given by the equations in Proposition 9.

Proposition 28. *The globally efficient allocation can be decentralized by the wedges of Proposition 9. The globally efficient government actions $g_{i,jk}$ (for either $i = j$ or $i = k$) are given by*

$$-\underbrace{\frac{\partial \mathcal{L}_i}{\partial g_{i,jk}(m)}}_{\text{Country } i \text{ Cost}} \geq \underbrace{\frac{\partial \mathcal{L}_j}{\partial \bar{a}_{jk}(m)}}_{\text{Country } j \text{ Benefit}} + \underbrace{\frac{\partial \mathcal{L}_k}{\partial a_k^A(m)}}_{\text{Country } k \text{ Spillover}} + \underbrace{\int_{i'} \frac{\partial \mathcal{L}_{i'}}{\partial a_k^A(m)} di'}_{\text{Foreign Spillovers}} \tag{39}$$

where $\frac{\partial \mathcal{L}_i}{\partial g_{i,jk}(m)} = \omega_i \frac{\partial U_i}{d g_{jk}(m)} + \Lambda_i \frac{\partial \Gamma_i}{\partial g_{jk}(m)}$ *and so on.*

Proof. The proof of the decentralizing wedges follows as in the proof of Proposition 9. The government action rules follow directly from the derivatives of the global Lagrangian. ∎

The globally efficient allocation of government actions is a generalization of the optimal bailout rule of Proposition 8, with analogous intuition. Note that for $j \neq i$, we have an action smoothing result: $\frac{\partial \mathcal{L}_i}{\partial g_{i,ji}(m)} = \frac{\partial \mathcal{L}_j}{\partial g_{j,ji}(m)}$, that is the marginal cost of providing the action is smoothed across countries. For example, this corresponds to bailout sharing.

From here, the non-cooperative results on quantity regulation follow as in the baseline model and bailouts section. Taking either $i = j$ or $i = k$, the neglected terms are always the terms that affect other countries, namely the foreign spillovers and either the spillover ($i = j$) or the benefit ($i = k$). For domestic actions, there are neglected foreign spillovers, while for domestic actions on foreign agents there is unequal treatment when the cost of providing the action is held fixed.

On the other hand, suppose that choices of foreign government actions $g_{i,ij}$ and $g_{i,ji}$ are delegated to agents, but can be taxed.[82] Once this is imposed and governments use Pigouvian taxation, these foreign government actions are no different from regular actions from a technical perspective,[83] and the efficiency of Pigouvian taxation is restored.

E.6 Preference Misalignment

We now suppose that there is a difference in preferences between country planners and multinational agents, that is country planners have a utility function $V_i(v_i(a_i), v_i^A(a_i, a^A))$. For example, preference differences may arise due to paternalism, control by special interest groups, or corruption. For simplicity, we incorporate the welfare weights into the planner utility function.

We define efficient policies with respect to those of country planners. This is a natural efficiency benchmark, as country planners agree to cooperative agreements.[84] Under this definition, globally efficient policy can be characterized as follows.

Proposition 29. *The globally efficient wedges are given by*

$$\tau_{ji}(m) = -\Delta_{ji}(m) - \Omega^v_{i,i}(m) - \int_{i'} \Omega^v_{i',i}(m) di' \tag{40}$$

where we have

$$\Delta_{ji}(m) = \frac{1}{\lambda_j^0} \left[\frac{\partial V_j}{\partial v_j} \frac{\partial v_{ji}}{\partial a_{ji}(m)} - \frac{\partial U_j}{\partial u_j} \frac{\partial u_{ji}}{\partial a_{ji}(m)} \right]$$

and where $\Omega^v_{i,j}$ are defined analogously to $\Omega_{i,j}$, but with the planner utility functions.

Proof. The proof follows as usual by writing country social welfare as $U_i + (V_i - U_i)$ and comparing the planner and agent first order conditions. ∎

Globally efficient policy accounts for spillovers onto the welfare of country planners in a standard way. However, it also must correct for the difference in preferences, yielding the first term $\Delta_{ji}(m)$.

[82] Notice that $g_{i,ij}$ is delegated to country i agents and $g_{i,ji}$ to country j agents.
[83] Excepting that there is a non-linear aggregate arising from u^g_{ii}, which is covered above.
[84] See Korinek (2017) for the same argument.

From here, characterization of optimal quantity regulation follows as in Section 6, except with the spillovers defined above. Regulation of domestic agents accounts for both the preference difference and spillovers to country planner welfare, but does not account for spillovers to foreign planners. Regulation of foreign agents allows them to conduct activities only to the point that it increases domestic planner welfare. The result is uninternalized spillovers and unequal treatment.

The result for Pigouvian taxation is more subtle. Considering tax revenue collections with no monopolist distortion, we have the tax revenue collection $\tau_{ji}(m)a_{ji}(m)$. Note first that differentiating in $a_{ji}(m)$, we obtain the total revenue impact (assuming no monopoly rents)

$$\tau_{ji}(m) + \int_{i'} \frac{\partial \tau_{i'i}}{\partial a_i^A(m)} a_{i'i}(m) = \tau_{ji}(m) + \int_{i'} \Omega_{i',i}(m) di'$$

where we note that $\tau_{i,ji}(m)$ is now the benefit to the foreign agent net of the wedge placed by the foreign planner, which unwinds the preference difference. This results in the difference $\Delta_{ji}(m)$ being correctly accounted for. However, the spillovers defined above are the spillovers to the agent, not the planner. This implies setting correct policy requires $\Omega^v_{j,i} = \Omega_{j,i}$ when $j \neq i$. The simplest way for this requirement to hold is if spillovers onto foreign agents are limited to constraint set spillovers, for example the fire sales of the baseline model.

Finally, it should be noted that these results imply that country planners can achieve the cooperative outcome using Pigouvian taxation. However, this section does not address whether the cooperative outcome is superior to the non-cooperative outcome. This latter claim requires a normative stand on whether the preferences of the planner or the agent are the normatively legitimate preferences, which depends on the source of preference difference. Although interesting for future work, such analysis is beyond the scope of this paper.

E.7 A Finite Country Game

We now consider a game with a finite number of countries, and show that the optimality of Pigouvian taxation is obtained up to a set of new external reoptimization effects. Provided that these external reoptimization effects are negligible, for example in the limit with a large number of countries, the results of the paper are obtained.

Suppose that rather than a continuum of countries, we have a finite set $I = \{1, ...I\}$ of countries, each of measure $\frac{1}{I}$. To simplify exposition, we assume that there is a single action $M = \{m\}$ and rule out constraint sets. As a result, we write

$$\max_{a_i} U_i \left(u_i(a_i, I), u_i^A(a_i, a^A, I), W_i \right)$$

where we have

$$u_i(a_i, I) = \sum_{j \in I} u_{ij}(a_{ij}, I)$$

$$a_i^A = \frac{1}{I} \sum_{j \in I} a_{ij}$$

and so on. We use the functional dependency on I to capture scaling as we take the limit $I \to +\infty$, which will allow for home bias and marginal foreign investment.

The following Proposition characterizes the equilibrium under non-cooperative Pigouvian taxation. For expositional purposes, we focus on the domestic tax rate $\tau_{i,ii}$.

Proposition 30. *Suppose that Assumption 10 holds. In the finite country game, the non-cooperative equilibrium under Pigouvian taxation has the following tax rate on the domestic activity of domestic agents*

$$\tau_{i,ii} = \underbrace{-\frac{1}{I} \sum_j \Omega_{j,i}}_{\text{Total Spillovers}} \underbrace{- \frac{1}{I} \frac{1}{\lambda_i^0} \sum_{j \neq i} \sum_k \mu_{i,jk} \left[\overbrace{\frac{d}{dW_j} \left[\frac{1}{\lambda_j^0} \frac{dU_j}{da_{jk}} \right] \Omega_{j,i}}^{\text{“Wealth Effect”}} + \overbrace{\frac{d}{du_{ji}^A} \left[\frac{1}{\lambda_j^0} \frac{dU_j}{da_{jk}} \right] \frac{du_{ji}^A}{da_i^A}}^{\text{“Price Effect”}} \right]}_{\text{External Reoptimization Effects}}.$$

where $\mu_{i,jk}$ is a Lagrange multiplier defined in the proof.

In the finite country game, the intuition behind the internalization of foreign spillovers is the same as the baseline model. However, there is also an additional set of *external reoptimization effects* that arise due to global monopoly power: the *entire contracts* of foreign agents are affected to first order by changes in domestic activities and aggregates, including allocations and aggregates outside the domestic economy.

These external reoptimization effects consist of two effects. The “Wealth Effect” arises because taxes on foreign agents reduce their wealth level, impacting their preferences over their entire contract. The “Price Effect” arises because a change in the domestic aggregate affects the benefit foreign agents get from activities, which in turn affects their entire contract. These additional forces amount to an additional form of monopolist distortion. When these monopolist distortions disappear, efficiency is restored.

In the baseline model, we have taken a continuous limit, where the marginal presence in foreign countries implies that the wealth effects and price effects are negligible.

Notice that if we characterized the tax rate $\tau_{i,ij}$ on foreign activities of agents, it would now account for the fact that agents' contribution to the foreign aggregate spills back to domestic agents. This would result in a form of excessive taxation, because the domestic planner is also taxing this externality. This term would disappear in limit, as the contribution to the foreign aggregate becomes negligible, so that excessive taxation disappears in limit, as in the baseline model.

E.7.1 Proof of Proposition 30

Given this setup, the demand functions of the country i multinational agent are given by the system of equations

$$\tau_{i,ii} = \frac{1}{\lambda_i^0}\frac{dU_i}{da_{ii}}$$

$$\tau_{i,ij} + \tau_{j,ij} = \frac{1}{\lambda_i^0}\frac{dU_i}{da_{ij}}$$

where we have defined $\lambda_i^0 = \frac{\partial U_i}{\partial W_i}$ and $\frac{dU_i}{da_{ij}} = \frac{\partial U_i}{\partial u_i}\frac{\partial u_{ij}}{\partial a_{ij}} + \frac{\partial U_i}{\partial u_i^A}\frac{\partial u_{ij}^A}{\partial a_{ij}}$.

Now, consider the optimization problem of country planner i, which is given by

$$\max_{a,\tau_i} U_i\left(u_i(a_i, I), u_i^A(a_i, a^A, I), A_i + \sum_{j\neq i}\left[\tau_{i,ji}a_{ji} - \tau_{j,ij}a_{ij}\right]\right)$$

subject to the above implementability conditions in all countries, taking as given τ_{-i}. Notice that tax collections do not need to be scaled by $\frac{1}{I}$ since countries have equal measure. We write the Lagrangian as

$$\mathcal{L}_i = U_i\left(u_i(a_i, I), u_i^A(a_i, a^A, I), A_i + \sum_{j\neq i}\left[\tau_{i,ji}a_{ji} - \tau_{j,ij}a_{ij}\right]\right) + \sum_{j,k}\mu_{i,jk}\text{FOC}_{jk}$$

where $\mu_{i,jk}$ is the Lagrange multiplier on the FOC of agent j for its action in country k.

From here, note that we have $\mu_{i,ik} = 0$, given the complete set of controls on domestic agents. Moreover, the FOC for the tax on foreign agents $\tau_{i,ji}$ is given by

$$\begin{aligned} 0 &= \lambda_i^0 a_{ji} - \mu_{i,ji} + \sum_k \mu_{i,jk} \frac{d\text{FOC}_{jk}}{dW_j} a_{ji} \\ &= \lambda_i^0 a_{ji} - \mu_{i,ji} + \sum_k \mu_{i,jk} \frac{d}{dW_j} \left[\frac{1}{\lambda_j^0} \frac{dU_j}{da_{jk}} \right] a_{ji} \end{aligned}$$

where the final term reflects wealth effects on foreign multinational agent k.

From here, let us take the FOC in the domestic action a_{ii}. We have

$$0 = \frac{dU_i}{da_{ii}} + \frac{1}{I} \frac{dU_i}{da_i^A} + \frac{1}{I} \sum_{j \neq i} \sum_k \mu_{i,jk} \frac{d\text{FOC}_{jk}}{da_i^A}.$$

Taking the derivatives, substituting in for $\mu_{i,ji}$, and applying Assumption 10, we obtain

$$0 = \frac{dU_i}{da_{ii}} + \frac{1}{I} \frac{dU_i}{da_i^A} + \frac{1}{I} \sum_{j \neq i} \left[\lambda_i^0 + \sum_k \mu_{i,jk} \frac{d}{dW_j} \left[\frac{1}{\lambda_j^0} \frac{dU_j}{da_{jk}} \right] \right] \frac{1}{\lambda_j^0} \frac{dU_j}{da_i^A} + \frac{1}{I} \sum_{j \neq i} \sum_k \mu_{i,jk} \frac{d}{du_{ji}^A} \left[\frac{1}{\lambda_j^0} \frac{dU_j}{da_{jk}} \right] \frac{du_{ji}^A}{da_i^A}$$

and finally, substituting in the tax rate,

$$\tau_{i,ii} = \underbrace{-\frac{1}{I} \sum_j \Omega_{j,i}}_{\text{Total Spillovers}} \underbrace{- \frac{1}{I} \frac{1}{\lambda_i^0} \sum_{j \neq i} \sum_k \mu_{i,jk} \left[\frac{d}{dW_j} \left[\frac{1}{\lambda_j^0} \frac{dU_j}{da_{jk}} \right] \Omega_{j,i} + \frac{d}{du_{ji}^A} \left[\frac{1}{\lambda_j^0} \frac{dU_j}{da_{jk}} \right] \frac{du_{ji}^A}{da_i^A} \right]}_{\text{External Reoptimization Effects}}.$$