Crisis Interventions in Corporate Insolvency

Samuel Antill* Christopher Clayton†

February 2021

Preliminary and Incomplete.

Abstract

We model the optimal resolution of insolvent firms in general equilibrium. Absent externalities, the optimal corporate-insolvency system encourages lending by letting banks assign liquidations ex-post. We show that a social planner optimally intervenes in such a system during a crisis because of two pecuniary externalities. First, liquidation waves create fire-sale discounts, motivating a subsidy for liquidation-preventing loans to insolvent firms. However, a loan-price externality arises when collateral-constrained banks allocate scarce capital to averting liquidations rather than bolstering healthier firms, motivating a subsidy for liquidating insolvent firms. Efficient intervention can thus encourage or discourage liquidation, depending on the crisis. Our model sheds light on recent crisis-motivated policy proposals.

*Antill: Harvard Business School. Email: santill@hbs.edu

†Clayton: Yale School of Management. Email: christopher.clayton@yale.edu.

We are grateful to Darrell Duffie, Sam Hanson, Jeremy Stein, and Adi Sunderam for helpful comments and suggestions.

1 Introduction

The COVID-19 pandemic has motivated a wave of proposals for government interventions into the process for resolving insolvent firms.[1] Most proposals aim to preserve firms that would otherwise be liquidated, begging the question of why the US legal system does not account for the social costs of liquidation to preempt such interventions. Using a general equilibrium model, we show that even an optimally designed system for resolving insolvent firms can be improved upon by crisis-specific interventions. Moreover, crisis interventions aimed at avoiding liquidations can be beneficial in one crisis and harmful in another, even if firm cashflows are identical in the two crises.

We study policies to deal with insolvent firms. In our three-period model, banks extend loans to firms via standard short-term debt contracts. At date one, an aggregate state realizes (e.g. normal times or crisis), and each firm realizes two idiosyncratic shocks: one determining a temporary cash-flow need and another determining a long-run value. Firms with high long-run value are able to roll over debt by obtaining a second loan from banks, at an endogenous loan price. The loan price depends on whether banks face binding collateral constraints. Firms with low long-run value become insolvent – the combination of their outstanding debt and cash-flow needs exceed their long-run values – and bear a loss on their assets, for example due to costs of a bankruptcy process. At date one, insolvent firms can be resolved in one of two ways. First, insolvent firms can be liquidated. Second, a firm can be restructured, with debtholders accepting a haircut, allowing the firm to continue operating.[2] We use the term "continuation" to refer to such a restructuring, which could be conducted out of court or in a Chapter 11 bankruptcy. Liquidation may be valuable relative to continuation for firms with sufficiently low long-run value. Insolvency procedures

[1] To paraphrase Greenwood, Iverson, and Thesmar (2020): (i) Hanson, Stein, Sunderam, and Zwick (2020b) recommend keeping firms solvent by funding fixed obligations like rent; (ii) Saez and Zucman (2020) recommend keeping firms solvent by funding all expenses; (iii) Brunnermeier and Krishnamurthy (2020) recommend subsidizing refinancing for small firms; (iv) Greenwood and Thesmar (2020) recommend extending a tax credit to claimants (i.e., landlords) that accept a haircut on loan obligations; (v) Iverson, Ellias, and Roe (2020) recommend hiring additional bankruptcy judges; (vi) Skeel (2020) recommend creating a standard "prepacked" restructuring process; (vii) Blanchard et al. (2020) recommend the government accept larger losses than creditors in reorganizations; (viii) DeMarzo, Krishnamurthy, and Rauh (2020) recommend a government-funded vehicle extend debtor-in-possession financing; (ix) the Bankruptcy and COVID-19 Working Group recommends extending deadlines for small businesses in Chapter 11.

[2] In our baseline model, a debt haircut and debt-equity conversion are equivalent.

are characterized by a *liquidation rule*, designating which insolvent firms face liquidation and which insolvent firms face continuation.

We begin in Section 3 by studying the private optimum when firms are able to commit to a liquidation rule in insolvency at date zero, when initially signing contracts. We show that the privately optimal liquidation rule is a threshold rule, whereby all firms below a threshold long-run value are liquidated while firms above the threshold are restructured. Moreover, this threshold rule is the *creditor-recovery-maximizing* liquidation rule, which maximizes the *ex-post* recovery value to the firms' creditors. This implies that the threshold is exactly the long-run firm value at which recovery to creditors from liquidation and continuation are equated. Intuitively, by maximizing ex-post recovery to creditors in insolvency, firms also maximize their own value from an investment perspective. As a result, the ex-ante preferences of firms and ex-post preferences of banks with respect to the liquidation rule are aligned.

However, two pecuniary externalities arise in our model from the liquidation-continuation decision. The first is a fire-sale externality: the equilibrium liquidation price may decline in the number of liquidations (Shleifer and Vishny, 1992). The second is a loan-price externality: when banks face a binding collateral constraint, continuations reduce the supply of loanable funds, affecting loan prices for all firms. This hinders the ability of firms to roll over debt and increases the number of insolvent firms. Private agents do not internalize their impact on either the date-one liquidation or loan price, leading to a potential role for intervention by a social planner to account for the pecuniary externalities.

We next study the social optimum, in which the planner not only manages the debt level of firms but also commits to a liquidation rule for insolvent firms. The planner internalizes the impact of her choices on prices, but must otherwise respect the constraints of private agents. We show that the socially optimal liquidation rule for insolvent firms differs from the privately optimal (creditor-recovery maximizing) rule in two key ways. First, the fire-sale externality incentivizes the social planner to avoid liquidations, relative to the privately optimal rule, in order to mitigate the fire sale. The result and intuition are closely in line to standard fire-sale externality problems.

On the other hand, the loan-price externality, arising when banks face binding collateral constraints, can lead the social planner to adopt a liquidation rule that implements *more* liquidations than the private optimum. Intuitively, when a firm with low long-run value continues rather than liquidates, banks' resources are tied up in the firm and cannot be lent out. By liquidating a low-value firm, bank resources are freed up and the supply of loanable funds increases, allowing firms to borrow at cheaper rates. This enhances the ability of high-value firms to roll over their debt, pushing more firms out of the insolvency region and avoiding insolvency costs. This in turn enhances bank solvency by increasing their recovery value from high-value firms. As a result, the social planner may find it optimal to sacrifice low-value firms in order to preserve high-value firms.

We further show that the social optimum can be implemented with a simple tax or subsidy scheme, even without knowledge by the planner of the long-term value of any individual firm. In particular, if the social optimum promotes more (fewer) liquidations of firms than the private optimum, then the planner can implement this optimum by providing a fixed subsidy (tax) of liquidations of insolvent firms, or equivalently by a fixed tax (subsidy) of continuations of insolvent firms. Under this fixed tax or subsidy, private agents are incentivized to implement the socially optimal threshold rule whether they choose it ex ante or ex post. Intuitively, because both the private and social optima consist of threshold rules, a social planner only has to move the threshold. Accordingly, the social planner calibrates a fixed tax or subsidy to move the threshold to the desired level.

Next, in Section 4 we study the implication of multiple (heterogeneous) creditors for the optimal seniority structure and liquidation rule. We show that the private optimum again implements the creditor-recovery-maximizing liquidation rule, and moreover implements a seniority structure that allocates proceeds to banks with the highest marginal value of wealth in any given aggregate state. This seniority structure tends to prioritize banks that face low collateral haircuts, who have the greatest ability to lend out additional funds in that state.

By contrast, a social planner jointly designing the liquidation rule and seniority structure not only alters the liquidation rule, but also the seniority structure. The socially optimal seniority structure trades off the private marginal value of wealth to banks against the social value of loanable funds, and prioritizes banks with greater lending capabilities when loanable funds are valuable (e.g. due to binding collateral constraints). However, the optimal seniority structure also downweights the magnitude of the loan-price externality in the liquidation rule. Intuitively, by allocating loanable funds to banks with the greatest lending capabilities, the planner increases the supply of loanable funds. This reduces the need to increase loanable funds by liquidating low-value firms, and so reduces planner intervention in the liquidation rule. As a result, planner intervention in the seniority structure of debt provides a partial substitute for intervention in the liquidation rule in insolvency.

Finally, in Section 5 we discuss our results in the context of various proposals for interventions in firm insolvency that have arisen in the wake of the COVID-19 pandemic.

Related Literature. This paper contributes to the theoretical literature studying the socially optimal resolution of insolvent firms. In early seminal work, Shleifer and Vishny (1992) show that fire-sale externalities create a motive for social planners to avoid liquidations. Corbae and D'Erasmo (forthcoming) estimate a general-equilibrium model with both reorganization and liquidation in bankruptcy, but do not consider fire-sale externalities or collateral constraints. Hanson, Stein, Sunderman, and Zwick (2020a) model an economy in which extending credit to otherwise insolvent firms helps to mitigate aggregate demand externalities. In their model, a social planner would optimally rescue some firms that the private sector would deem nonviable in order to preserve option value: nonviable firms support aggregate demand, a public good, if the economy recovers quickly.

In recent work, Donaldson, Morrison, Piacentino, and Yu (2020) develop an elegant model of the complementarities between bankruptcy and out-of-court restructurings. Like our paper, they use a model to analyze recent proposed interventions into the resolution of insolvent firms. We contribute by modeling how a liquidation of one firm can impose externalities on unrelated insolvent firms. We thus complement Donaldson et al. (2020) by analyzing recent policy proposals in light of different market failures.

We also relate to a substantial literature on optimal interventions for financial intermediaries, including macroprudential regulation, bailout recapitalizations, and orderly resolution ("bail-ins").[3] Whereas this literature principally studies interventions in financial intermediaries, our focus is on insolvency interventions in non-financial corporates who contract with intermediaries.

Finally, this paper relates to the literature on zombie loans: subsidized bank loans to insolvent firms.[4] Prior work studies models in which banks impose a negative externality on

[3]For example, see Bianchi (2011), Bianchi and Mendoza (2018), Caballero and Krishnamurthy (2001), Chari and Kehoe (2016), Clayton and Schaab (2020), Dávila and Korinek (2018), Farhi and Tirole (2012), Lorenzoni (2008), and Stein (2012).

[4]For empirical evidence of zombie lending and the economic impact of zombie loans, see Caballero, Hoshi, and Kashyap (2008); Acharya, Eisert, Eufinger, and Hirsch (2019); Blattner, Farinha, and Rebelo (2019);

other agents by preserving firms through zombie loans. Caballero, Hoshi, and Kashyap (2008) and Acharya, Crosignani, Eisert, and Eufinger (2020) show theoretically and empirically that, by keeping insolvent firms alive to compete in product markets, zombie loans lead to lower product prices and markups, reducing entry and productivity. We contribute to this literature by modeling a tradeoff between two externalities: one similar externality associated with a misallocation of credit due to socially suboptimal firm preservation, and an opposing externality associated with inefficient liquidation.[5]

2 Model

There are three dates, $t = 0, 1, 2$. The economy consists of a unit continuum of ex-ante identical firms, a banking sector, and a representative arbitrageur. We model the banking sector using a representative bank, although for brevity we refer to the representative bank as "banks." Firms obtain funding from the representative bank to finance investment. Arbitrageurs purchase investment projects that are liquidated prior to maturity. Firms and banks are risk neutral and do not discount the future.

We model aggregate uncertainty with an aggregate state that takes the value $s \in S$ with probability $f(s)$. We also model idiosyncratic firm-specific uncertainty, described below. All uncertainty in the model is resolved at date one.

2.1 Firms

At date zero, ex-ante identical firms have initial assets $A_0^f > 0$. Firms use their initial assets and also raise financing from banks via standard short-term debt contracts, with face value denoted by D_0, in order to finance an investment project.[6] A project of scale I requires $\Phi(I)$ dollars of funding, where $\Phi(\cdot)$ is an exogenous increasing function. The initial financing (budget) constraint of firms is

$$c_0^f + \Phi(I) \leq A_0^f + q_0 D_0 \tag{1}$$

Acharya, Borchert, Jager, and Steffen (2021) and Acharya, Crosignani, Eisert, and Eufinger (2020).

[5] See Antill (2020b) and Ayotte and Morrison (2009) for empirical evidence of inefficient liquidations and a review of the empirical literature on inefficient liquidations.

[6] We could obtain similar results with more general liabilities under incomplete markets.

where $q_0 D_0$ is total financing obtained from banks, and where c_0^f is date 0 firm consumption. The total value of financing $q_0 D_0$ will be pinned down by a bank participation constraint, defined below.

At date one, the aggregate state is realized. In addition, each firm realizes an idiosyncratic viability state $v \in V$ and an idiosyncratic date-one expense cash-flow shock $\epsilon \geq 0$. Each firm requires an idiosyncratic cash-flow infusion $\epsilon \geq 0$ per unit of project scale in order to reach maturity. If the firm reaches maturity, it receives a payout v per unit of project scale at date two. A firm that does not pay the expense ϵI is rendered non-viable and must be liquidated. We assume that ϵ and v are jointly distributed according to a continuous density $g(v, \epsilon | s)$.

At date one, firms have access to bank financing, which can be used to roll over outstanding debt as well as to pay the required date-one expense ϵ. In particular, in state s, a firm can borrow $q_1(s)$ dollars by promising to repay one dollar at date two. A firm is therefore solvent at date one if and only if

$$-\epsilon I + q_1(s) v I \geq D_0.$$

It follows that there is a threshold rule $v^*(\epsilon, s)$ so that a firm is solvent at date one in state s if and only if $v \geq v^*(\epsilon, s)$. Note that this threshold depends on $q_1(s)$, with $\frac{\partial v^*(\epsilon,s)}{\partial q_1(s)} < 0$ so that higher prices enhance solvency. A solvent firm funds its cash-flow need by issuing $\epsilon I / q_1(s)$ units of new debt, in addition to rolling over its outstanding debt.[7]

We let $\mathcal{R}_1(v, \epsilon, s) D_0$ denote the debt recovery value that a bank receives in state s from a loan to a firm with viability v and expense ϵ. For solvent firms, we have $\mathcal{R}_1(v, \epsilon, s) D_0 = D_0$.

2.2 Resolving insolvent firms

Insolvent firms with $v < v^*(\epsilon, s)$ enter an insolvency-resolution process that leads to one of two outcomes: liquidation or continuation. Liquidation entails a sale of all of the firm's assets. This could be achieved through either an out-of-court foreclosure and sale or through

[7] We have assumed incomplete markets not only on firm value v (e.g. due to asymmetric information), but also on the aggregate state s and temporary cost ϵ. Naturally, firms and banks in our model would wish to make debt contingent on (ϵ, s), even if they can't make it contingent on the private value v. Our qualitative results extend to the case where firms can write contingent debt contracts $D_0(\epsilon, s)$, since there is still a spectrum of insolvency outcomes based on the long-term value v.

a bankruptcy filing under Chapter 7 or Chapter 11 of the US bankruptcy code. Alternatively, in continuation, the firm's creditors agree to pay the firm's date-one expenses in exchange for the firm's future cashflows. Continuation could represent a "zombie loan" in which a bank lends to a firm at a subsidized rate, an out-of-court restructuring, or a reorganization under Chapter 11 of the US bankruptcy code.

In our model, a liquidation rule is defined by a probability $\rho(v, \epsilon, s) \in [0, 1]$ that a firm with viability v and cash-flow need ϵ is liquidated in state s. The probability of continuation is $1 - \rho(v, \epsilon, s)$.

In either liquidation or continuation, each insolvent firm experiences a proportional loss χ of its project scale, so that its scale in insolvency is $\tilde{I} = (1-\chi)I$. We interpret χ as reflecting the costs of financial distress. For example, a firm that enters bankruptcy incurs direct costs associated with the bankruptcy process.[8] Even a firm that does not enter bankruptcy likely incurs indirect financial distress costs associated with a loss of employees or customers.

In continuation, the bank covers the firm's date-one expenses $\epsilon\tilde{I}$. This could take the form of either a zombie loan, new financing provided as part of an out-of-court restructuring, or debtor-in-possession (DIP) financing in a Chapter 11 bankruptcy.[9] In exchange, the bank receives all of the equity in the firm.[10] Continuation payoffs are received at date two, leaving the bank with a date-two cash flow $v\tilde{I}$. The date-one value of this equity is $q_1(s)v\tilde{I}$, where $q_1(s)$ is the date-one market value of a date-two dollar. It will be convenient to define the date-one value of all equity from all continuations as

$$\mathcal{E}(s) \equiv q_1(s) \int_{v \leq v^*(\epsilon,s)} \big(1 - \rho(v, \epsilon, s) \big) \, v\tilde{I} dG(s), \tag{2}$$

where $G(s)$ is the joint probability law of v, ϵ conditional on state s.[11]

In summary, the payoff from resolving an insolvent firm with viability v through continu-

[8] Professional fees in large Chapter 11 cases are, on average, 1% to 6% of the estate value (LoPucki and Doherty, 2004). Trustees in Chapter 7 bankruptcy cases regularly receive 25% of the estate in fees (Antill, 2020a).

[9] Many bankrupt firms do not require DIP financing. We allow ϵ to be arbitrarily close to 0 to capture this case.

[10] Notice that this is equivalent to assuming that outstanding debt is converted to equity, and then new funding to cover the expense is raised from other banks (not necessarily the outstanding equity holders).

[11] Formally, for any s and any measurable $A \subset \mathbb{R}^2$, $G(A|s) \equiv \mathbb{P}\Big((v, \epsilon) \in A | s \Big)$, where $\mathbb{P}$ is the probability measure for the space over which our model is defined.

ation, in state s, is $\mathcal{R}_1(v,\epsilon,s)D_0 = (-\epsilon + q_1(s)v)\tilde{I}$. On the other hand, if a firm is liquidated, the arbitrageur buys the capital $\tilde{I}$ at price $\gamma(s)$ per unit, where $\gamma(s)$ is the endogenous equilibrium resale price of assets.[12] In this case, the bank receives a total date-one payout $\mathcal{R}_1(v,\epsilon,s)D_0 = \gamma(s)\tilde{I}$.

2.3 Banks

Banks are risk neutral and have initial wealth A_0^b. A bank has two options: it can consume its endowment immediately, or it can contract with a firm to provide a loan. Firms have full bargaining power in that relationship, so that there is a participation constraint

$$A_0^b \leq E[c^b],$$

where c^b is the date-two consumption value to the bank of the firm loan.[13] We interpret each bank as contracting simultaneously with a cross-section of firms, where A_0^b is both the total equity capital of the bank and the amount it allocates to each firm, so that by law of large numbers each bank receives the expected payoff across firms.[14]

If a bank contracts with a firm, the contract specifies a debt level D_0 and a payment $q_0 D_0$ to the firm. A bank may finance an amount $q_0 D_0 > A_0^b$ by raising debt B_0 from households at exogenous price $\underline{q}$, so that its debt level is $B_0 = \frac{q_0 D_0 - A_0^b}{\underline{q}}$.[15]

A bank that signs a contract with a firm receives its payout $\mathcal{R}_1(s)D_0$ from the firm and repays households B_0. The bank at date one then has a choice to lend out its funds again to firms. Solvent firms rolling over their debt demand bank funds at date one, with total (dollar-value) demand

$$Q^D(s) \equiv \int_{v \geq v^*(\epsilon,s)} \bigg(D_0 + \epsilon I \bigg) \, dG(s). \tag{3}$$

[12]We assume that $\gamma(s)$ is sufficiently low that solvent firms never find it optimal to liquidate assets rather than continue.

[13]We assume without loss of generality that banks forgo date-one consumption to instead lend again and consume at date two.

[14]This assumption is made to ensure that banks do not default on household debt.

[15]Because a bank contracts simultaneously with multiple firms, B_0 here is the debt issued for the loan in the specific firm. Because the bank does not default on household debt in aggregate and each individual firm is marginal, each firm contracting with the bank can treat its own contract with the bank as an expected value. This leads to a representative agent like representation.

To meet this demand, banks can raise debt $B_1(s)$ from households, such that the supply of bank funds is

$$Q^S(s) \equiv B_1(s) - B_0 + \int_{v \geq v^*(\epsilon,s)} D_0 dG(s) + \int_{v \leq v^*(\epsilon,s)} \left(-(1-\rho(v,\epsilon,s))\epsilon + \rho(v,\epsilon,s)\gamma(s) \right) \tilde{I} dG(s), \tag{4}$$

where $-(1-\rho(v,\epsilon,s))\epsilon$ accounts for funds that are unavailable because they are used for DIP financing of bankrupt firms. Moreover, note that equity from continuations does not directly contribute to either supply or demand at date one, because the associated cashflows are received at date two.[16] However, banks can pledge equity in reorganized firms to borrow at date one. We assume that banks face a collateral constraint

$$B_1(s) \leq \phi(s) \left(Q^S(s) + \mathcal{E}(s) \right), \tag{5}$$

where $1-\phi(s)$ is a state-dependent haircut on the market value of collateral. The representative bank's date-two cashflows arise from equity in reorganized firms ($\mathcal{E}(s)$) and funds invested at date one ($Q^S(s)$) in loans to firms. Substituting the collateral constraint, which can be taken to hold with equality without loss of generality,[17] into equation (4), we obtain total loan supply

$$\begin{aligned} Q^S(s) = \frac{1}{1-\phi(s)} \Bigg[& \phi(s)\mathcal{E}(s) - B_0 + \int_{v \geq v^*(\epsilon,s)} D_0 dG(s) \\ & + \int_{v \leq v^*(\epsilon,s)} \left(-(1-\rho(v,\epsilon,s))\epsilon + \rho(v,\epsilon,s)\gamma(s) \right) \tilde{I} dG(s) \Bigg]. \end{aligned} \tag{6}$$

The participation constraint of banks, which requires that banks are indifferent at date zero between consuming their initial wealth A_0^b and consuming their equilibrium date-two cashflow, is thus given by

[16] Likewise, the reorganized firm's capital structure can be determined at date two without affecting the date one equilibrium.

[17] Maximal leverage is strictly optimal when $q_1(s) < 1$ and weakly optimal when $q_1(s) = 1$.

$$
\begin{aligned}
A_0^b &\equiv \mathbb{E}^S\left[\frac{1}{q_1(s)}\Big(Q^S(s)+\mathcal{E}(s)\Big)-B_1\right] \\
&= \mathbb{E}^S\left[\frac{1-\phi(s)q_1(s)}{q_1(s)}\ \Big(\ Q^S(s)+\mathcal{E}(s)\ \Big)\right].
\end{aligned} \tag{7}
$$

This participation constraint is taken as a constraint by firms in their decision problem. It is immediate that the participation constraint always binds.

2.4 Loan Market Clearing

The equilibrium loan price $q_1(s)$ is set to ensure that the date-one loan-market clearing constraint is satisfied, that is

$$Q^S(s) - Q^D(s) \geq 0, \tag{8}$$

with inequality only if $q_1(s) = 1$.[18] Note that $q_1(s)$ affects $Q^D(s)$ and $Q^S(s)$ through the solvency threshold $v^*(\epsilon, s)$, and affects $\mathcal{E}(s)$ through both $v^*(\epsilon, s)$ and the equity valuation.

2.5 Arbitrageurs

Arbitrageurs are second-best users with a technology $\mathcal{F}(L(s), s)$ that converts bank projects into the consumption good. As a result, the equilibrium liquidation price is given by

$$\gamma(s) = \gamma(L(s), s) \equiv \frac{\partial \mathcal{F}(L(s), s)}{\partial L(s)}, \qquad L(s) = \int_{v \leq v^*(\epsilon,s)} \rho(v, \epsilon, s)\tilde{I} dG(s), \tag{9}$$

so that there is a fire sale spillover whenever $\frac{\partial \mathcal{F}(L(s),s)}{\partial L(s)}$ decreases in $L(s)$.

At date zero, arbitrageurs have wealth A_0^a, but are borrowing constrained and cannot borrow against future income. Date-zero arbitrageur welfare is given by

$$u^a(A_0^a) + E\Big[\mathcal{F}(L(s), s) - \gamma(s)L(s)\Big], \tag{10}$$

where $u^{a\prime} > 1$ so that the borrowing constraint binds. The arbitrageur borrowing constraint

[18] Recall that by convention, we have defined $Q^S(s)$ to be loan supply under maximal leverage. If $q_1(s) = 1$, then loan market clearing may be obtained without maximal leverage.

is a simple method of eliminating arbitrageur surplus at date one from welfare considerations when the social planner assigns a welfare weight of zero, which simplifies analysis. However, given the bank collateral constraint, we could obtain similar welfare results without the borrowing constraint and welfare weight of zero.

Although we use the terminology "liquidation," in our model sales of firms to arbitrageurs may not involve liquidation in the sense of selling assets and closing the firm. Instead, we can think about it as selling distressed loans to arbitrageurs (e.g. non-banks), who may not have the expertise of banks in dealing with an insolvent firm. An alternative interpretation is that liquidation corresponds to a transfer of stakes and control rights from banks to less efficient arbitrageurs. Moreover, the fire sale corresponds to the idea that as more banks sell their positions, they sell to increasingly inefficient arbitrageurs.

3 Optimal Liquidation Rules

In this section, we characterize the privately optimal liquidation rule ρ that would be set by firms and banks as part of their optimal contract. We compare it with the socially optimal liquidation rule that would be set by a social planner, who internalizes effects on both the liquidation prices γ and the loan prices q_1. We show that both the privately and socially optimal rules take the form of threshold rules: firms with high values v are reorganized with probability one, whereas firms with low values v are liquidated with probability one. However, the privately and socially optimal rules differ, and in particular show that the socially optimal rule may imply more liquidations or fewer liquidations than the privately optimal rule.

For the results that follow, it is helpful to define the *creditor-recovery-maximizing liquidation rule* as follows.

Definition 1. *The* creditor-recovery-maximizing liquidation rule *is a threshold rule for liquidation*

$$\rho(v, \epsilon, s) = \left\{ \begin{array}{ll} 0, & v \geq v^L(\epsilon, s) \\ 1, & v < v^L(\epsilon, s) \end{array} \right\}, \tag{11}$$

where the threshold $v^L(\epsilon, s)$ is given by

$$\underbrace{-\epsilon + q_1(s)v^L(\epsilon, s)}_{\text{Date 1 Continuation Value}} = \underbrace{\gamma(s)}_{\text{Date 1 Liquidation Value}}. \tag{12}$$

The creditor-recovery-maximizing liquidation rule is the rule that maximizes the discounted payoff to creditors in insolvency. In other words, it is the liquidation rule that would be adopted if banks decided at date one whether to reorganize or liquidate insolvent firms. The indifference point $v^L(\epsilon, s)$ is the point at which the date-one present value of reorganizing the firm and liquidating the firm are equated. At higher values of v, the present value of continuation is higher than liquidation, and firms are always reorganized. At lower values of v, the present value of liquidation is higher, and firms are always liquidated.

3.1 Privately Optimal Liquidation

We begin by studying the liquidation rule ρ that arises in a competitive equilibrium in which firms may determine ρ ex-ante through contracts. Formally, firms solve

$$\max_{D_0, I, c_0^f, \rho} u_0^f(c_0^f) + E\left[\int_{v \geq v^*(\epsilon,s)} \left[vI - \frac{1}{q_1(s)}\left[\epsilon I + D_0\right]\right] dG(s)\right]$$

subject to the date-zero budget constraint (1) and to the bank participation constraint (7). Recall that $\epsilon \geq 0$. Firms take equilibrium prices q_1 and γ as given. Note that in this environment, firms are choosing a liquidation rule at date zero.

The following result characterizes the privately optimal liquidation rule.

Proposition 1. *The privately optimal liquidation rule is the creditor-recovery-maximizing liquidation rule.*

Although the privately optimal liquidation rule is chosen at date zero by firms (debtors), it nevertheless coincides with the creditor-recovery-maximizing liquidation rule that would be chosen at date one by banks (creditors). The intuition is that higher recovery values to banks in insolvency relaxes the date-zero participation constraint, and allows firms to obtain more financing from banks ex ante. As a result, the optimal liquidation rule from the perspective of firms coincides with the optimal rule from the perspective of banks.

All else equal, in the competitive equilibrium, the liquidation threshold $v^L(\epsilon, s)$ decreases in $q_1(s)$ and increases in $\gamma(s)$. Higher loan prices $q_1(s)$ increase the date-one present value of continuation, and lead banks to reorganize more firms rather than liquidate them. Conversely, higher liquidation prices $\gamma(s)$ increase the recovery value from liquidating a firm, discouraging continuation.

3.2 Socially Optimal Liquidation

We next study the socially optimal liquidation rule that is designed by a social planner who internalizes the determination of equilibrium prices, but must otherwise respect the same constraints faced by private agents. For expositional simplicity, we assume the planner places a welfare weight of zero on arbitrageurs.

As a result, the constrained efficient planning problem is to maximize firm welfare subject to the firm date-zero budget constraint (1), the bank participation constraint (7), and also to the loan and liquidation market clearing conditions (8 and 9). The following proposition characterizes the socially optimal liquidation rule.

Proposition 2. *The socially optimal liquidation rule is a threshold rule, with*

$$q_1(s)v^L(\epsilon, s) = \left(1 + \underbrace{\frac{\mu(s)}{\beta(s)}}_{\text{Loan Price Externalities}}\right)\left(\gamma(s) + \epsilon - \underbrace{\left|\frac{\partial\gamma(L(s), s)}{\partial L(s)}\right| L(s)}_{\text{Liquidation Price Externalities}}\right),$$

where Λ and $\mu(s)$ are Lagrange multipliers on the bank participation constraint and the market clearing constraint in state s, respectively, and where by definition

$$\beta(s) \equiv \Lambda \frac{1 - \phi(s)q_1(s)}{(1 - \phi(s))q_1(s)} + \mu(s)\frac{\phi(s)}{1 - \phi(s)}.$$

The multiplier $\mu(s)$, when non-zero, is given by

$$\mu(s)\frac{\partial[Q^D(s)-Q^S(s)]}{\partial q_1(s)} = \underbrace{\frac{1-\Lambda}{q_1(s)^2}Q^D(s)}_{\text{Distributive Externality. } \leq 0} + \underbrace{\Lambda\frac{1-\phi(s)q_1(s)}{(1-\phi(s))q_1(s)}\int_{\epsilon\geq 0}\left|\frac{\partial v^*(\epsilon,s)}{\partial q_1(s)}\right|\chi\mathcal{V}^*(\epsilon,s)d\epsilon}_{\text{Solvency Externality. } \geq 0}$$
$$+\underbrace{\frac{\Lambda}{q_1(s)^2}\frac{\phi(s)(1-q_1(s))}{1-\phi(s)}\mathcal{E}(s)}_{\text{Revaluation Externality. } \geq 0}.$$

where $\mathcal{V}^*(\epsilon,s) = D_0 g\left(v^*(\epsilon,s),\epsilon|s\right)$ is the value of debt repaid to banks by the marginally solvent firm with expense ϵ.

Like the privately optimal liquidation rule, the socially optimal liquidation rule also takes the form of a threshold rule. However, the socially optimal threshold $v^L(\epsilon,s)$ differs from the privately optimal threshold in two important ways.

First, there is an externality arising from fire sales of firm assets. Absent externalities from the loan price $q_1(s)$, this pushes the threshold downward, and the socially optimal liquidation rule encourages more continuations than the privately optimal liquidation rule does.

Second, there is an externality arising from the loan price when banks are collateral constrained and hence $q_1(s) < 1$. Absent liquidation-price externalities, when the multiplier $\mu(s)$ is positive (negative), the socially optimal liquidation rule discourages (encourages) continuation relative to the privately optimal rule because there is social value to higher (lower) prices $q_1(s)$.

There are three core externalities underlying the determination of the welfare impact of an increase in $q_1(s)$, and hence the determination of $\mu(s)$. First, there is a distributive externality: a higher price redistributes resources at date one to firms and away from banks. This externality is *negative* from the firm perspective when repayment to banks is more valuable than repayment to firms due to the participation constraint, that is $\Lambda > 1$. It pushes for $\mu(s) < 0$ and a more continuation-friendly liquidation rule in order to reduce the equilibrium price and redistribute repayment towards banks. This externality naturally scales in the loan demand of solvent firms.

The second externality is a solvency externality: a higher price enhances the ability of firms to roll over debt at date one, and so increases the solvency threshold $v^*(\epsilon, s)$. This results in a positive externality for firms, and so contributes towards $\mu(s) > 0$ and a liquidation-friendly liquidation rule. The intuition is that by shutting more bankrupt firms, more bank resources are freed up for lending to solvent firms both because banks no longer need to cover the DIP loan ϵ and because they receive the immediate liquidation value $\gamma(s)$ at date one rather than the final value v at date two. As a result, liquidation increases the supply of loanable funds to solvent firms, and so increases the loan price and boosts firm solvency.

The final externality is a revaluation of bankrupt firms: the present value of bank claims on insolvent but reorganized firms rises with the price, increasing bank net worth and resulting in a positive externality.

From here, we obtain the following corollary.

Corollary 1. *Suppose that in state s, $\frac{\partial[\mathcal{Q}^D - \mathcal{Q}^S]}{\partial q_1(s)} > 0$ and $\frac{\partial \gamma(L(s), s)}{\partial L(s)} = 0$. Then, the socially optimal liquidation rule implies more liquidations than the privately optimal liquidation rule iff*

$$\left(1 - \frac{1}{\Lambda}\right) Q^D(s) \leq \frac{1 - \phi(s) q_1(s)}{(1 - \phi(s))} \int_{\epsilon \geq 0} \frac{1}{q_1(s)} \left| \frac{\partial v^*(\epsilon, s)}{\partial q_1(s)} \right| \chi \mathcal{V}^*(\epsilon, s) d\epsilon + \frac{\phi(s)(1 - q_1(s))}{1 - \phi(s)} \mathcal{E}(s).$$

Corollary 1 highlights three conditions under which the socially optimal liquidation rule may lead to more liquidations than the privately optimal rule would. The first is that loan demand of bankrupt firms or of the marginally solvent firm is high relative to demand by solvent firms. In this case, the revaluation and solvency externalities are large relative to the distributive externality. The second is cases where the solvency threshold is particularly sensitive to the price. In this case, the welfare cost of the solvency externality is large. The third is cases where the insolvency cost χ is large, in which case once again the solvency externality is large.

3.3 Implementation

We next turn to the question of how a social planner might implement the socially optimal liquidation rule in practice. In characterizing the socially optimal liquidation rule, we have assumed that the social planner has full information over the realization of (v, ϵ) at any given firm, and so can enforce the socially optimal rule. In practice, however, although the financing needs ϵ may be known, the insolvency process must uncover the long-term value of the firm, v, which may be known to the firm and its creditors but not necessarily to the planner. As such, we must look to implement the socially optimal rule under an incentive-compatible mechanism.

It turns out that there is a simple incentive-compatible implementation of the socially optimal liquidation rule using taxes or subsidies. We now characterize two possible implementations.

Proposition 3. *Suppose that ϵ and s are observable to the social planner, but v is not. The following two approaches implement the socially optimal liquidation rule of Proposition 2.*[19]

1. *The social planner levies a tax on continuation, with proceeds remitted lump sum to banks. The tax per unit of firm scale is*

$$\tau(\epsilon, s) = \frac{\mu(s)}{\beta(s)}\left(\gamma(s) + \epsilon\right) - \left(1 + \frac{\mu(s)}{\beta(s)}\right)\left|\frac{\partial \gamma(L(s), s)}{\partial L(s)}\right| L(s)$$

2. *The social planner provides a subsidy for liquidation, financed by a lump sum tax on banks. The subsidy per unit of firm scale is $\tau(\epsilon, s)$.*

Proposition 3 provides two possible implementations of the optimal mechanism, which do not require the social planner to directly observe long-run firm value v. The first implementation has the social planner tax continuation, charging a fixed tax at date 1 to any firm that reorganizes. The size of this fixed tax is calibrated so that the marginally indifferent firm under the socially optimal liquidation rule remains the marginally indifferent firm once the tax is charged. Moreover because the tax does not depend on v, it does not change the threshold property of the optimal liquidation rule, and leads private agents to implement the socially optimal rule. The second implementation accomplishes the same result with a subsidy on liquidation, rather than a tax on continuation.

Both approaches require a lump-sum levy on banks to finance the scheme. Under the first implementation, taxes are collected, so that the revenue is remitted lump sum back to

[19]For exposition, we adopt language suggestive of $\tau(\epsilon, s) > 0$. Proposition 3 also applies when $\tau(\epsilon, s) < 0$, except that continuations are subsidized (and a lump sum tax levied on banks) whereas liquidations are taxed (and proceeds remitted lump sum).

banks. Under the second implementation, providing a subsidy requires raising lump sum taxes on banks. In this sense, the first approach may be simpler to implement at date 1, as it implements a Pigouvian tax that is remitted lump sum to the agents it collects it from. Nevertheless, it does require implementing a tax on banks during times of crisis.

As a result, an alternate possible implementation is for the planner to finance a subsidy at date one but levying a tax on banks at date zero. This is a natural implementation that most closely respects the desire of a planner to transfer resources to banks ("bailouts") at date one. However, provided that such insurance is sufficiently costly, for example because it requires distortionary costs of raising money from taxpayers at date one, a social planner may only desire to partially correct the liquidation rule. In this case, a planner would partially subsidize liquidations or continuations, but would not be able to achieve the socially optimal rule.

Finally, it should be noted that even if private agents had the ability to contract privately for bailout revenues from taxpayers, at a price greater than one, private agents would generically fail to achieve efficient policies on two fronts. First, private agents would not set the correct liquidation rule, for the reasons outlined above. Second, private agents would not set the correct level of bailout transfers, not internalizing the loan and liquidation-price externalities. A social planner therefore not only alters the level of bailouts to banks, but also makes them contingent on the liquidation rule by providing subsidies based on the outcome of the insolvency process.

4 Heterogeneous Banks

In this section, we study the effect of multiple (heterogeneous) bank creditors facing different collateral constraints in the design of the liquidation rule as well as the optimal seniority structure among creditors. We show that a social planner can improve welfare by subordinating the claims of banks that, because of their idiosyncratic collateral constraints, struggle to lend against securities of bankrupt firms. Moreover, planner intervention in the seniority structure can serve as a partial substitute for intervention in liquidation decisions.

Because we explicitly model seniority, our leading application for this section is to a traditional bankruptcy process, such as Chapter 11, with changes in the liquidation rule reflecting different outcomes of the bankruptcy process.[20] Nevertheless, we preserve the notation and terminology of previous sections for consistency.

4.1 Novel assumptions in the heterogeneous-banks extension

We assume that there are N^B distinct banks, each of equal measure, indexed by $b = 1, 2, ..., N^b$. Bank b has initial wealth A_0^b. At date zero, bank b raises debt B_0^b from households at an exogenous price $\underline{q}^b$. For simplicity, we assume that each dollar that bank b lends is divided equally among all firms, ensuring there are multiple creditors for each firm. Bank b lends $q_0^b D_0^b$ to firms at date zero, where q_0^b is the endogenous price of debt owned by bank b. Although different firms will offer the same debt price to the same bank b, different bank types b may face different debt prices not only because they have (potentially) different participation constraints, but also because they may have endogenously different claim seniority.[21]

At date one, firms realize idiosyncratic shocks (v, ϵ) as in Section 2. Firms roll over their debt, leading to a total date-one demand for bank funds equal to

$$Q^D(s) = \int_{v \geq v^*(\epsilon, s)} \left(\epsilon I + \sum_{b=1}^{N^B} D_0^b \right) dG(s), \tag{13}$$

so that total firm demand is determined by total debt, $D_0 = \sum_{b=1}^{N^B} D_0^b$. As in Section 2.2, firms with idiosyncratic shocks (v, ϵ) are liquidated with probability $\rho(v, \epsilon, s)$ in state s. The cashflows associated with liquidation and continuation are identical to those described in Section 2.2. However, we now assume that the cashflow in insolvency is divided among banks according to an endogenous seniority structure. Specifically, if a firm with idiosyncratic shocks (v, ϵ) is insolvent in state s, we assume that bank b receives a fraction $S^b(v, \epsilon, s) \in [0, 1]$ of the cashflows associated with outcome of the liquidation rule. It follows that the date-one value of all of bank b's equity in all continued firms is equal to

$$\mathcal{E}^b(s) \equiv q_1(s) \int_{v \leq v^*(\epsilon, s)} S^b(v, \epsilon, s) \left(1 - \rho(v, \epsilon, s) \right) v \tilde{I} dG(s). \tag{14}$$

[20] For example, accomplished by implementations similar to those of Proposition 3.

[21] Notice that even if firms were not given full bargaining power in contracting with banks at date zero, differences in seniority would still lead to differences in debt prices.

In addition to this equity associated with continuation, bank b receives total date-one insolvency cashflows equal to

$$\int_{v\leq v^*(\epsilon,s)} S^b(v,\epsilon,s)\left(-\left(\ 1-\rho(v,\epsilon,s)\ \right)\epsilon+\rho(v,\epsilon,s)\gamma(s)\right)\tilde{I}dG(s).$$

Finally, at date one, bank b borrows B_1^b from households, so that total lending from bank b to firms is given by

$$\begin{aligned} Q^{S,b}(s) = B_1^b - B_0^b + &\int_{v\geq v^*(\epsilon,s)} D_0^b dG(s) \\ &+\int_{v\leq v^*(\epsilon,s)} S^b(v,\epsilon,s)\left(-\left(\ 1-\rho(v,\epsilon,s)\ \right)\epsilon+\rho(v,\epsilon,s)\gamma(s)\right)\tilde{I}dG(s) \end{aligned} \tag{15}$$

Bank b's date-one borrowing from households is constrained by a collateral constraint

$$B_1^b \leq \phi^b(s)\left(\ Q^{S,b}(s)+\mathcal{E}^b(s)\ \right), \tag{16}$$

where $1-\phi^b(s)$ is a state-dependent collateral haircut specific to bank b. As in the baseline model, Without loss of generality, we can assume that equation (16) holds with equality. Combining this with equation (15), the total date-one supply of bank funds by bank b is

$$\begin{aligned} Q^{S,b}(s) = \frac{1}{1-\phi^b(s)}\Bigg[&\phi^b(s)\mathcal{E}^b(s) - B_0^b + \int_{v\geq v^*(\epsilon,s)} D_0^b dG(s) \\ &+\int_{v\leq v^*(\epsilon,s)} S^b(v,\epsilon,s)\left(-\left(\ 1-\rho(v,\epsilon,s)\ \right)\epsilon+\rho(v,\epsilon,s)\gamma(s)\right)\tilde{I}dG(s)\Bigg]. \end{aligned}$$

Notice that this equation is of the same form as in the baseline model, except that it is now the bank-specific loan supply of bank b.

Finally, the bank-specific participation constraint is defined and derived analogously to Section 2.3, and is given by

$$A_0^b = \mathbb{E}^S\left[\frac{1-\phi^b(s)q_1(s)}{q_1(s)}\left(\ Q^{S,b}(s)+\mathcal{E}^b(s)\ \right)\right]. \tag{17}$$

In particular, there is now a participation constraint for each bank b.

4.2 Privately optimal liquidation and seniority

We begin by studying the liquidation rule ρ and seniority structure $\{S^b(v,\epsilon,s)\}_{b=1}^{N^b}$ that arises in a competitive equilibrium. Formally, firms solve

$$\max_{I, c_0^f, \rho, \{S^b, D_0^b\}_{b=1}^{NB}} u_0^f(c_0^f) + E\left[\int_{v \geq v^*(\epsilon,s)} \left[vI - \frac{1}{q_1(s)}\left[\epsilon I + \sum_{b=1}^{N^b} D_0^b\right]\right] dG(s)\right] \tag{18}$$

subject to the date-zero budget constraint (1) and bank-specific participation constraints (17). As in Section 3.1, firms take equilibrium prices q_1 and γ as given as they choose a liquidation rule and seniority structure at date zero.

The following result characterizes the privately optimal liquidation rule ρ and the optimal seniority structure S in the presence of heterogeneous banks.

Proposition 4. *The privately optimal liquidation rule is the creditor-recovery-maximizing liquidation rule. Moreover, define*

$$\beta^b_{private}(s) \equiv \Lambda^b \frac{1 - \phi^b(s) q_1(s)}{(1 - \phi^b(s)) q_1(s)}.$$

Bank b receives positive recovery in state s only if $b \in \operatorname{argmax}_{\hat{b}} \beta^{\hat{b}}_{private}(s)$, that is

$$\beta^b_{private}(s) < \max \beta^b_{private}(s) \Rightarrow \sup_{v,\epsilon} S^b(v,\epsilon,s) = 0.$$

As in Proposition 1, the optimal liquidation rule ρ chosen privately by firms is the rule that maximizes total creditor recovery. Just as before, such a liquidation rule is optimal because it maximizes each firm's borrowing potential at date zero.

Additionally, in choosing the seniority structure of claims, Proposition 4 reveals that firms prioritize banks with high values of $\beta^b_{private}(s)$. In particular, $\beta^b_{private}(s)$ is the marginal value of a unit of repayment to bank b at date one from the perspective of the firm at date zero. When bank b receives a unit of repayment at date one, it uses it to borrow from households and lend, receiving total surplus $\left(\ 1 - \phi^b(s) q_1(s)\ \right) / \left(\ (1 - \phi^b(s)) q_1(s)\ \right)$. This surplus is weighted by Λ^b, the shadow value to the firm of relaxing the bank b participation constraint. As a result, $\beta^b_{private}$ provides a stochastic discount factor (SDF) used to price payoffs to bank b from the firm's perspective, with the seniority structure allocating claims to banks with the highest SDF.

The optimal seniority structure for firms prioritizes seniority for banks with a high shadow value of relaxing the participation constraint (high Λ^b) to encourage such banks to lend more at date zero. Moreover, we have

$$\frac{\partial \beta^b_{private}(s)}{\partial \phi^b(s)} = \Lambda^b \frac{1 - q_1(s)}{(1 - \phi^b(s))^2 q_1(s)} > 0,$$

so the privately optimal seniority structure also prioritizes banks that are best able to capitalize on funds at date one (high pledgeability $\phi^b(s)$, or equivalently low haircuts $1 - \phi^b(s)$). Intuitively, banks facing the lowest collateral haircuts are best able to engage in lending at date one by borrowing against collateral, and so gain the most value from receiving payoffs in that state.

4.3 Socially optimal bankruptcy and seniority

We now consider the problem of a social planner that chooses a liquidation rule $\rho(v, \epsilon, s)$ and a seniority structure $\{S^b(v, \epsilon, s)\}_{b=1}^{N^b}$, along with a liquidation price, debt prices, debt levels, and firm consumption and investment policies, to maximize firm welfare (18) subject to the firm date-zero budget constraint (1), bank participation constraints (17), the liquidation-market-clearing condition (9), and a loan-market-clearing condition

$$\sum_{b=1}^{N^B} Q^{S,b}(s) \geq Q^D(s), \tag{19}$$

with inequality only if $q_1(s) = 1$. Thus as before, the planner is subject to the same constraint as firms and banks, but internalizes the effects of her decisions on equilibrium loan and liquidation prices.

The following proposition characterizes the socially optimal liquidation rule and seniority structure in the presence of heterogeneous banks.

Proposition 5. *Let Λ^b denote the Lagrange multiplier in the social planner's problem associated with bank b's participation constraint. Let $\mu(s)$ denote the Lagrange multiplier on the loan-market-clearing condition in state s and define*

$$\beta^b(s) \equiv \Lambda^b \frac{1-\phi^b(s)q_1(s)}{(1-\phi^b(s))q_1(s)} + \mu(s)\frac{\phi^b(s)}{1-\phi^b(s)}.$$

Suppose that in some state $\hat{s}$, the social planner never reorganizes firms with a negative recovery value, that is

$$q_1(\hat{s})v < \epsilon \Rightarrow \rho(v,\epsilon,\hat{s}) = 0. \tag{20}$$

Then the following statements are true in state $\hat{s}$:

1. *The socially optimal liquidation rule is a threshold rule, given by*

$$q_1(\hat{s})v^L(\epsilon,\hat{s}) = \Bigg(1 + \underbrace{\frac{\mu(\hat{s})}{\max_b \beta^b(\hat{s})}}_{\text{Loan Price Externalities}}\Bigg)\Bigg(\gamma(\hat{s}) + \epsilon - \underbrace{\left|\frac{\partial\gamma(L(\hat{s},\hat{s})}{\partial L(\hat{s})}\right| L(\hat{s})}_{\text{Liquidation Price Externalities}}\Bigg).$$

2. *Bank b receives positive recovery in state $\hat{s}$ only if $b \in \operatorname{argmax}_{\hat{b}} \beta^{\hat{b}}(\hat{s})$, that is*

$$\beta^b(\hat{s}) < \max_{\hat{b}} \beta^{\hat{b}}(\hat{s}) \Rightarrow \sup_{v,\epsilon} S^b(v,\epsilon,\hat{s}) = 0.$$

The condition (20) states that, in state $\hat{s}$, the social planner finds it optimal to choose liquidation whenever a continuation implies a negative recovery, that is the discounted value of firm cash flows under continuation is negative. In principle, it is conceivable that a social planner might implement a negative-payoff continuation if an additional liquidation would dramatically lower liquidation payoffs through the liquidation-price externality. However, equation (20) follows naturally in a setting in which bankruptcy participants enjoy limited liability.[22]

[22] A continuation plan that assigns a creditor a negative payoff would almost certainly fail the "best-interest-of-the-creditors" test (11 U.S.C. §1129(a)7), which requires that each creditor receive at least as much as it would in a Chapter 7 liquidation. However, a creditor could nonetheless receive a negative payoff

Comparing Proposition 5 to Proposition 4, we see that the social planner not only chooses a different liquidation rule than private firms, but also chooses a different seniority structure. Under condition (20), the social planner assigns positive recovery to banks with the highest value of $\beta^b(s)$, where

$$\beta^b(s) - \beta^b_{private}(s) = \mu(s)\frac{\phi^b(s)}{1-\phi^b(s)}.$$

If $\mu(s) > 0$, the social planner thus assigns lower priority to banks that are constrained at date one (low $\phi^b(s)$) than firms would. Intuitively, the social planner internalizes the ability of banks with high $\phi^b(s)$ values to supply additional funds using equity in reorganized firms as collateral. If $\mu(s) > 0$, then the planner benefits from a higher supply of loanable funds, which increases the price at which banks lend and so boosts firm solvency. The loan-price-externality motivation for altering the seniority structure among banks is therefore precisely the same loan-price-externality motivation for intervening in liquidation decisions, with the same intuition.

Notably, in a given aggregate state, it can be optimal to assign all payoffs from all bankruptcies to one creditor. This is true both in the private problem and the social planner's problem (under condition (20)).[23] Thus, given complete contracts, all proceeds might optimally be allocated to a unique creditor in each state. Absent complete contracts, the initial contract may have to average across aggregate states.

Recall that whenever $\mu(s) > 0$, the social planner has an incentive to avoid continuations in order to save funds for solvent firms. Formally, Proposition 5 shows that the liquidation threshold is inflated by a factor of $\mu(s)/\max_b \beta^b(s)$, where $\beta^b(s) > 0$ for all b if $\mu(s) > 0$. Thus, holding everything else fixed in a state s such that $\mu(s) > 0$, increases in $\max_b \beta^b(s)$ imply more continuations. In particular, to the extent that greater heterogeneity in $\beta^b(s)$ values leads to a larger maximum value $\max_b \beta^b(s)$, bank heterogeneity promotes continuation. Intuitively, the social planner can exploit bank heterogeneity by prioritizing those banks that can most easily redirect bankruptcy payoffs to solvent firms, so the social planner can assign more continuations without crowding out funding to solvent firms. This suggests

in a bankruptcy if the judge refuses to reimburse its legal expenses, or if it is sued by the trustee for a fraudulent transfer or preference.

[23]Without condition (20), the social planner would assign all positive payoffs to one creditor and all negative payoffs to another creditor.

that an efficient seniority structure can bring the socially efficient liquidation rule closer to the privately optimal rule, providing a partial substitute for direct intervention in the liquidation rule.

5 Discussion

In the wake of the COVID-19 pandemic, academics have proposed government interventions aimed at mitigating social losses caused by corporate defaults. Blanchard et al. (2020) propose several such measures, one of which aims to facilitate the restructuring of insolvent firms. Specifically, Blanchard et al. (2020) propose that in any restructuring that allows an insolvent firm to continue operating, the government could accept a larger write down or "haircut" on its claims than the haircut paid by private creditors:

> "*If a firm continues but needs restructuring, the government automatically accepts a haircut on its claims (deferred taxes plus guaranteed credits) equal to the haircut agreed by private creditors of the same rank plus a fixed continuation premium.*"

Such a policy amounts to subsidizing creditors in any restructuring that results in the continuation of an insolvent firm. Our model (Proposition 2) implies that such a policy is socially optimal if banks are not financially constrained and fire-sale externalities are nontrivial.[24] Given the health of the banking sector during the COVID-19 pandemic (Greenwood, Iverson, and Thesmar, 2020), it is likely that the social planner in our model would enact a similar policy in a crisis resembling the COVID-19 pandemic.

DeMarzo, Krishnamurthy, and Rauh (2020) propose that the Federal Reserve and the Treasury create a special purpose vehicle (SPV) to provide DIP loans to bankrupt firms. The SPV would provide highly subsidized loans to bankrupt firms, which would be fully collateralized, priming existing liens if necessary (11 U.S.C. §364(d)). While we do not formally model such a policy, our analysis nonetheless illustrates the potential benefits of the policy proposed by DeMarzo, Krishnamurthy, and Rauh (2020). Recall that in states in which banks are financially constrained, the social planner faces a tradeoff between reorganizing viable firms and preserving funding to solvent firms at reasonable credit spreads.

[24] Indeed, the government accepting a larger haircut on distressed debt is analogous to the implementation that we describe in Proposition 3.

If the government were to exogenously increase the supply of date-one loans to bankrupt firms at subsidized rates, then more reorganizations could be achieved without exacerbating the loan-price externality, in which privately supplied DIP loans crowd out funding to solvent firms. As such, our model suggests that this proposal may be especially effective when banks are also in financial distress and hence loan price externalities are strong. Outside of our model, it is possible that creditors may force marginally solvent firms into bankruptcy, increasing deadweight losses, in order to take advantage of subsidized government funding. However, this incentive for inefficient behavior would be mitigated by the fact that government DIP loans would be senior to all unsecured debt, and even potentially secured debt through priming liens.

Our heterogeneous banks extension (Section 4) suggests a related but different policy intervention into the market for DIP loans. The social planner of Section 4 may wish to impose a bankruptcy seniority structure in which higher priority is given to those banks that are less constrained in their lending in a crisis. Such a seniority structure could be achieved by encouraging courts to approve more DIP loans and grant more priming liens. Such a policy would increase the seniority of DIP lenders, who by revealed preference are sufficiently unconstrained in their lending to provide financing to bankrupt firms. Even outside of a crisis, this socially optimal seniority structure provides a positive explanation for why DIP lenders enjoy high priority in bankruptcy.

Finally, other academics have proposed policies aimed at paying the debt of all firms to prevent deadweight losses associated with liquidation (Saez and Zucman, 2020; Hanson et al., 2020b). Specifically, Saez and Zucman (2020) recommend expanding unemployment insurance and paying a fraction of the maintenance costs of businesses in sectors that are affected by pandemic-induced shutdowns. Hanson et al. (2020b) propose an intervention in which the government would "provide payment assistance to enable impacted businesses to meet their recurring fixed obligations—including interest, rent, lease, and utility payments." We do not formally model interventions like these that entail paying the expenses of all operating firms in troubled sectors. Such an intervention is analogous to the implementation we describe in Proposition 3, in which the government subsidizes the continuation of insolvent businesses. Unlike the policy described in Proposition 3, these proposed interventions also subsidize the continuation of solvent businesses, making them potentially expensive. However, a policy like ours that makes aid contingent on demonstrated insolvency could incentivize firms to file for bankruptcy, incurring deadweight losses, just to take advantage of the government subsidy. In deciding how to allocate assistance, the government thus faces a tradeoff between the expense of helping solvent firms and the greater incentive for firms to demonstrate their insolvency through value-destroying bankruptcy filings.

6 Conclusion

We study optimal liquidation-continuation decisions for firms facing correlated distress in a crisis. Firms privately choose the liquidation rule that maximizes recovery to creditors in order to maximize the value of their securities. By contrast, a social planner intervenes in the optimal liquidation rule due to two pecuniary externalities. First, a fire sale of firm assets under liquidation motivates the planner to adopt a rule that encourages more continuations than the private optimum. Second, a loan price externality arising due to collateral-constrained banks can promote a planner to adopt a rule that encourages more liquidations, sacrificing low-value firms to promote solvency of high-value firms. Our model sheds light on recent policy proposals designed to promote firm solvency in the wake of a crisis.

References

Acharya, V. V., L. Borchert, M. Jager, and S. Steffen. 2021. Kicking the Can Down the Road: Government Interventions in the European Banking Sector. *The Review of Financial Studies, Forthcoming* .

Acharya, V. V., M. Crosignani, T. Eisert, and C. Eufinger. 2020. Zombie credit and (dis-) inflation: evidence from europe. Working Paper, National Bureau of Economic Research.

Acharya, V. V., T. Eisert, C. Eufinger, and C. Hirsch. 2019. Whatever it takes: The real effects of unconventional monetary policy. *The Review of Financial Studies* 32:3366–411.

Antill, S. 2020a. Are bankruptcy professional fees excessively high? Working Paper, Harvard Business School.

———. 2020b. Do the right firms survive bankruptcy? Working Paper, Harvard Business School.

Ayotte, K. M., and E. R. Morrison. 2009. Creditor control and conflict in chapter 11. *Journal of Legal Analysis* 1:511–51.

Bianchi, J. 2011. Overborrowing and systemic externalities in the business cycle. *American Economic Review* 101:3400–26. ISSN 00028282. doi:10.1257/aer.101.7.3400.

Bianchi, J., and E. G. Mendoza. 2018. Optimal Time-Consistent Macroprudential Policy. *Journal of Political Economy* 126:588–634. ISSN 0022-3808. doi:10.1086/696280.

Blanchard, O., T. Philippon, J. Pisani-Ferry, et al. 2020. A new policy toolkit is needed as countries exit covid-19 lockdowns. *Peterson Institute for International Economics Policy Brief* 20–8.

Blattner, L., L. Farinha, and F. Rebelo. 2019. When losses turn into loans: The cost of undercapitalized banks. Working Paper, Stanford Graduate School of Business.

Brunnermeier, M., and A. Krishnamurthy. 2020. Corporate debt overhang and credit policy. In *BPEA conference*.

Caballero, R. J., T. Hoshi, and A. K. Kashyap. 2008. Zombie lending and depressed restructuring in japan. *American economic review* 98:1943–77.

Caballero, R. J., and A. Krishnamurthy. 2001. International and domestic collateral constraints in a model of emerging market crises. *Journal of Monetary Economics* 48:513–48. ISSN 03043932. doi:10.1016/S0304-3932(01)00084-8.

Chari, V. V., and P. J. Kehoe. 2016. Bailouts, Time Inconsistency, and Optimal Regulation: A Macroeconomic View. *American Economic Review* 106:2458–93. ISSN 0002-8282. doi:10.1257/aer.20150157.

Clayton, C., and A. Schaab. 2020. Bail-Ins, Optimal Regulation, and Crisis Resolution.

Corbae, D., and P. D'Erasmo. forthcoming. Reorganization or liquidation: Bankruptcy choice and firm dynamics. *Review of Economic Studies* .

Dávila, E., and A. Korinek. 2018. Pecuniary Externalities in Economies with Financial Frictions. *The Review of Economic Studies* 85:352–95. ISSN 0034-6527. doi:10.1093/restud/rdx010.

DeMarzo, P. M., A. Krishnamurthy, and J. Rauh. 2020. Debtor-in-possession financing facility. Working Paper, Stanford University Working Paper.

Donaldson, J. R., E. R. Morrison, G. Piacentino, and X. Yu. 2020. Restructuring vs. bankruptcy. Working Paper, Washington University in St. Louis and Columbia University.

Farhi, E., and J. Tirole. 2012. Collective moral hazard, maturity mismatch, and systemic bailouts. *American Economic Review* 102:60–93. ISSN 00028282. doi:10.1257/aer.102.1.60.

Greenwood, R., B. Iverson, and D. Thesmar. 2020. Sizing up corporate restructuring in the covid crisis. *Brookings Papers of Economic Activity, forthcoming* .

Greenwood, R., and D. Thesmar. 2020. Sharing the economic pain of the coronavirus. Working Paper, Working paper, Harvard Business School.

Hanson, S., J. Stein, A. Sunderman, and E. Zwick. 2020a. Business credit programs in the pandemic era. *Brookings Papers of Economic Activity, forthcoming* .

Hanson, S. G., J. C. Stein, A. Sunderam, and E. Zwick. 2020b. Business continuity insurance: Keeping america's lights on during the pandemic. *Becker Friedman Institute White Paper* .

Iverson, B. C., J. A. Ellias, and M. J. Roe. 2020. Estimating the need for additional bankruptcy judges in light of the covid-19 pandemic. *Available at SSRN 3624529* .

LoPucki, L. M., and J. W. Doherty. 2004. The determinants of professional fees in large bankruptcy reorganization cases. *Journal of Empirical Legal Studies* 1:111–41.

Lorenzoni, G. 2008. Inefficient credit booms. *Review of Economic Studies* 75:809–33. ISSN 00346527. doi:10.1111/j.1467-937X.2008.00494.x.

Saez, E., and G. Zucman. 2020. Keeping business alive: the government will pay. *Social Europe* 18:2020–.

Shleifer, A., and R. W. Vishny. 1992. Liquidation values and debt capacity: A market equilibrium approach. *The Journal of Finance* 47:1343–66.

Skeel, D. 2020. Bankruptcy and the coronavirus. *Brookings Working Paper* .

Stein, J. C. 2012. Monetary Policy as Financial Stability Regulation. *The Quarterly Journal of Economics* 127:57–95. ISSN 0033-5533. doi:10.1093/qje/qjr054.

A Proofs

A.1 Proof of Proposition 1

Conjecturing a threshold rule $\rho(v,\epsilon,s) = \mathbf{1}(v < v^L(\epsilon,s))$, for any function $f(v,\epsilon)$ such that $f(v,\epsilon)g(v,\epsilon|s)$ is integrable, we write

$$\begin{aligned}
\frac{\partial}{\partial\rho(v,\epsilon,s)}\int_{v\le v^L(\epsilon,s)} f(v,\epsilon)dG(s) &= \frac{\partial}{\partial v^L(\epsilon,s)}\int_{\epsilon\ge 0}\int_0^{v^L(\epsilon,s)} f(v,\epsilon)g(v,\epsilon|s)dvd\epsilon \\
&= \int_{\epsilon\ge 0}\frac{\partial}{\partial v^L(\epsilon,s)}\int_0^{v^L(\epsilon,s)} f(v,\epsilon)g(v,\epsilon|s)dvd\epsilon \\
&= \int_{\epsilon\ge 0} f(v^L(\epsilon,s),\epsilon)g\left(v^L(\epsilon,s),\epsilon|s\right)d\epsilon, \qquad (21)
\end{aligned}$$

where the first equality is the definition of $g(v,\epsilon|s)$, the second equality applies dominated convergence and the third equality is an application of the Leibniz rule.

The private firm Lagrangian is given by

$$\begin{aligned}
\mathcal{L} =& u_0^f(c_0^f) + E\left[\int_{v\ge v^*(\epsilon,s)}\left[vI - \frac{1}{q_1(s)}[\epsilon I + D_0]\right]dG(s)\right] \\
&+ \lambda\left[A_0^f + q_0 D_0 - c_0^f - \Phi(I)\right] + \Lambda\left[E\left[\frac{1-\phi(s)q_1(s)}{q_1(s)}\left(Q^S(s) + \mathcal{E}(s)\right)\right] - A_0^b\right]
\end{aligned}$$

Differentiating in the bankruptcy code $\rho(v,\epsilon,s)$, we obtain

$$\frac{\partial\mathcal{L}}{\partial\rho(v,\epsilon,s)} = \Lambda\frac{1-\phi(s)q_1(s)}{q_1(s)}\frac{\partial Q^S(s)+\mathcal{E}(s)}{\partial\rho(v,\epsilon,s)}f(s).$$

From equation (21) and equation (6),

$$\frac{\partial Q^S(s)+\mathcal{E}(s)}{\partial\rho(v,\epsilon,s)} = \frac{1}{1-\phi(s)}\left[\int_{\epsilon\ge 0}\left[-q_1(s)v^L(\epsilon,s)+\gamma(s)+\epsilon\right]g\left(v^L(\epsilon,s),\epsilon|s\right)\tilde{I}d\epsilon\right]$$

from which the threshold rule $-\epsilon + q_1(s)v^L(\epsilon,s) = \gamma(s)$ follows immediately.

A.2 Proof of Proposition 2

The Lagrangian of the social planning problem is given by

$$
\begin{aligned}
\mathcal{L} =& u(c_0^f) + E\Bigg[\int_{v\geq v^*(\epsilon,s)}\Bigg[vI - \frac{1}{q_1(s)}[\epsilon I + D_0]\Bigg]dG(s)\Bigg] \\
&+ \lambda\Bigg[A_0^f + q_0 D_0 - c_0^f - \Phi(I)\Bigg] + \Lambda\Bigg[E\Bigg[\frac{1-\phi(s)q_1(s)}{q_1(s)}\Bigg(\ Q^S(s) + \mathcal{E}(s)\ \Bigg)\ \Bigg] - A_0^b\Bigg] \\
&+ E\Bigg[\mu(s)\Bigg(Q^S(s) - Q^D(s)\Bigg)\Bigg] + E^S\Bigg[\zeta(s)\Bigg(\gamma(s) - \frac{\partial \mathcal{F}(L(s),s)}{\partial L(s)}\Bigg)\Bigg]
\end{aligned}
$$

where $L(s) = \int_{v\leq v^*(\epsilon,s)} \rho(v,\epsilon,s)\tilde{I}dG(s)$. Differentiating in $\rho(v,\epsilon,s)$ at an interior point, we have

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \rho(v,\epsilon,s)} =& \Lambda\frac{1-\phi(s)q_1(s)}{(1-\phi(s))q_1(s)}\int_{\epsilon\geq 0}\Bigg[-q_1(s)v^L(\epsilon,s) + \gamma(s) + \epsilon\Bigg]g\ \Big(\ v^L(\epsilon,s),\epsilon|s\ \Big)\ \tilde{I}d\epsilon f(s) \\
&+ \mu(s)\frac{\partial\Bigg(Q^S(s) - Q^D(s)\Bigg)}{\partial \rho(v,\epsilon,s)}f(s) - \zeta(s)\frac{\partial \gamma(s)}{\partial L(s)}\frac{\partial L(s)}{\partial \rho(v,\epsilon,s)}f(s)
\end{aligned}
$$

From here, we have

$$
\begin{aligned}
\frac{\partial\Bigg(Q^S(s) - Q^D(s)\Bigg)}{\partial \rho(v,\epsilon,s)} &= \frac{\partial Q^S(s)}{\partial \rho(v,\epsilon,s)} \\
&= \frac{1}{1-\phi(s)}\Bigg[\int_{\epsilon\geq 0}\Bigg[-\phi(s)q_1(s)v^L(\epsilon,s) + \gamma(s) + \epsilon\Bigg]g\ \Big(\ v^L(\epsilon,s),\epsilon|s\ \Big)\ \tilde{I}d\epsilon\Bigg]
\end{aligned}
$$

and

$$
\frac{\partial L(v,s)}{\partial \rho(v,\epsilon,s)} = \int_{\epsilon\geq 0}\tilde{I}g\ \Big(\ v^L(\epsilon,s),\epsilon|s\ \Big)\ d\epsilon,
$$

so that we have

$$\frac{1}{\tilde{I}f(s)}\frac{\partial\mathcal{L}}{\partial\rho(v,\epsilon,s)} = \int_{\epsilon\geq 0}\Bigg(\Lambda\frac{1-\phi(s)q_1(s)}{(1-\phi(s))q_1(s)}\Bigg[-q_1(s)v^L(\epsilon,s)+\gamma(s)+\epsilon\Bigg]$$
$$+\mu(s)\frac{1}{1-\phi(s)}\Bigg[-\phi(s)q_1(s)v^L(\epsilon,s)+\gamma(s)+\epsilon\Bigg]$$
$$+\zeta(s)\left|\frac{\partial\gamma(s)}{\partial L(s)}\right|\Bigg)\, g\,\big(\,v^L(\epsilon,s),\epsilon|s\,\big)\; d\epsilon.$$

From this, we obtain a threshold rule, given by

$$q_1(s)v^L(\epsilon,s) = \left(1+\frac{\mu(s)}{\Lambda\frac{1-\phi(s)q_1(s)}{(1-\phi(s))q_1(s)}+\mu(s)\frac{\phi(s)}{1-\phi(s)}}\right)\Big(\epsilon+\gamma(s)\Big)+\frac{\zeta(s)}{\Lambda\frac{1-\phi(s)q_1(s)}{(1-\phi(s))q_1(s)}+\mu(s)\frac{\phi(s)}{1-\phi(s)}}\left|\frac{\partial\gamma(s)}{\partial L(s)}\right|$$

From here, we need to characterize the multipliers. Beginning with $\zeta(s)$, we have

$$0 = \frac{1}{f(s)}\frac{\partial\mathcal{L}}{\partial\gamma(s)} = \left(\Lambda\frac{1-\phi(s)q_1(s)}{q_1(s)}+\mu(s)\right)\frac{\partial Q^S(s)}{\partial\gamma(s)}+\zeta(s)$$

and using from here $\frac{\partial Q^S(s)}{\partial\gamma(s)} = \frac{1}{1-\phi(s)}\int_{v\leq v^*(\epsilon,s)}\rho(v,\epsilon,s)\tilde{I}dG(s) = \frac{1}{1-\phi(s)}L(s)$, we obtain

$$\zeta(s) = -\left(\Lambda\frac{1-\phi(s)q_1(s)}{(1-\phi(s))q_1(s)}+\frac{\mu(s)}{1-\phi(s)}\right)L(s)$$

which substituting into the threshold rule yields

$$q_1(s)v^L(\epsilon,s) = \left(1+\frac{\mu(s)}{\Lambda\frac{1-\phi(s)q_1(s)}{(1-\phi(s))q_1(s)}+\mu(s)\frac{\phi(s)}{1-\phi(s)}}\right)\left(\epsilon+\gamma(s)-\left|\frac{\partial\gamma(s)}{\partial L(s)}\right|L(s)\right).$$

Finally, we obtain $\mu(s)$ from

$$
\begin{aligned}
0 &= \frac{1}{f(s)}\frac{\partial \mathcal{L}}{\partial q_1(s)} \\
&= \int_{v\geq v^*(\epsilon,s)} \frac{1}{q_1(s)^2}[\epsilon I + D_0]dG(s) \\
&\quad + \Lambda \frac{\partial}{\partial q_1(s)}\left[\frac{1-\phi(s)q_1(s)}{q_1(s)}\left(Q^S(s)+\mathcal{E}(s)\right)\right] + \mu(s)\frac{\partial\left(Q^S(s)-Q^D(s)\right)}{\partial q_1(s)}
\end{aligned}
$$

where we have used the threshold rule property to drop the term involving liquidation market clearing. From here, note that we have

$$
\begin{aligned}
\frac{\partial \mathcal{E}(s)}{q_1(s)} &= \int_{\epsilon\geq 0} \frac{\partial v^*(\epsilon,s)}{\partial q_1(s)} q_1(s)v^*(\epsilon,s)\tilde{I}g(v^*(\epsilon,s)|s)d\epsilon + \int_{v\leq v^*(\epsilon,s)}(1-\rho(v,\epsilon,s))v\tilde{I}dG(s) \\
\frac{\partial Q^S(s)}{\partial q_1(s)} &= \frac{1}{1-\phi(s)}\left[\phi(s)\frac{\partial \mathcal{E}(s)}{\partial q_1(s)} + \int_{\epsilon\geq 0}\frac{\partial v^*(\epsilon,s)}{\partial q_1(s)}\left(-D_0-\epsilon\tilde{I}\right)d\epsilon\right] \\
\frac{\partial Q^D(s)}{\partial q_1(s)} &= \int -\frac{\partial v^*(\epsilon,s)}{\partial q_1(s)}\left(D_0+\epsilon I\right)g(v^*(\epsilon,s)|s)d\epsilon.
\end{aligned}
$$

from which we obtain, noting that by definition $D_0 = \left(-\epsilon + q_1(s)v^*(\epsilon,s)\right)I$ and that $\frac{\partial v^*(\epsilon,s)}{\partial q_1(s)} < 0$,

$$
\frac{\partial(Q^S(s)+\mathcal{E}(s))}{\partial q_1(s)} = \frac{1}{1-\phi(s)}\left[\int_{\epsilon\geq 0}\left|\frac{\partial v^*(\epsilon,s)}{\partial q_1(s)}\right|\left(I-\tilde{I}\right)\frac{D_0}{I}g(v^*(\epsilon,s)|s)d\epsilon + \frac{1}{q_1(s)}\mathcal{E}(s)\right].
$$

Now, substituting back in above, and rearranging, we have

$$\mu(s)\frac{\partial\Big(Q^D(s)-Q^S(s)\Big)}{\partial q_1(s)}$$
$$=\int_{v\geq v^*(\epsilon,s)}\frac{1}{q_1(s)^2}[\epsilon I+D_0]dG(s)-\Lambda\frac{1}{q_1(s)^2}\ \Big(\ Q^S(s)+\mathcal{E}(s)\ \Big)$$
$$+\Lambda\frac{1-\phi(s)q_1(s)}{q_1(s)}\frac{1}{1-\phi(s)}\left[\int_{\epsilon\geq 0}\left|\frac{\partial v^*(\epsilon,s)}{\partial q_1(s)}\right|\Big(I-\tilde{I}\ \Big)\ \frac{D_0}{I}g(v^*(\epsilon,s)|s)d\epsilon+\frac{1}{q_1(s)}\mathcal{E}(s)\right]$$

and using that $Q^S(s)=Q^D(s)$ when $\mu(s)\neq 0$, we obtain

$$\mu(s)\frac{\partial\Big(Q^D(s)-Q^S(s)\Big)}{\partial q_1(s)}=\frac{1-\Lambda}{q_1(s)^2}Q^D(s)+\frac{\Lambda}{q_1(s)^2}\frac{\phi(s)(1-q_1(s))}{1-\phi(s)}\mathcal{E}(s)$$
$$+\Lambda\frac{1-\phi(s)q_1(s)}{(1-\phi(s))q_1(s)}\int_{\epsilon\geq 0}\left|\frac{\partial v^*(\epsilon,s)}{\partial q_1(s)}\right|\Big(I-\tilde{I}\ \Big)\ \frac{D_0}{I}g(v^*(\epsilon,s)|s)d\epsilon.$$

Finally, it is helpful to define

$$\mathcal{V}^*(\epsilon,s)\equiv(v^*(\epsilon,s)+\epsilon)Ig\ \big(\ v^*(\epsilon,s),\epsilon|s\ \big)=D_0g\ \big(\ v^*(\epsilon,s),\epsilon|s\ \big)$$

as the total value of threshold solvent firms. Then, we have

$$\mu(s)\frac{\partial\Big(Q^D(s)-Q^S(s)\Big)}{\partial q_1(s)}=\frac{1-\Lambda}{q_1(s)^2}Q^D(s)+\frac{\Lambda}{q_1(s)^2}\frac{\phi(s)(1-q_1(s))}{1-\phi(s)}\mathcal{E}(s)$$
$$+\Lambda\frac{1-\phi(s)q_1(s)}{(1-\phi(s))q_1(s)}\int_{\epsilon\geq 0}\left|\frac{\partial v^*(\epsilon,s)}{\partial q_1(s)}\right|\chi\mathcal{V}^*(\epsilon,s)d\epsilon.$$

Finally, note that we have

$$\frac{\partial\Big(Q^D(s) - Q^S(s)\Big)}{\partial q_1(s)} = \frac{1}{1-\phi(s)} \int_{\epsilon \geq 0} \left| \frac{\partial v^*(\epsilon, s)}{\partial q_1(s)} \right| [\epsilon - \phi(s) q_1(s) v^*(\epsilon, s)] \Big(I - \tilde{I}\Big)\, g(v^*(\epsilon, s)|s) d\epsilon$$
$$- \frac{\phi(s)}{1-\phi(s)} \frac{1}{q_1(s)} \mathcal{E}(s),$$

which is positive for $\phi(s)$ close to zero.

A.3 Proof of Proposition 4

Define $D_0 \equiv \sum_{b=1}^{NB} D_0^b$. The Lagrangian of the private problem is given by

$$\mathcal{L} = u(c_0^f) + E\Bigg[\int_{v \geq v^*(\epsilon, s)} \Bigg[vI - \frac{1}{q_1(s)}[\epsilon I + D_0]\Bigg] dG(s)\Bigg]$$
$$+ \lambda\Big[A_0^f + q_0 D_0 - c_0^f - \Phi(I)\Big] + \sum_b \Lambda^b \Bigg[E\Bigg[\frac{1-\phi^b(s) q_1(s)}{q_1(s)} \Big(Q^{S,b}(s) + \mathcal{E}^b(s) \Big)\Bigg] - A_0^b\Bigg].$$

A.3.1 Optimal seniority

The components of $\mathcal{L}$ that depend on S^b may be isolated as

$$\sum_b \Lambda^b \Bigg[E\Bigg[\frac{1-\phi^b(s) q_1(s)}{q_1(s)} \Big(Q^{S,b}(s) + \mathcal{E}^b(s) \Big)\Bigg]\Bigg].$$

Further dropping terms not involving S^b in the definition of $Q^{S,b}(s)$,

$$\sum_b E\Bigg[\beta^b_{private}(s) \int_{v \leq v^*(\epsilon, s)} S^b(v, \epsilon, s) \Big((1-\rho(v,\epsilon,s))(q_1(s)v - \epsilon) + \rho(v,\epsilon,s)\gamma(s) \Big) \tilde{I} dG(s)\Bigg]. \tag{22}$$

Fixing ϵ, s, v, we see that an optimal seniority structure must set $S^b(v, \epsilon, s) = 0$ whenever $\beta^b_{private}(s) \neq J(v, \epsilon, s)$, where

$$J(v,\epsilon,s) \equiv \begin{cases} \max_b \beta^b_{private}(s) & \text{if} \quad q_1(s)v \geq \epsilon \ \text{ or } \ \rho(v,\epsilon,s)=1 \\ \min_b \beta^b_{private}(s) & \text{if} \quad q_1(s)v < \epsilon \ \text{ and } \ \rho(v,\epsilon,s)=0. \end{cases} \tag{23}$$

A.3.2 Optimal bankruptcy rule

Differentiating $\mathcal{L}$ in $\rho(v,\epsilon,s)$ at an interior point, we have

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \rho(v,\epsilon,s)} = & \sum_b \Lambda^b \frac{1-\phi^b(s)q_1(s)}{(1-\phi^b(s))q_1(s)} \int_{\epsilon \geq 0} S^b(v^L(\epsilon,s),\epsilon,s)\Big[-q_1(s)v^L(\epsilon,s)+\gamma(s)+\epsilon\Big] \\ & \times g\ \big(\ v^L(\epsilon,s),\epsilon|s\ \big)\ \tilde{I} d\epsilon f(s). \end{aligned}$$

It is immediate that this derivative is equal to zero for the creditor-maximizing rule

$$q_1(s)v^L(\epsilon,s) = \gamma(s)+\epsilon.$$

Moreover, if $q_1(s)v < \epsilon$ for some v,ϵ,s, then

$$\begin{aligned} q_1(s)v &< \epsilon \\ &< \epsilon + \gamma \\ &= q_1(s)v^L(\epsilon,s), \end{aligned}$$

implying a liquidation under the creditor-maximizing rule. It follows from equation (23) that for any v,ϵ,s, we have $J(v,\epsilon,s) = \max_b \beta^b_{private}(s)$, completing the proof.

A.4 Proof of Proposition 5

Define $D_0 \equiv \sum_{b=1}^{NB} D_0^b$. The Lagrangian of the social planning problem is given by

$$
\begin{aligned}
\mathcal{L} =& u(c_0^f) + E\Bigg[\int_{v \geq v^*(\epsilon,s)} \Big[vI - \frac{1}{q_1(s)}[\epsilon I + D_0]\Big] dG(s)\Bigg] \\
&+ \lambda\Big[A_0^f + q_0 D_0 - c_0^f - \Phi(I)\Big] + \sum_b \Lambda^b \Bigg[E\Bigg[\frac{1-\phi^b(s)q_1(s)}{q_1(s)}\Bigg(Q^{S,b}(s) + \mathcal{E}^b(s)\Bigg)\Bigg] - A_0^b\Bigg] \\
&+ E\Bigg[\mu(s)\Bigg(-Q^D(s) + \sum_b Q^{S,b}(s)\Bigg)\Bigg] + E^S\Bigg[\zeta(s)\Bigg(\gamma(s) - \frac{\partial \mathcal{F}(L(s),s)}{\partial L(s)}\Bigg)\Bigg].
\end{aligned}
$$

A.4.1 Optimal seniority

The components of $\mathcal{L}$ that depend on S^b may be isolated as

$$
\sum_b \Lambda^b \Bigg[E\Bigg[\frac{1-\phi^b(s)q_1(s)}{q_1(s)}\Bigg(Q^{S,b}(s) + \mathcal{E}^b(s)\Bigg)\Bigg]\Bigg] + \sum_b E\Bigg[\mu(s)\Bigg(Q^{S,b}(s)\Bigg)\Bigg].
$$

Further dropping terms not involving S^b in the definition of $Q^{S,b}(s)$,

$$
\begin{aligned}
&\sum_b \Lambda^b E\Bigg[\frac{1-\phi^b(s)q_1(s)}{(1-\phi^b(s))q_1(s)} \int_{v \leq v^*(\epsilon,s)} S^b(v,\epsilon,s)\Bigg((1-\rho(v,\epsilon,s))(q_1(s)v - \epsilon) + \rho(v,\epsilon,s)\gamma(s)\Bigg)\tilde{I} dG(s)\Bigg] \\
&+ \sum_b E\Bigg[\frac{\mu(s)}{1-\phi^b(s)} \int_{v \leq v^*(\epsilon,s)} S^b(v,\epsilon,s)\Bigg((1-\rho(v,\epsilon,s))(q_1(s)\phi^b(s)v - \epsilon) + \rho(v,\epsilon,s)\gamma(s)\Bigg)\tilde{I} dG(s)\Bigg].
\end{aligned}
\tag{24}
$$

Rearranging the second line,

$$\frac{\mu(s)}{1-\phi^b(s)}\int_{v\leq v^*(\epsilon,s)} S^b(v,\epsilon,s)\left(\ (1-\rho(v,\epsilon,s))(q_1(s)\phi^b(s)v-\epsilon)+\rho(v,\epsilon,s)\gamma(s)\ \right)\tilde{I}dG(s)$$
$$=\frac{\mu(s)\phi^b(s)}{1-\phi^b(s)}\int_{v\leq v^*(\epsilon,s)} S^b(v,\epsilon,s)\left(\ (1-\rho(v,\epsilon,s))(q_1(s)v-\epsilon)+\rho(v,\epsilon,s)\gamma(s)\ \right)\tilde{I}dG(s)$$
$$+\frac{\mu(s)}{1-\phi^b(s)}(1-\phi^b(s))\int_{v\leq v^*(\epsilon,s)} S^b(v,\epsilon,s)\left(\ (1-\rho(v,\epsilon,s))(-\epsilon)+\rho(v,\epsilon,s)\gamma(s)\ \right)\tilde{I}dG(s).$$

Applying the definition of $\beta^b(s)$, we can thus rewrite equation (24) as

$$\sum_b E\Bigg[\beta^b(s)\int_{v\leq v^*(\epsilon,s)} S^b(v,\epsilon,s)\left(\ (1-\rho(v,\epsilon,s))(q_1(s)v-\epsilon)+\rho(v,\epsilon,s)\gamma(s)\ \right)\tilde{I}dG(s)\Bigg]$$
$$+\sum_b E\Bigg[\mu(s)\int_{v\leq v^*(\epsilon,s)} S^b(v,\epsilon,s)\left(\ (1-\rho(v,\epsilon,s))(-\epsilon)+\rho(v,\epsilon,s)\gamma(s)\ \right)\tilde{I}dG(s)\Bigg].$$

Pulling the finite sum inside the expectation, the second line simplifes to

$$E\Bigg[\mu(s)\int_{v\leq v^*(\epsilon,s)}\left(\ (1-\rho(v,\epsilon,s))(-\epsilon)+\rho(v,\epsilon,s)\gamma(s)\ \right)\tilde{I}\times\sum_b S^b(v,\epsilon,s)dG(s)\Bigg]$$
$$=E\Bigg[\mu(s)\int_{v\leq v^*(\epsilon,s)}\left(\ (1-\rho(v,\epsilon,s))(-\epsilon)+\rho(v,\epsilon,s)\gamma(s)\ \right)\tilde{I}dG(s)\Bigg],$$

where we used the fact that $\sum_b S^b = 1$ by definition. The second line thus doesn't depend on the seniority structure. The optimal seniority structure thus maximizes

$$\sum_b E\Bigg[\beta^b(s)\int_{v\leq v^*(\epsilon,s)} S^b(v,\epsilon,s)\left(\ (1-\rho(v,\epsilon,s))(q_1(s)v-\epsilon)+\rho(v,\epsilon,s)\gamma(s)\ \right)\tilde{I}dG(s)\Bigg].$$

For any ϵ, s, v, we see that any optimal seniority structure must set $S^b(v,\epsilon,s)=0$ whenever $\beta^b(s)\neq J(v,\epsilon,s)$, where

$$J(v,\epsilon,s)\equiv\begin{cases}\max_b\beta^b(s) & \text{if}\quad q_1(s)v\geq\epsilon\ \text{ or }\ \rho(v,\epsilon,s)=1\\ \min_b\beta^b(s) & \text{if}\quad q_1(s)v<\epsilon\ \text{ and }\ \rho(v,\epsilon,s)=0.\end{cases}\tag{25}$$

A.4.2 Optimal bankruptcy rule

Differentiating $\mathcal{L}$ in $\rho(v, \epsilon, s)$ at an interior point, we have

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \rho(v,\epsilon,s)} =& \sum_b \Lambda^b \frac{1-\phi^b(s)q_1(s)}{(1-\phi^b(s))q_1(s)} \int_{\epsilon \geq 0} S^b(v^L(\epsilon,s),\epsilon,s)\Big[-q_1(s)v^L(\epsilon,s)+\gamma(s)+\epsilon\Big] \\
& \times g\ \Big(\ v^L(\epsilon,s),\epsilon|s\ \Big)\ \tilde{I} d\epsilon f(s) + \mu(s) \frac{\partial\Big(Q^S(s)-Q^D(s)\Big)}{\partial \rho(v,\epsilon,s)} f(s) - \zeta(s) \frac{\partial \gamma(s)}{\partial L(s)} \frac{\partial L(s)}{\partial \rho(v,\epsilon,s)} f(s),
\end{aligned}
$$

where $Q^S(s) \equiv \sum_b Q^{S,b}(s)$. From here, we have

$$
\begin{aligned}
\frac{\partial\Big(Q^S(s)-Q^D(s)\Big)}{\partial \rho(v,\epsilon,s)} &= \frac{\partial Q^S(s)}{\partial \rho(v,\epsilon,s)} \\
&= \sum_b \frac{1}{1-\phi^b(s)} \Bigg[\int_{\epsilon \geq 0} S^b(v^L(\epsilon,s),\epsilon,s) \Big[-\phi^b(s)q_1(s)v^L(\epsilon,s)+\gamma(s)+\epsilon\Big] \\
&\times g\ \Big(\ v^L(\epsilon,s),\epsilon|s\ \Big)\ \tilde{I} d\epsilon \Bigg]
\end{aligned}
$$

and

$$
\frac{\partial L(v,s)}{\partial \rho(v,\epsilon,s)} = \int_{\epsilon \geq 0} \tilde{I} g\ \Big(\ v^L(\epsilon,s),\epsilon|s\ \Big)\ d\epsilon,
$$

so that we have

$$
\begin{aligned}
\frac{1}{\tilde{I} f(s)} \frac{\partial \mathcal{L}}{\partial \rho(v,\epsilon,s)} = \int_{\epsilon \geq 0} g\ \Big(\ v^L(\epsilon,s),\epsilon|s\ \Big) \Bigg[\zeta(s)\left|\frac{\partial \gamma(s)}{\partial L(s)}\right| + \sum_b S^b(v^L(\epsilon,s),\epsilon,s)\ \Bigg(\\
\Lambda^b \frac{1-\phi^b(s)q_1(s)}{(1-\phi^b(s))q_1(s)} \Big[-q_1(s)v^L(\epsilon,s)+\gamma(s)+\epsilon\Big] \\
+\mu(s)\frac{1}{1-\phi^b(s)} \Big[-\phi^b(s)q_1(s)v^L(\epsilon,s)+\gamma(s)+\epsilon\Big]\ \Bigg)\Bigg] d\epsilon.
\end{aligned}
$$

To characterize $\zeta(s)$, we differentiate with respect to $\gamma(s)$:

$$0 = \frac{1}{f(s)} \frac{\partial \mathcal{L}}{\partial \gamma(s)} = \sum_b \left(\Lambda^b \frac{1 - \phi^b(s) q_1(s)}{q_1(s)} + \mu(s) \right) \frac{\partial Q^{S,b}(s)}{\partial \gamma(s)} + \zeta(s)$$

and using $\frac{\partial Q^{S,b}(s)}{\partial \gamma(s)} = \frac{1}{1-\phi^b(s)} \int_{v \leq v^*(\epsilon,s)} S^b(v,\epsilon,s) \rho(v,\epsilon,s) \tilde{I} dG(s)$, from here we obtain

$$\begin{aligned} \zeta(s) &= -\sum_b \left(\Lambda^b \frac{1 - \phi^b(s) q_1(s)}{(1 - \phi^b(s)) q_1(s)} + \frac{\mu(s)}{1 - \phi^b(s)} \right) L^b(s) \\ &= -\sum_b \left(\beta^b(s) + \mu(s) \right) L^b(s), \end{aligned}$$

where

$$L^b(s) \equiv \int_{v \leq v^*(\epsilon,s)} S^b(v,\epsilon,s) \rho(v,\epsilon,s) \tilde{I} dG(s)$$

is the fraction of liquidated assets with proceeds accruing to bank b.

Now, fix some s such that the limited liability condition holds. From equation (25), we see that $S^b(v,\epsilon,s) = 0$ unless $\beta^b(s) = \max_j \beta^j(s)$. Thus, for any b, either $S^b(v,\epsilon,s) = 0$ for all v, ϵ or $\beta^b(s) = \max_j \beta^j(s)$. Plugging this in,

$$\begin{aligned} \zeta(s) &= -\sum_b \left(\max_j \beta^j(s) + \mu(s) \right) L^b(s) \\ &= -\left(\max_j \beta^j(s) + \mu(s) \right) \sum_b L^b(s) \\ &= -\left(\max_j \beta^j(s) + \mu(s) \right) L(s), \end{aligned}$$

where the final equality follows from the fact that $\sum^b S^b(v,\epsilon,s) = 1$. Setting $\frac{\partial \mathcal{L}}{\partial \rho(v,\epsilon,s)} = 0$ and plugging this expression in for $\zeta(s)$,

$$0 = \int_{\epsilon \geq 0} g\ \Big(\ v^L(\epsilon, s), \epsilon | s\ \Big) \Bigg[- \Bigg(\max_j \beta^j(s) + \mu(s) \Bigg) L(s) \Bigg| \frac{\partial \gamma(s)}{\partial L(s)} \Bigg| + \sum_b S^b(v^L(\epsilon, s), \epsilon, s)\ \Bigg($$

$$\Lambda^b \frac{1 - \phi^b(s) q_1(s)}{(1 - \phi^b(s)) q_1(s)} \Bigg[- q_1(s) v^L(\epsilon, s) + \gamma(s) + \epsilon \Bigg] + \mu(s) \frac{1}{1 - \phi^b(s)} \Bigg[- \phi^b(s) q_1(s) v^L(\epsilon, s) + \gamma(s) + \epsilon \Bigg]\ \Bigg) \Bigg] d\epsilon.$$

Noting that

$$\frac{\mu(s)}{1 - \phi^b(s)} = \mu(s) + \frac{\phi^b(s)\mu(s)}{1 - \phi^b(s)},$$

and applying the definition of $\beta^b(s)$, this is

$$0 = \int_{\epsilon \geq 0} g\ \Big(\ v^L(\epsilon, s), \epsilon | s\ \Big) \Bigg[- \Bigg(\max_j \beta^j(s) + \mu(s) \Bigg) L(s) \Bigg| \frac{\partial \gamma(s)}{\partial L(s)} \Bigg| + \sum_b S^b(v^L(\epsilon, s), \epsilon, s)\ \Bigg($$

$$\beta^b(s) \Bigg[- q_1(s) v^L(\epsilon, s) + \gamma(s) + \epsilon \Bigg] + \mu(s) \Bigg[\gamma(s) + \epsilon \Bigg]\ \Bigg) \Bigg] d\epsilon.$$

Recalling that either $S^b(v, \epsilon, s) = 0$ for all v, ϵ or $\beta^b(s) = \max_j \beta^j(s)$,

$$0 = \int_{\epsilon \geq 0} g\ \Big(\ v^L(\epsilon, s), \epsilon | s\ \Big) \Bigg[- \Bigg(\max_j \beta^j(s) + \mu(s) \Bigg) L(s) \Bigg| \frac{\partial \gamma(s)}{\partial L(s)} \Bigg|$$

$$+ \Bigg(\max_j \beta^j(s) \Bigg[- q_1(s) v^L(\epsilon, s) + \gamma(s) + \epsilon \Bigg] + \mu(s) \Bigg[\gamma(s) + \epsilon \Bigg]\ \Bigg) \Bigg] \sum_b S^b(v^L(\epsilon, s), \epsilon, s) d\epsilon.$$

Noting that $\sum_b S^b(v, \epsilon, s)$, this implies the threshold rule

$$q_1(s) v^L(\epsilon, s) = \Bigg(\ 1 + \frac{\mu(s)}{\max_j \beta^j(s)}\ \Bigg) \Bigg[\gamma(s) + \epsilon - L(s) \Bigg| \frac{\partial \gamma(s)}{\partial L(s)} \Bigg| \Bigg],$$

completing the proof.