Crisis Interventions in Corporate Insolvency

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Abstract

We model the optimal resolution of insolvent firms in general equilibrium. Privately optimizing agents choose to let banks assign liquidations, encouraging ex-ante lending. A social planner optimally intervenes during a crisis because of two pecuniary externalities. A fire-sale externality motivates subsidies for liquidation-preventing loans to insolvent firms. However, a loan-price externality arises when constrained banks allocate scarce capital to averting liquidations rather than bolstering healthier firms, motivating liquidation subsidies. Efficient intervention can thus encourage or discourage liquidation, depending on the crisis, shedding light on recent crisis-motivated policy proposals. Interventions in seniority structures can substitute for interventions in liquidation decisions.

Keywords: Corporate Insolvency, Bankruptcy, Crisis Intervention, Pecuniary Externality, Zombie Lending

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1 Introduction

The COVID-19 pandemic motivated a wave of proposals for government interventions into existing processes for resolving insolvent firms.\(^1\) Most proposals aim to preserve firms that would otherwise be liquidated under existing insolvency laws. Using a general equilibrium model, we show that even an optimally designed system for resolving insolvent firms can be improved upon by crisis-specific interventions. Interventions that deter liquidations can be beneficial in one crisis and harmful in another, even if firm cashflows are identical in the two crises.

We study crisis-motivated interventions in existing corporate insolvency systems. In our three-period model, banks extend loans to firms via standard short-term debt contracts. At date one, the realization of an aggregate state determines whether a crisis occurs. Each firm realizes two idiosyncratic date-one shocks: one determining a temporary cash-flow need and another determining a long-run value. A firm with high long-run value can roll over its debt by obtaining a second loan from banks at an endogenous loan price. The loan price depends on whether banks face binding collateral constraints. A firm with low long-run value becomes insolvent – the combination of its outstanding debt and cash-flow needs exceeds its long-run value – and bears a loss on its assets due to financial distress costs. At date one, insolvent firms can be resolved in one of two ways. First, insolvent firms can be liquidated. Second, a firm can be restructured, with debtholders accepting a haircut, allowing the firm to continue operating.\(^2\) We use the term “continuation” to refer to such a restructuring, which could be conducted out of court or in a Chapter 11 bankruptcy. Liquidation may be valuable relative to continuation for firms with sufficiently low long-run value. Insolvency procedures are characterized by a liquidation rule, designating which insolvent firms

\(^1\)To paraphrase Greenwood, Iverson, and Thesmar (2020): (i) Hanson, Stein, Sunderam, and Zwick (2020b) recommend keeping firms solvent by funding fixed obligations like rent; (ii) Saez and Zucman (2020) recommend keeping firms solvent by funding all expenses; (iii) Brunnermeier and Krishnamurthy (2020) recommend subsidizing refinancing for small firms; (iv) Greenwood and Thesmar (2020) recommend extending a tax credit to claimants (i.e., landlords) that accept a haircut on loan obligations; (v) Iverson, Ellias, and Roe (2020) recommend hiring additional bankruptcy judges; (vi) Skeel (2020) recommend creating a standard “prepacked” restructuring process; (vii) Blanchard, Philippon, and Pisani-Ferry (2020) recommend the government accept larger losses than creditors in reorganizations; (viii) DeMarzo, Krishnamurthy, and Rauh (2020) recommend a government-funded vehicle extend debtor-in-possession financing; (ix) the Bankruptcy and COVID-19 Working Group recommends extending deadlines for small businesses in Chapter 11.

\(^2\)In our baseline model, a debt haircut and debt-equity conversion are equivalent.
We begin in Section 3 by studying the private optimum when firms are able to commit, at date zero when signing contracts, to a liquidation rule in insolvency. We show that the privately optimal liquidation rule is a threshold rule: all insolvent firms below a threshold long-run value are liquidated while firms above the threshold are restructured. Moreover, this threshold rule is the creditor-recovery-maximizing liquidation rule, which maximizes ex-post recovery for the firms’ creditors. In particular, the threshold is exactly the long-run firm value at which recovery to creditors from liquidation and continuation are equated. Intuitively, by maximizing ex-post recovery to creditors in insolvency, firms also maximize their own value ex ante through initial funds raised. As a result, the ex-ante preferences of firms and ex-post preferences of banks with respect to the liquidation rule are aligned.

However, two pecuniary externalities arise in our model from the choice between liquidation and continuation. The first is a fire-sale externality: the equilibrium liquidation price may decline in the number of liquidations (Shleifer and Vishny, 1992). The second is a loan-price externality: when banks face a binding collateral constraint, continuations reduce the supply of loanable funds, affecting loan prices for all firms. This results in misallocation of scarce bank funds, hindering the ability of firms to roll over debt and increasing the number of insolvent firms. Private agents do not internalize their impact on either the date-one liquidation or loan price, leading to a potential role for intervention by a social planner to account for the pecuniary externalities.

We next study the social optimum, in which a social planner chooses both the debt level and liquidation rule of firms. The planner internalizes the impact of her choices on prices, but must otherwise respect the constraints of private agents. We show that the socially optimal liquidation rule for insolvent firms differs from the privately optimal (creditor-recovery maximizing) rule in two key ways. First, the fire-sale externality incentivizes the social planner to avoid liquidations, relative to the privately optimal rule, in order to mitigate fire sales. The result and intuition are closely in line with standard fire-sale externality problems.

On the other hand, the loan-price externality, arising when banks face binding collateral con-
straints, can lead the social planner to adopt a liquidation rule that implements more liquidations than the private optimum. Intuitively, when a firm with low long-run value continues rather than liquidates, banks’ resources are tied up in the firm and cannot be lent out. By liquidating a low-value firm, bank resources are freed up and the supply of loanable funds increases, allowing firms to borrow at cheaper rates. This enhances the ability of high-value firms to roll over their debt, pushing more firms out of the insolvency region and avoiding insolvency costs. This in turn enhances bank solvency by increasing recovery values from high-value firms. By committing to liquidate low value firms, the social planner thus alleviates the problem of misallocation by reallocating scarce bank funds to high-value firms.

We further show that feasible interventions can incentivize private agents to use the efficient liquidation rule. Specifically, the social optimum can be implemented with a simple subsidy scheme, even if the planner does not observe the long-run value of any individual firm. In particular, if the social optimum promotes more (fewer) liquidations of firms than the private optimum, then the planner can implement this optimum by providing a fixed subsidy for liquidations (restructurings) of insolvent firms. Under this fixed subsidy, private agents are incentivized to implement the socially optimal insolvency rule even when they make the decision ex post based on creditor recovery maximization. Intuitively, because both the private and social optima consist of threshold rules, a social planner only has to move the threshold. Accordingly, the social planner can feasibly calibrate a fixed subsidy to move the threshold to the desired level.

In Section 4, we discuss our results in the context of various proposals for interventions in firm insolvency that have arisen in the wake of the COVID-19 pandemic. We explain how a socially optimal subsidy for liquidations or continuations could be practically implemented in a crisis without a lengthy legislative process. We also characterize situations in which various policy proposals coincide with optimal interventions in our model.

Finally, in Section 5 we consider model extensions. We show that our results are robust to allowing for acquisitions, which have features of both liquidation and continuation. Our results also continue to hold when the planner optimizes the welfare of both firms and the purchasers of
liquidated assets. We further extend the model to allow for heterogeneous creditors (banks), and we derive the socially optimal seniority structure and liquidation rule in this setting. We show that the private optimum again implements the creditor-recovery-maximizing liquidation rule. Moreover, private agents optimally implement a seniority structure that allocates proceeds to banks with the highest marginal value of wealth in any given aggregate state. This seniority structure tends to prioritize banks that face low collateral haircuts, who have the greatest ability to lend out additional funds in that state. We show that a social planner optimally intervenes in both the liquidation rule and the seniority structure. The socially optimal seniority structure trades off the private marginal value of wealth to banks against the social value of loanable funds, prioritizing banks with greater lending capabilities when loanable funds are valuable (e.g. due to binding collateral constraints). However, the optimal seniority structure also downweights the magnitude of the loan-price externality in the liquidation rule. Intuitively, by allocating loanable funds to banks with the greatest lending capabilities, the planner increases the supply of loanable funds. This reduces the need to increase loanable funds by liquidating low-value firms. As a result, planner intervention in the seniority structure of debt provides a partial substitute for intervention in the liquidation rule in insolvency.

**Related literature.** This paper contributes to the theoretical literature studying the socially optimal resolution of insolvent firms. In early seminal work, Shleifer and Vishny (1992) show that fire-sale externalities create a motive for social planners to avoid liquidations. Corbae and D’Erasmo (2021) estimate a general-equilibrium model with both reorganization and liquidation in bankruptcy, but do not consider fire-sale externalities or collateral constraints. Hanson, Stein, Sunderman, and Zwick (2020a) model an economy in which extending credit to otherwise insolvent firms helps to mitigate aggregate demand externalities. In their model, a social planner would optimally rescue some firms that the private sector would deem nonviable in order to preserve option value: nonviable firms support aggregate demand, a public good, if the economy recovers quickly. Philippon (2020) models a mechanism-design problem in which the government seeks
to prevent inefficient liquidations without resorting to an indiscriminate bailout. Chari and Kehoe (2016) study optimal policy with time inconsistent bailouts in a costly state verification framework, and show that restrictions on debt and size constitute optimal policy and that intervention in resolution is not required. This happens because with costly state verification, debt contracts are the only renegotiation proof contracts, and because liquidation never increases firm value in their setting. Clayton and Schaab (2020) show that an orderly bank resolution (bail-in) regime is a socially optimal policy in a optimal contracting model with an incentive problem, but show that the social optimum always promotes continuation given the fire sale externality studied. Colliard and Gromb (2018) and Keister and Mitkov (2021) study how the prospect of bailouts distorts incentives of banks to privately bail-in or renegotiate with their creditors. Glode and Opp (2021) study complementarities in debt renegotiation when businesses are connected in a debt chain, and study how government interventions can prevent waves of defaults. Li and Li (2021) show that public liquidity interventions during crises preserve low-quality firms and mitigate the cleansing effect of crises. Our paper contributes to this literature by showing in a problem of optimal insolvency rule design that optimal intervention can favor either liquidation or continuation, depending on the strength of externalities in the banking sector versus the nonfinancial sector.

In recent work, Donaldson, Morrison, Piacentino, and Yu (2020) develop an elegant model of the complementarities between bankruptcy and out-of-court restructurings. Like our paper, they use a model to analyze recent proposed interventions into the resolution of insolvent firms. We contribute by modeling how a liquidation of one firm can impose externalities on unrelated insolvent firms. We thus complement Donaldson et al. (2020) by analyzing recent policy proposals in light of different market failures.

A substantial literature has studied optimal macroprudential and bailout interventions for financial intermediaries in the presence of pecuniary externalities (e.g. Bianchi (2016), Bianchi and Mendoza (2018), Caballero and Krishnamurthy (2001), Dávila and Korinek (2018), Lorenzoni (2008), Stein (2012)) and fiscal externalities (e.g. Farhi and Tirole (2012), Chari and Kehoe (2016)). Our model complements this literature by focusing on ex-post interventions in the in-
solvency process for nonfinancials, which is a rule governing liquidation versus continuation, in a model with two-sided pecuniary externalities. Insolvency intervention differs from macroprudential regulation or bailouts because the debt level (net of bailouts) of the firm determines the region of insolvency but not the decision to liquidate or continue conditional on insolvency. We also show that macroprudential interventions and insolvency interventions are complements in our model.

Finally, this paper relates to the literature on zombie loans: subsidized bank loans to insolvent firms. 3 Prior work studies models in which banks impose a negative externality on other agents by preserving firms through zombie loans. Caballero, Hoshi, and Kashyap (2008) and Acharya, Crosignani, Eisert, and Eufinger (2020) show theoretically and empirically that, by keeping insolvent firms alive to compete in product markets, zombie loans lead to lower product prices and markups, reducing entry and productivity. We contribute to this literature by modeling a tradeoff between two externalities: one similar externality associated with a misallocation of credit due to socially suboptimal firm preservation, and an opposing externality associated with inefficient liquidation. 4

2 Model

There are three dates, \( t = 0, 1, 2 \). The economy consists of three types of agents: firms, banks, and arbitrageurs. There is a unit continuum of each type of agent. All agents are risk neutral and do not discount the future.

Ex-ante identical firms obtain funding from banks at date zero to finance investment. At date one, all uncertainty in the model is resolved. Macroeconomic conditions are determined by an aggregate state that takes the value \( s \in S \) with probability \( f(s) \). Firms also realize idiosyncratic shocks, leading some firms to become insolvent. Arbitrageurs purchase investment projects from

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3 For empirical evidence of zombie lending and the economic impact of zombie loans, see Caballero, Hoshi, and Kashyap (2008); Acharya, Eisert, Eufinger, and Hirsch (2019); Blattner, Farinha, and Rebelo (2019); Acharya, Borchert, Jager, and Steffen (2021) and Acharya, Crosignani, Eisert, and Eufinger (2020).

4 See Antill (2020b) and Ayotte and Morrison (2009) for empirical evidence of inefficient liquidations and a review of the empirical literature on inefficient liquidations.
the subset of insolvent firms that are liquidated. Other firms obtain new loans from banks, allowing
them to continue operating and realize a final payoff at date two.

We now describe each agent in detail. The section concludes with a definition of a competitive
equilibrium.

2.1 Firms

At date zero, ex-ante identical firms have exogenous initial assets $A_f^0 > 0$. Firms use initial assets and bank financing in order to fund investment projects. A project of scale $I$ requires $\Phi(I)$ dollars of funding, where $\Phi(\cdot)$ is an exogenous function. Firms raise financing from banks through standard short-term debt.\footnote{We could obtain similar results with more general liabilities under incomplete markets. In Section 2.4, we provide an interpretation of debt financing and bankruptcy in the context of costly state verification.} Firms raise total financing $q_0D_0$ from banks at date zero, where $D_0$ is the promised date-one repayment. The total value of financing $q_0D_0$ will be pinned down by a bank participation constraint, defined below. Letting $c_0^f$ denote date-zero firm consumption, firms thus face an initial budget constraint

$$c_0^f + \Phi(I) \leq A_0^f + q_0D_0.$$  (1)

At date one, the aggregate state is realized. In addition, each firm realizes an idiosyncratic viability state $v \in V$ and an idiosyncratic date-one expense shock $\epsilon \geq 0$. Each firm needs a cash infusion of $\epsilon$ per unit of project scale in order to pay expenses and continue operating through date two. If the firm reaches date two, it receives a payout $v$ per unit of project scale at that time. A firm that does not pay the expense $\epsilon I$ is rendered non-viable and must be liquidated. We assume that $\epsilon$ and $v$ are jointly distributed according to an exogenous continuous density $g(v, \epsilon|s)$.

At date one, firms have access to additional bank financing, which can be used to roll over outstanding debt as well as to pay the required date-one expense $\epsilon$. Specifically, in state $s$, a firm can borrow $q_1(s)$ dollars by promising to repay one dollar at date two, where the price $q_1(s)$ will be endogenously determined. A firm is solvent at date one if and only if
\[ q_1(s)v \geq D_0 + \epsilon I. \]  

(2)

It follows that there is a threshold rule \( v^*(\epsilon, s) \) such that a firm is solvent at date one in state \( s \) if and only if \( v \geq v^*(\epsilon, s) \equiv \left( D_0 + \epsilon I \right) / \left( q_1(s)I \right) \). Note that this threshold decreases in \( q_1(s) \) so that higher prices enhance solvency. A solvent firm funds its cash-flow need by issuing \( \epsilon I / q_1(s) \) units of new debt, obtaining \( \epsilon I \) date-one dollars, and rolling over its outstanding debt.\(^6\) The total loan demand, in date-one dollars, from solvent firms is thus

\[
Q^D(s) \equiv \int_{v \geq v^*(\epsilon, s)} \left( D_0 + \epsilon I \right) dG(s).
\]

(3)

We let \( R_1(v, \epsilon, s)D_0 \) denote the debt recovery value that a bank receives in state \( s \) from a loan to a firm with viability \( v \) and expense \( \epsilon \). For solvent firms, we have \( R_1(v, \epsilon, s)D_0 = D_0 \): creditors receive full recovery on their claim.

### 2.2 Resolving insolvent firms

Firms with \( v < v^*(\epsilon, s) \) are insolvent. Each insolvent firm experiences an exogenous proportional loss \( \chi \) of its project scale, so that its scale in insolvency is \( \tilde{I} \equiv (1 - \chi)I \). We interpret \( \chi \) as reflecting the costs of financial distress. For example, a firm that enters bankruptcy incurs direct costs associated with the bankruptcy process.\(^7\) Even a firm that does not enter a formal bankruptcy proceeding likely incurs indirect financial distress costs associated with a loss of employees or customers.

We model the insolvency-resolution process as a rule for sorting firms into one of two outcomes: liquidation or continuation.\(^8\) An insolvency-resolution process is thus defined by a probability \( \rho(v, \epsilon, s) \in [0, 1] \) that an insolvent firm with viability \( v \) and cash-flow need \( \epsilon \) is liquidated in

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\(^6\)We have assumed incomplete markets not only on firm value \( v \) (e.g., due to asymmetric information), but also on the aggregate state \( s \) and temporary cost \( \epsilon \). Naturally, firms and banks in our model would wish to make debt contingent on \( (\epsilon, s) \), even if they can’t make it contingent on the private value \( v \). Our qualitative results extend to the case where firms can write contingent debt contracts \( D_0(\epsilon, s) \), since there is still a spectrum of insolvency outcomes based on the long-run value \( v \).

\(^7\)Professional fees in large Chapter 11 cases are, on average, 1% to 6% of the estate value (LoPucki and Doherty, 2004). Trustees in Chapter 7 bankruptcy cases regularly receive 25% of the estate in fees (Antill, 2020a).

\(^8\)Section 5.2 shows that our results are robust to allowing for acquisitions as well.
state $s$. The probability of continuation is $1 - \rho(v, \varepsilon, s)$. We now define liquidation and continuation.

### 2.2.1 Liquidation definition

Liquidation entails a sale of all of the firm’s assets. This could be achieved through either an out-of-court foreclosure and sale or through a bankruptcy filing under Chapter 7 or Chapter 11 of the US bankruptcy code. If a firm is liquidated, the arbitrageur buys the capital $\tilde{I}$ at price $\gamma(s)$ per unit, where $\gamma(s)$ is the endogenous equilibrium resale price of assets.\(^9\) The arbitrageur may be interpreted as a strategic buyer that will use the liquidated assets in its business operations. Because the arbitrageur uses the assets in its own business, the liquidation payoff $\gamma(s)\tilde{I}$ does not depend on the insolvent firm’s viability or expenses. We thus define the bank’s total date-one recovery in liquidation as $\mathcal{R}_1(v, \varepsilon, s)D_0 = \gamma(s)\tilde{I}$.

### 2.2.2 Continuation definition

In continuation, the firm’s creditors agree to pay the firm’s date-one expenses in exchange for the firm’s future cashflows. Continuation could represent a “zombie loan” in which a bank lends to a firm at a subsidized rate, an out-of-court restructuring, or a reorganization under Chapter 11 of the US bankruptcy code. Notably, all uncertainty in our model is resolved at date one, so it is irrelevant whether the bank receives debt or equity from the insolvent firm at date one.

In continuation, the bank covers the firm’s date-one expenses $\varepsilon\tilde{I}$. This could take the form of a zombie loan, new financing provided as part of an out-of-court restructuring, or debtor-in-possession (DIP) financing in a Chapter 11 bankruptcy.\(^10\) In exchange, the bank receives all of the equity in the firm.\(^11\) Continuation payoffs are received at date two, leaving the bank with a date-two cash flow $v\tilde{I}$. The date-one value of this equity is $q_1(s)v\tilde{I}$, where $q_1(s)$ is the date-one

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\(^9\)We assume that $\gamma(s)$ is sufficiently low that solvent firms never find it optimal to liquidate assets rather than continue.

\(^10\)Many bankrupt firms do not require DIP financing. We allow $\varepsilon$ to be arbitrarily close to 0 to capture this case.

\(^11\)Notice that this is equivalent to assuming that outstanding debt is converted to equity, and then new funding to cover the expense is raised from other banks (not necessarily the outstanding equity holders).
market value of a date-two dollar. It will be convenient to define the date-one value of all equity from all continuations as

\[ E(s) \equiv q_1(s) \int_{v \leq v^*(\epsilon,s)} \left( 1 - \rho(v,\epsilon,s) \right) v \tilde{I} dG(s), \]  

(4)

where \( G(s) \) is the joint probability law of \( v, \epsilon \) conditional on state \( s \).\(^{12}\)

In summary, the payoff from resolving an insolvent firm with viability \( v \) through continuation, in state \( s \), is \( R_1(v,\epsilon,s)D_0 = (-\epsilon + q_1(s)v)\tilde{I} \).

### 2.3 Banks

We assume that there is a continuum of identical banks that are risk neutral and have exogenous initial wealth \( A^b_0 \). Each bank has two options: it can consume its endowment immediately, or it can contract with firms to provide loans. Firms have full bargaining power in that relationship and choose loan terms subject to a bank participation constraint,

\[ A^b_0 \leq \mathbb{E}^S[c^b(s)], \]  

(5)

where \( \mathbb{E}^S[\cdot] \) denotes expectation over the date-one aggregate state \( s \). Banks must be willing to forgo consuming \( A^b_0 \) at date zero in order to instead lend those funds out and obtain the expected date-two consumption value \( c^b(s) \) associated with the loans.\(^{13}\) We assume that each bank contracts simultaneously with a cross-section of firms, so that the payoff to each bank from its contract is the cross-sectional expected value, contingent on the aggregate state. This assumption eases exposition but does not change results when banks do not default on their debt, as we assume.\(^{14}\)

A bank contract with a firm specifies a face value of debt \( D_0 \) and a payment \( q_0D_0 \) to the firm.

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\(^{12}\)Formally, for any \( s \) and any measurable \( A \subset \mathbb{R}^2 \), \( G(A|s) \equiv \mathbb{P} \left( (v,\epsilon) \in A | s \right) \), where \( \mathbb{P} \) is the probability measure for the space over which our model is defined.

\(^{13}\)We assume without loss of generality that banks forgo date-one consumption to instead lend again and consume at date two.

\(^{14}\)An alternative assumption along these lines would be that banks can hedge idiosyncratic firm risk, and so receive the expected repayment \( \mathbb{E}[R_1(v,\epsilon,s)]D_0 \).
A bank may finance an amount \( q_0 D_0 > A_0^b \) by raising debt \( B_0 \) from households at exogenous price \( q \), so that its debt level is

\[
B_0 = \frac{q_0 D_0 - A_0^b}{q}. \tag{6}
\]

At date one, banks that contract with firms receive total payout \( \mathbb{E}[\mathcal{R}_1(v, \epsilon, s)]D_0 \) and repay households \( B_0 \).\(^{15}\) Thus, all banks are identical at date one. We relax this assumption in Section 5.1.

After receiving its payout, each bank can choose to re-lend to firms at price \( q_1(s) \). Banks can borrow again from households at a price of one. We let \( B_1(s) \) denote the quantity borrowed, which may be negative if a bank stores funds with households to consume at date two.\(^{16}\) A bank’s date-one loan supply is thus

\[
Q^S(s) \equiv B_1(s) - B_0 + \int_{v \geq v^*(\epsilon, s)} D_0 dG(s) + \int_{v \leq v^*(\epsilon, s)} \left( - (1 - \rho(v, \epsilon, s)) \epsilon + \rho(v, \epsilon, s) \gamma(s) \right) \tilde{I} dG(s), \tag{7}
\]

where \(- (1 - \rho(v, \epsilon, s)) \epsilon\) accounts for funds that are unavailable because they are used to restructure insolvent firms. Note that equity from continuations does not appear in loan supply at date one because the associated cashflows are received at date two.\(^{17}\)

Bank borrowing from households at date one is limited by a collateral constraint, given by

\[
B_1(s) \leq \phi(s) \left( Q^S(s) + \bar{\epsilon}(s) \right), \tag{8}
\]

where \( 1 - \phi(s) \) is an exogenous state-dependent haircut on the market value of collateral and

\(^{15}\)We assume that banks do not default on household debt.

\(^{16}\)Since banks do not discount the future, they are indifferent between consuming unused funds at date one and storing them with households.

\(^{17}\)Likewise, the reorganized firm’s capital structure can be determined at date two without affecting the date-one equilibrium.
\( \varepsilon(s) \) is the equity in restructured firms. Low values of \( \phi(s) \) thus correspond to large haircuts, constraining bank borrowing.

Banks’ date-two cashflows arise from equity in restructured firms and loans extended at date one, net of repayment to households (or alternatively, funds received from households if \( B_1(s) < 0 \)),

\[
c^b(s) = \frac{1}{q_1(s)} \left( Q^S(s) + \varepsilon(s) \right) - B_1(s). \tag{9}
\]

At date one, the decision problem of banks is therefore to choose household debt \( B_1(s) \) and lending supply \( Q^S(s) \) to maximize final consumption, subject to the budget constraint (7) and the collateral constraint (8).

This maximization turns out to be trivial. Banks choose not to lend if \( q_1(s) > 1 \), so we can focus on cases \( q_1(s) \leq 1 \) since firms have positive loan demand. If \( q_1(s) = 1 \), then banks are indifferent between any lending scales. If \( q_1(s) < 1 \), then banks choose maximal leverage: the collateral constraint binds. We can therefore focus on the case of maximal leverage.\(^{19}\) Given maximal leverage, we can combine the collateral constraint with the budget constraint to obtain loan supply

\[
Q^S(s) = \frac{1}{1 - \phi(s)} \left[ \phi(s)\varepsilon(s) - B_0 + \int_{v \geq v^*(\varepsilon, s)} D_0dG(s) \right. \\
+ \left. \int_{v \leq v^*(\varepsilon, s)} \left( -\rho(v, \varepsilon, s)\varepsilon + \rho(v, \varepsilon, s)\gamma(s) \right) \tilde{I}dG(s) \right]. \tag{10}
\]

Finally, we can apply equation (8) to back out the debt level \( B_1(s) \). Substituting \( B_1(s) \) back into

\(^{18}\)Recall that banks do not discount cashflows. Banks lend to liquidity constrained firms to obtain \( 1/q_1(s) \) dollars at date two for each dollar provided at date one.

\(^{19}\)Note that if this results in a violation of market clearing then it must be that \( q_1(s) = 1 \), in which case banks receive the same final consumption regardless of their leverage choice. Thus, characterizing lending and final consumption under maximal leverage is sufficient for solving the model.
the date-zero bank participation constraint, we obtain

\[
A_0^b = \mathbb{E}^S \left[ \frac{1}{q_1(s)} \left( Q^S(s) + \bar{\gamma}(s) \right) - B_1(s) \right]
\]

\[
= \mathbb{E}^S \left[ \frac{1 - \phi(s)q_1(s)}{q_1(s)} \left( Q^S(s) + \bar{\gamma}(s) \right) \right].
\]

(11)

This participation constraint is taken as a constraint by firms in their decision problem. It is immediate that the participation constraint always binds.

### 2.4 Firm optimization

Price-taking firms take date-one loan prices \( q_1(s) \) and liquidation prices \( \gamma(s) \) as given. At date zero, firms choose investment, consumption, and debt-contract terms to maximize utility subject to the date-zero budget constraint (1) and bank participation constraint (11):

\[
\sup_{I, c_f^0, D_0, q_0} u_0^f(c_f^0) + \mathbb{E}^S \left[ \int_{v \geq v^*(\varepsilon, s)} \left[ vI - \frac{1}{q_1(s)} \left( \varepsilon I + D_0 \right) \right] dG(s) \right].
\]

(12)

Equation (12) embeds three key assumptions. First, we assume that insolvent firms’ date-one cashflows do not enter date-zero firm utility. Each firm correctly anticipates that if it is insolvent at date one, then banks receive all of its cashflows. Second, firms may choose any debt level and interest rate subject to bank participation constraints. Intuitively, we assume that the banking sector is perfectly competitive so firms extract all surplus from date-zero loans. Third, firms are able to perfectly commit to a future liquidation or continuation through debt-contract terms (i.e., liquidation-enabling security interests or continuation-enabling credit lines). While this is an unusual assumption, we show in Section 3.1 that firms’ optimal choices correspond to a realistic system in which banks choose between continuation and liquidation at date one to maximize recovery. Thus, we show as a result that firms would commit to a creditor-recovery-maximizing liquidation rule if it were feasible.
Interpretation as Costly State Verification. Our model could be interpreted as a model in which
the long-run value \( v \) is subject to costly state verification, where \( \chi \) is the resource cost of verifying
the state.\(^{20}\) It is well known that with costly state verification, debt-equity contracts are the only
renegotiation-proof contracts (Townsend (1979), Chari and Kehoe (2016)). The insolvency system
in our model is the joint process of (i) verifying the state after insolvency has been declared, and
(ii) the creditors using that verification to decide whether to liquidate or restructure the firm. We
assume that the liquidation rule can be committed to by the firm, contingent on the verification
of the state in insolvency, but we show that the privately optimal liquidation rule is equivalent
to allowing the bank to choose between liquidation and continuation ex post. In this sense, we
could think of our model as either a model of ex ante contractual provisions or of ex post creditor
decisions. We will show that the social optimum can be implemented using an ex post subsidy on
liquidation or continuation, which does not require the planner to observe \( v \). This gives some appeal
to the interpretation of an ex-post formal bankruptcy system in which the planner can intervene in
existing contracts.

2.5 Arbitrageurs

Arbitrageurs are the second-best users of firms’ liquidated investment projects. We interpret ar-
bitrageurs as strategic buyers, such as new entrants, who repurpose firms’ liquidated assets for
another business. Arbitrageurs have a technology that converts \( L(s) \) dollars of liquidated assets
into \( F(L(s), s) \) units of consumption.

At date one, arbitrageurs take the price \( \gamma(s) \) of liquidated assets as given and choose a quantity
\( L(s) \) of liquidated assets to purchase to maximize their utility in each state,\(^{21}\)

\(^{20}\)Technically, our model makes \( D_0 \) wholly non-contingent, but our results can be extended to the case where
\( D_0(\epsilon, s) \) depends on the verifiable states \((\epsilon, s)\) and not on the non-verifiable state \( v \).

\(^{21}\)While arbitrageurs’ date-zero utility is irrelevant for the competitive equilibrium definition, some assumptions are
necessary to formalize the social planner’s treatment of arbitrageurs. At date zero, arbitrageurs have exogenous initial
wealth \( A_0^a \). Arbitrageurs are borrowing constrained when maximizing a date-zero exogenous utility function \( u^a(\cdot) \) and
cannot borrow against future income. We assume that \( u'' > 1 \) so that the borrowing constraint binds. The arbitrageur
borrowing constraint is a simple method of eliminating arbitrageur surplus at date one from welfare considerations
when the social planner assigns a welfare weight of zero, which simplifies analysis. However, given the bank collateral
constraint, we could obtain similar welfare results without the borrowing constraint and welfare weight of zero. See
The first-order condition for equation (13) implies that the equilibrium quantity \( L(s) \) of liquidated assets must satisfy:

\[
\gamma(s) = \frac{\partial \bar{F}(L(s), s)}{\partial L(s)}.
\] (14)

### 2.6 Market clearing and competitive equilibrium

Our model has two state contingent equilibrium prices, the date-one loan price \( q_1(s) \) and the date-one liquidation price \( \gamma(s) \). We now state the market-clearing conditions of the economy that determine these prices. We then define a competitive equilibrium.

#### 2.6.1 Date-one loan-market clearing

At date one, the market for loans must clear, that is

\[
Q^S(s) \geq Q^D(s),
\] (15)

with inequality only if \( q_1(s) = 1 \). Recall that by convention, we have defined \( Q^S(s) \) to be loan supply under maximal bank leverage. If \( q_1(s) = 1 \), then loan market clearing may be obtained without maximal bank leverage, and banks are indifferent to any leverage.

#### 2.6.2 Date-one liquidation market clearing

At date one, the market for liquidations must also clear, with insolvent firms supplying liquidated assets and arbitrageurs demanding them. We therefore have

\[
L(s) = \int_{v \leq v^*(\varepsilon, s)} \rho(v, \varepsilon, s) \tilde{I} dG(s).
\] (16)

Appendix C.3.2 for further discussion.
It is convenient to directly define the equilibrium price by substituting market clearing into the demand function (14) of arbitrageurs. Doing so, we obtain

$$\gamma(L(s), s) \equiv \frac{\partial \mathcal{F}(L(s), s)}{\partial L(s)}. \quad (17)$$

There is thus a fire-sale spillover, in which additional liquidations lower the proceeds from all liquidations, whenever $\partial^2 \mathcal{F}(L(s), s)/\partial^2 L(s) < 0$.

2.6.3 Competitive equilibrium

A competitive equilibrium consists of allocations $(D_0, c_0^f, I, \rho, L, Q^S, q_0D_0, B_0, B_1)$ and prices $\gamma(s), q_1(s)$ satisfying the following conditions:

1. Given prices, firms choose $(I, c_0^f, D_0, q_0D_0, \rho)$ to solve (12).

2. Banks’ date-zero participation constraint (11) and budget constraint (6) are satisfied, given $B_0$ and the equilibrium contract.

3. Banks choose $(B_1, Q^S)$ to maximize utility (9) at date one given prices.

4. Arbitrageurs choose $L(s)$ to maximize utility (13) at date one given prices.

5. The date-one loan-market-clearing conditions (15) and liquidation-market-clearing conditions (16) are satisfied.

3 Optimal liquidation rules

In this section, we characterize the privately optimal liquidation rule $\rho$ that would be set by firms and banks as part of their optimal contract in a competitive equilibrium. We compare it with the socially optimal liquidation rule that would be set by a social planner, who internalizes effects on both the liquidation prices $\gamma(s)$ and the loan prices $q_1(s)$. We show that both the privately and socially optimal rules take the form of threshold rules: firms with idiosyncratic $v$ values below a
threshold are liquidated. However, the privately and socially optimal thresholds differ. In particular, we show that the socially optimal rule may imply more liquidations or fewer liquidations than the privately optimal rule.

For the results that follow, it is helpful to define the creditor-recovery-maximizing liquidation rule as follows.

**Definition 1.** The creditor-recovery-maximizing liquidation rule is a threshold rule for liquidation

\[
\rho(v, \epsilon, s) = \begin{cases} 
0, & v \geq v^L(\epsilon, s) \\
1, & v < v^L(\epsilon, s)
\end{cases},
\]

where the threshold \( v^L(\epsilon, s) \) is given by

\[
-\epsilon + q_1(s)v^L(\epsilon, s) = \gamma(s).
\]

The creditor-recovery-maximizing liquidation rule is the rule that maximizes the discounted payoff to creditors in insolvency. In other words, it is the liquidation rule that would be adopted if banks decided at date one whether to reorganize or liquidate insolvent firms. The indifference point \( v^L(\epsilon, s) \) is the point at which the date-one present value of reorganizing the firm and liquidating the firm are equated. At higher values of \( v \), the present value of continuation is higher than liquidation, and firms are always reorganized. At lower values of \( v \), the present value of liquidation is higher, and firms are always liquidated.

### 3.1 Privately optimal liquidation

We begin by studying the privately optimal liquidation rule \( \rho \) that arises in a competitive equilibrium, which is characterized in the following proposition.

**Proposition 1.** The privately optimal liquidation rule in competitive equilibrium is the creditor-recovery-maximizing liquidation rule.
Although the privately optimal liquidation rule is chosen at date zero by firms (debtors), it nevertheless coincides with the creditor-recovery-maximizing liquidation rule that would be chosen at date one by banks (creditors). The intuition is that higher recovery values to banks in insolvency relax the date-zero bank participation constraint. This allows firms to obtain more financing from banks ex ante. As a result, the optimal liquidation rule from the perspective of firms coincides with the optimal rule from the perspective of banks.

All else equal, in the competitive equilibrium, the liquidation threshold $v^L(\mathcal{E}, s)$ decreases in $q_1(s)$ and increases in $\gamma(s)$. Higher loan prices $q_1(s)$ increase the date-one present value of continuation, and lead banks to restructure more firms rather than liquidate them. Conversely, higher liquidation prices $\gamma(s)$ increase the recovery value from liquidating a firm, discouraging continuation.

### 3.2 Socially optimal liquidation

We next study the socially optimal liquidation rule that is designed by a social planner who internalizes the determination of equilibrium prices, but must otherwise respect the same constraints faced by private agents. For expositional simplicity, we assume the planner places a welfare weight of zero on arbitrageurs. We relax this assumption in Appendix C.3. Both banks and households break even in expectation, and so do not feature in welfare calculations.

The optimal regulatory problem of the social planner is to choose a complete set of Pigouvian wedges $\tau$ on the firm decision problem, with revenues remitted lump-sum back to firms, to maximize firm welfare (12). The complete set of wedges available are therefore a wedge $\tau^D_0$ on debt issuance, $\tau^I$ on investment scale, $\tau^c_0$ on consumption, and fully contingent wedges $\tau^\rho(v, \mathcal{E}, s)$ on the liquidation rule $\rho$. Given that the bank decision at date zero is pinned down by the firm participation constraint, wedges on firms are sufficient from a regulatory perspective — controlling firm debt is equivalent to controlling bank debt.

Given the planner possesses a complete set of controls over firm behavior, we can instead

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22Appendix C.3 considers alternative welfare criteria.
directly solve the constrained efficient planning problem of the planner in terms of allocations, and then back out the implementing wedges. In particular, the constrained efficient planning problem is to maximize firm welfare subject to the firm date-zero budget constraint (1), the bank participation constraint (11), and also to the liquidation and loan market clearing conditions (14) and (15).

The social optimum involves a full specification \((I, c^f_0, D_0, q_0D_0, \rho)\) of the firms’ allocations. For the moment, we focus on the socially optimal liquidation rule, which is characterized in the following proposition.

**Proposition 2.** The socially optimal liquidation rule is a threshold rule, with

\[
q_1(s)v_L(\epsilon, s) = \left(1 + \frac{\mu(s)}{\beta(s)}\right) \left(\gamma(s) + \epsilon - \frac{\partial \gamma(L(s), s)}{\partial L(s)} L(s)\right),
\]

where \(\Lambda\) and \(\mu(s)\) are Lagrange multipliers on the bank participation constraint and the loan-market clearing constraint in state \(s\), respectively, and where

\[
\beta(s) \equiv \Lambda \frac{1 - \phi(s)q_1(s)}{(1 - \phi(s))q_1(s)} + \mu(s) \frac{\phi(s)}{1 - \phi(s)}
\]

is the marginal social value of bank wealth at date one in state \(s\).

Like the privately optimal liquidation rule, the socially optimal liquidation rule also takes the form of a threshold rule. However, the socially optimal threshold \(v_L(\epsilon, s)\) differs from the privately optimal threshold in two important ways.

First, there is an externality arising from fire sales of firm assets. Absent externalities from the loan price \(q_1(s)\), this pushes the threshold downward, and the socially optimal liquidation rule encourages more continuations than the privately optimal liquidation rule does.

Second, there is an externality arising from the loan price when banks are collateral constrained and hence \(q_1(s) < 1\). Absent liquidation-price externalities, when the multiplier \(\mu(s)\) is positive, the socially optimal liquidation rule discourages continuation relative to the privately optimal rule.
because there is social value to higher prices $q_1(s)$. A positive multiplier $\mu(s) > 0$ indicates a positive social value to loanable funds at date one. In other words, $\mu(s) > 0$ indicates that a government would like to create a unit of loanable funds if it were able to, even if it earned no direct economic surplus by doing so. For example, it implies that there is potentially positive social value to capital injections into banks, or to macroprudential regulation of bank debt. In fact, we show below that optimal policy also involves positive taxes on bank debt when $\mu(s) > 0$ to increase loanable funds and relax the bank collateral constraint, but would involve subsidies of bank debt when $\mu(s) < 0$ in order to tighten the collateral constraint.

There are three core externalities underlying the determination of the welfare impact of an increase in $q_1(s)$ and hence the social value $\mu(s)$ of loanable funds. First, there is a distributive externality: a higher price redistributes resources at date one to firms and away from banks. This externality is negative from the firm perspective when repayment to banks is more valuable than repayment to firms due to the participation constraint, that is $\Lambda > 1$. It pushes for $\mu(s) < 0$ and a more continuation-friendly liquidation rule in order to reduce the equilibrium price and redistribute repayment towards banks. This externality naturally scales in the loan demand of solvent firms.

The second externality is a solvency externality: a higher price enhances the ability of firms to roll over debt at date one, and so increases the solvency threshold $v^*(\varepsilon, s)$. This results in a positive externality for firms, and so contributes towards $\mu(s) > 0$ and a liquidation-friendly liquidation rule. The intuition is that by closing more insolvent firms, more bank resources are freed up for lending to solvent firms: banks no longer need to cover the expense $\varepsilon$ and banks receive the immediate liquidation value $\gamma(s)$ at date one rather than the final value $v$ at date two. As a result, liquidation increases the supply of loanable funds to solvent firms, and so increases the loan price and boosts firm solvency.

The final externality is a revaluation of insolvent firms: the present value of bank claims on insolvent but restructured firms rises with the price, increasing bank net worth and resulting in a positive externality.
Formally, these externalities can be seen in the multiplier, which when non-zero is given by

\[
\mu(s) \frac{\partial [Q^D(s) - Q^S(s)]}{\partial q_1(s)} = \underbrace{\frac{1 - \Lambda}{q_1(s)^2} Q^D(s)}_{\text{Distributive Externality, } \leq 0} + \underbrace{\frac{\Lambda}{q_1(s)^2} \frac{\phi(s)(1 - q_1(s))}{1 - \phi(s)} \bar{c}(s)}_{\text{Revaluation Externality, } \geq 0} \\
+ \Lambda \frac{1 - \phi(s)q_1(s)}{(1 - \phi(s))q_1(s)} \int_{\varepsilon \geq 0} \left| \frac{\partial v^*(\varepsilon, s)}{\partial q_1(s)} \right| \chi^* v^*(\varepsilon, s) d\varepsilon, \tag{22}
\]

where \( v^* (\varepsilon, s) \equiv D_0 g \left( v^*(\varepsilon, s), \varepsilon | s \right) \) is the value of debt repaid to banks by the marginally solvent firm with expense \( \varepsilon \). Equation (22) highlights three economic conditions under which the social value of loanable funds is particularly high at date one (\( \mu(s) \) is large). The first is that loan demand of insolvent firms or of the marginally solvent firm is high relative to demand by solvent firms. In this case, the revaluation and solvency externalities are large relative to the distributive externality. The second is that the solvency threshold is particularly sensitive to the price. In this case, the welfare cost of the solvency externality is large. The third is that the distress cost \( \chi \) is large, in which case once again the solvency externality is large.

### 3.3 Implementation

We next turn to the question of how a social planner might implement the socially optimal liquidation rule in practice. In solving the constrained efficient planning problem underlying Proposition 2, we have assumed that the social planner possesses a complete set of contingent wedges \( \tau^\rho (v, \varepsilon, s) \), which can be used to influence the decision of a firm who realizes state \( (v, \varepsilon) \) over whether to liquidate or continue in aggregate state \( s \). In practice, the firm financing needs \( \varepsilon \) and the aggregate state \( s \) may both be known to the planner. However, the long-run value \( v \) of an insolvent firm must generally be estimated. While a firm and its creditors may eventually learn the value of \( v \), it could prove difficult to credibly convey this information to a social planner due to incentives for manipulating the planner’s actions. Implementing the socially optimal rule thus requires an incentive-compatible mechanism for eliciting long-run values. We now characterize a feasible
incentive-compatible implementation based on subsidies.

**Proposition 3.** Suppose that \( \varepsilon \) and \( s \) are observable to the social planner, but \( v \) is not. Define

\[
\tau(\varepsilon, s) \equiv \frac{\mu(s)}{\beta(s)} \left( \gamma(s) + \varepsilon \right) - \left( 1 + \frac{\mu(s)}{\beta(s)} \right) \left| \frac{\partial \gamma(L(s), s)}{\partial L(s)} \right| L(s). \tag{23}
\]

Then the following approach implements the socially optimal liquidation rule of Proposition 2.

1. If \( \tau(\varepsilon, s) < 0 \), the social planner provides a subsidy for continuation, financed by a lump sum tax on banks. The subsidy per unit of firm scale is \( |\tau(\varepsilon, s)| \).

2. If \( \tau(\varepsilon, s) \geq 0 \), the social planner provides a subsidy for liquidation, financed by a lump sum tax on banks. The subsidy per unit of firm scale is \( \tau(\varepsilon, s) \).

Proposition 3 provides an implementation of the optimal mechanism that does not require the social planner to directly observe long-run firm value \( v \). The implementation has the social planner subsidize continuation or liquidation, providing a subsidy at date one to agents choosing the desired outcome. The size of this fixed subsidy is calibrated so that creditors facing this subsidy are indifferent between liquidating or restructuring a firm if and only if the social planner is indifferent as well. Moreover, because the subsidy does not depend on \( v \), it does not change the threshold property of the optimal liquidation rule and leads private agents to implement the socially optimal rule.

This approach of providing a subsidy requires a lump-sum levy on banks to finance the scheme. It could be difficult to implement a tax on banks during times of crisis. An alternative possible implementation is for the planner to finance a subsidy at date one by levying a tax on banks at date zero. This is a natural implementation that most closely respects the desire of a planner to transfer resources to banks (“bailouts”) at date one. However, provided that such insurance is sufficiently costly, for example because it requires distortionary costs of raising money from taxpayers at date one, a social planner may only desire to partially correct the liquidation rule. In this case, a
planner would partially subsidize liquidations or continuations, but would not be able to achieve the socially optimal rule.

Finally, it should be noted that even if private agents had the ability to contract privately for bailout revenues from taxpayers at a price greater than one, private agents would generically fail to achieve efficient policies on two fronts. First, private agents would not set the correct liquidation rule, for the reasons outlined above. Second, private agents would not set the correct level of bailout transfers, not internalizing the loan and liquidation-price externalities. A social planner therefore not only alters the level of bailouts to banks, but also makes them contingent on the liquidation rule by providing subsidies based on the outcome of the insolvency process.

3.4 Relationship with macroprudential regulation

We have focused so far on characterizing the socially optimal liquidation rule. The social optimum studied also features macroprudential policies governing the other aspects of the contracting relationship, including bank debt $B_0$ and firm debt $D_0$. We now characterize socially optimal macroprudential controls over firm and bank debt.

**Proposition 4.** The socially optimal macroprudential debt taxes at date zero are given by

\[
\tau^B_0 = \frac{1}{\lambda} \mathbb{E}^S \left[ \mu(s) \frac{1 - \phi(s)}{1 - \phi(s)} \right] \tag{24}
\]

\[
\tau^D_0 = \frac{1}{\lambda} \mathbb{E}^S \left[ \phi(s) \mu(s) \int \left| \frac{\partial v^*(\varepsilon, s)}{\partial D_0} \right| q_1(s)v^*(\varepsilon, s)\chi I g(v^*(\varepsilon, s), \varepsilon|s) d\varepsilon - \int_{v \geq v^*(\varepsilon, s)} dG(s) \right] \]. \tag{25}

Macroprudential regulation of firm and bank debt both revolve around the loan price externality. The fact that the liquidation price externality does not feature is because even though marginal increases in debt change the solvency threshold $v^*$, the marginal insolvent firm is restructured rather
than liquidated. As a result, although the optimal liquidation rule trades off both externalities, optimal macroprudential regulation focuses on the loan price externality.

Consider first the optimal tax on bank debt, $\tau^B_0$. This is positive when the social value of loanable funds at date one is positive, that is $\mu(s) > 0$. This is because an increase in debt, all else equal, tightens the banks’ collateral constraints and forces a reduction in credit supply, in common with many models featuring collateral constraints.

Consider by contrast the optimal tax on firm debt. The optimal tax on firm debt is zero if $\phi(s) = 0$, and hence banks cannot borrow against their net worth at date one. The optimal tax on firm debt is thus related to the product of the loan price externality, $\mu(s)$, times the change in bank net worth at date zero. When the loan price externality is positive, the tax on firm debt is thus positive when an increase in firm debt, all else equal, reduces bank net worth. There are two competing forces at play. First, an increase in firm debt increases the solvency threshold, pushing more firms into insolvency and resulting in insolvency losses. This reduces net worth and loan supply and pushes for a positive tax on firm debt. However, there is also a distributive externality: an increase in firm debt redistributes resources at date one from solvent firms to banks. The distributive externality pushes for a subsidy on firm debt.

Proposition 4 therefore shows that conventional macroprudential policies and insolvency intervention pair together in the socially optimal regime. A positive loan price externality, $\mu(s) > 0$, pushes for more liquidations ex post, but also pushes for a positive tax on bank debt. It also pushes for a positive tax on firm debt when the solvency externality on firms dominates. All three of these policies synergize in increasing the supply of loanable funds to firms at date one.

### 3.5 Comparative statics in an illustrative example

In this section, we demonstrate how the socially optimal liquidation rule depends on the parameters of the model. Beginning with parametric assumptions described in Appendix B, we change certain parameter values to provide intuition for the model solution. This exercise is purely illustrative — our propositions do not rely on the assumptions in Appendix B and we do not claim that these
assumed parameter values are representative of any crisis.

To begin, we show that the social planner liquidates more firms when banks face tighter collateral constraints. We start with the assumptions described in Appendix B. Notably, we assume that there is a unique state $s$, so we suppress $s$ from our notation in this exercise. We vary the parameter $\phi$ across a grid of potential values and we calculate the corresponding socially optimal liquidation threshold $v^L(\bar{\epsilon})$ for the highest possible realization $\bar{\epsilon}$ of $\epsilon$. We also calculate the creditor-recovery-maximizing threshold $v^{\text{private}}(\bar{\epsilon}) = (\gamma + \bar{\epsilon})/q_1$ from equation (19). In Figure 1(a), the blue line plots the difference between the socially optimal threshold $v^L(\bar{\epsilon})$ and the privately optimal threshold $v^{\text{private}}(\bar{\epsilon})$. The x-axis shows the assumed value of the collateral haircut $1 - \phi$. When $\phi$ is close to one, the collateral haircut is $1 - \phi$ is small. Banks are thus able to easily supply date-one loans by borrowing against date-two cash flows. This corresponds to a crisis in which bank lending is relatively unconstrained. In this case, the market-clearing constraint (15) does not bind for the social planner. Indeed, this constraint does not bind for any of the $1 - \phi$ values to the left of the vertical line in Figure 1(a). Since the parameter $\phi$ only affects the social planner’s problem by constraining loan supply (equation (8)), it follows that the value of $\phi$ does not affect the planner’s problem in this region.

When the market-clearing constraint (15) does not bind, Figure 1(a) shows that the socially optimal threshold is lower than the private threshold — the blue line is negative. When bank lending is unconstrained, the planner thus liquidates fewer firms than privately optimizing agents would. This follows directly from Proposition 2: $\mu = 0$ implies that liquidation externalities push the efficient threshold of Proposition 2 below the private threshold from equation (19). Thus, when bank lending is unconstrained, the social planner optimally discourages liquidations relative to a competitive equilibrium. However, once the market-clearing constraint binds, increases in the collateral haircut $1 - \phi$ make the constraint bind tighter, increasing the Lagrange multiplier $\mu$. Comparing Proposition 2 to equation (19) shows that increases in $\mu$ increase the socially optimal liquidation threshold relative to the private threshold. This can be seen in Figure 1(a), in which the blue line increases in $1 - \phi$ when the market-clearing constraint binds. When $1 - \phi$ is very large,
banks become so constrained that the blue line rises above zero, meaning the planner encourages liquidations relative to a competitive equilibrium. The planner does this because liquidations lower the market-clearing interest rate by increasing the net supply of bank loans, allowing more firms to roll over debt and avoid insolvency (equation (2)).

Figure 1(a) demonstrates the key tradeoff faced by the planner. Frequent liquidations lead to fire sales that lower bank recovery, raising the cost of capital at date zero. However, restructurings deplete bank capital, preventing some firms from rolling over debt to avoid insolvency. The planner thus encourages liquidations when bank lending is constrained and discourages liquidations otherwise.

Finally, we show that this intuition leads to a surprising comparative static: extreme fire sales can actually motivate more liquidations under the efficient policy. To demonstrate this, we begin once again with the assumptions of Appendix B, which imply that liquidation prices are affine in the quantity of liquidated assets. Specifically, we assume that \[ \gamma(L) = \gamma_0 + \gamma_1 L \] for parameters \( \gamma_0 > 0, \gamma_1 < 0 \). Thus, the magnitude of \( \gamma_1 \) determines the severity of fire sales: \( \| \partial \gamma(L) / \partial L \| = |\gamma_1| \). We vary the parameter \( \gamma_1 \) across a grid of potential values and, as above, we calculate the corresponding socially optimal liquidation threshold and creditor-recovery-maximizing liquidation threshold. In Figure 1(b), the blue line once again plots the difference between the socially optimal threshold \( v^L(\bar{\epsilon}) \) and the privately optimal threshold \( v^{private}(\bar{\epsilon}) \). The x-axis shows the assumed magnitude \( |\gamma_1| \) of the fire-sale externality. When \( |\gamma_1| \) is small, bank recovery in liquidations high. Banks can lend out the proceeds from liquidations to satisfy loan demand. Thus, for low fire-sale magnitudes to the left of the vertical line in Figure 1(b), the market-clearing constraint does not bind. In this region, increases in the fire-sale magnitude \( |\gamma_1| \) lead to lower efficient liquidation thresholds. The planner reduces the frequency of liquidations when the negative externality associated with liquidations increases. However, once fire sales become extreme, bank recovery in liquidations falls so low that banks no longer have excess funds to satisfy loan demand. Once the market-clearing constraint binds, an increase in fire-sale severity tightens bank lending constraints. By the same intuition described above, tighter constraints can actually lead the planner to increase the
liquidation threshold and liquidate more firms. This can be seen in Figure 1(b), in which the blue line increases once the market-clearing constraint binds.

4 Policy discussion

Proposition 3 shows that a social planner can induce private creditors to efficiently resolve insolvent firms by subsidizing liquidations or continuations, depending on the nature of the crisis. The planner does not need to observe the long-run viability of individual firms to calculate the optimal subsidy. In this section, we discuss the practical implementation of such a policy. In Section 4.1, we discuss a potential policy tool for flexibly subsidizing liquidations or continuations. We discuss related policies that have been implemented and proposed in Sections 4.2 and 4.3, respectively.

4.1 Practical implementation

To effectively mitigate the crisis externalities that we model, policymakers need tools that can be quickly implemented. Ideally, a crisis response should not require a lengthy legislative process or new government fundraising, which could create delays. Further, our results imply that policy tools must be able to subsidize liquidation or continuation, depending on the nature of the crisis. We argue below that conditional tax forgiveness for bankrupt firms could feasibly and quickly implement the optimal policy in our model without immediate fundraising.23

In response to the COVID-19 pandemic, Blanchard, Philippon, and Pisani-Ferry (2020) propose that governments could deter liquidations by subsidizing restructurings that allow insolvent firms to continue operating.24 Specifically, Blanchard, Philippon, and Pisani-Ferry (2020) propose that governments could accept larger write downs or “haircuts” on tax claims than the haircuts accepted by private creditors. Such a policy amounts to subsidizing creditors in any restructuring that results in the continuation of an insolvent firm. Our model (Proposition 2) implies that such

23 Of course, forgiven taxes would eventually need to be offset for the government to balance its budget.
24 Similarly, Greenwood and Thesmar (2020) propose that the government could create a tax credit for lenders and landlords that agree to a firm-preserving restructuring.
a policy is socially optimal if banks are not financially constrained and fire-sale externalities are nontrivial. However, this form of government subsidy could just as easily be applied to incentivize liquidations. This policy tool is thus an attractive means of implementing the optimal policy of Proposition 3, in which either liquidation or continuation is subsidized.

Implementing the optimal policy of Proposition 3 through the approach proposed by Blanchard, Philippon, and Pisani-Ferry (2020) would be especially feasible in the US Chapter 11 bankruptcy system. In the US, bankrupt firms frequently owe money to the Internal Revenue Services (IRS) for unpaid taxes. According to 11 U.S.C. §507(a)8, IRS claims receive priority over general unsecured claims. The government could thus increase unsecured creditor recovery by announcing that it would accept plans in which IRS claims receive zero recovery. The government could subsidize a specific bankruptcy outcome, such as continuation or liquidation, by announcing that the subsidy only applies to plans implementing that outcome. In each bankruptcy, the U.S. Trustee could determine whether a plan meets the desired criteria.

The strength of this implementation approach is its flexibility. The government could subsidize liquidation or continuation without novel legislation or fundraising. In the context of our model, these subsidies can improve social welfare in a crisis. However, several caveats are in order. First, targeted tax forgiveness could incentivize nonbankrupt firms to distort their behavior in anticipation of future tax forgiveness. This distortion could lead to suboptimal firm investment and affect government tax revenues. Second, to the extent that tax forgiveness for bankrupt firms must be offset by the government forgoing some future spending, the welfare benefit of the bankruptcy subsidy could be more than offset by the welfare cost of the future forgone policy. Third and perhaps most importantly, implementable subsidy amounts would be limited by the size of the government’s

\[25\] Indeed, the government accepting a larger haircut on distressed debt is analogous to the implementation that we describe in Proposition 3.

\[26\] For example, the IRS held a $9.5 million claim in Guitar Center’s bankruptcy, a $22 million claim in J. Crew’s bankruptcy, and a $9.5 million claim in GNC’s bankruptcy. See https://cases.primeclerk.com/GNC/Home-ClaimDetails?id=NDQzNDU1MQ==; and https://cases.primeclerk.com/guitarcenter/Home-ClaimDetails?id=NDk2OTYyNQ==; and https://casedocs.omniagentsolutions.com/pocvol1/JCrew/Claim%20Scan/Claims/20-32181/7095000322.pdf.

\[27\] The government could theoretically do this without hindering plan confirmation because the fair and equitable standard only applies to creditors that do not accept a plan (11 U.S.C. §1129(b)1).

\[28\] The U.S. Trustee is already tasked with reviewing reorganization plans, see 28 U.S.C. §586(a)3.
claim in a particular bankruptcy. Thus, while conditional tax forgiveness could feasibly implement the optimal policy in our model, we cannot recommend such a policy without further empirical evidence.

4.2 Related policies

In response to previous crises, governments have implemented policies to deter liquidations. In the wake of the COVID-19 pandemic, numerous state and local governments instituted moratoriums on the eviction of commercial tenants. For example, the “COVID-19 Emergency Protect Our Small Businesses Act of 2021,” which was signed into law on March 6th 2021, banned evictions and foreclosure actions relating to certain small commercial properties in the state of New York. These moratoriums are analogous to an extreme version of the policy of Proposition 3 in which an infinite tax is levied on liquidations.

Similarly, Section 4013 of the Coronavirus Aid, Relief, and Economic Security (CARES) Act of 2020 encouraged banks to engage in restructurings, rather than liquidations, with distressed borrowers. Prior to this intervention, if banks engaged in troubled debt restructurings (TDRs) with distressed borrowers to avoid liquidations, banks had to categorize the borrowers’ loans as impaired. The value of an impaired loan must be revised downward to the expected discounted value of the future cashflows generated by the loan. Such a downward revision of loan value could harm a bank by depleting its regulatory capital, potentially forcing the bank to sell other assets at fire-sale prices to reduce its leverage and satisfy capital requirements (Laux and Leuz, 2010). Thus, the CARES Act encouraged the continuation of insolvent firms by alleviating negative regulatory consequences for banks negotiating with firms that were adversely affected by COVID.

Historically, governments have been more hesitant to subsidize or encourage liquidations during a crisis. However, outside of crises, governments have provided incentives for liquidations.

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For example, asset sales that are part of a liquidating Chapter 11 bankruptcy plan are exempt from any transfer or “stamp” taxes (11 U.S.C. §1146(a)). This tax exemption is effectively a subsidy for liquidations. Other policies have simply made it easier for lenders to liquidate firms. For example, a 2001 reform of Article 9 of the Uniform Commercial Code made it easier for secured lenders to foreclose on assets (Benmelech, Kumar, and Rajan, 2020). Likewise, antirecharacterization passed in states like Texas and Louisiana made it easier for creditors to seize assets associated with bankrupt firms (Ersahin, 2020). Prior to these laws, bankruptcy judges would sometimes recharacterize a debtor’s bankruptcy remote assets, typically held in a nonbankrupt affiliated special purpose vehicle, as part of the bankrupt firm, subjecting the assets to the automatic stay. By banning this practice of recharacterization, these laws effectively encouraged liquidations.

Internationally, the Czech Republic, France, Germany, Italy, Luxembourg, Portugal, Spain, Switzerland, and Turkey responded to COVID-19 by implementing temporary bankruptcy moratoriums. These interventions prevented creditors from initiating involuntary bankruptcy proceedings against insolvent firms (Gómez et al., 2020). Some of these moratoriums also suspended existing laws imposing personal liability on managers who fail to file for bankruptcy when their firms become insolvent.

4.3 Recent related proposals

The COVID-19 pandemic also led to academic proposals for government interventions aimed at mitigating social losses caused by liquidations. For example, as discussed above, Blanchard, Philippon, and Pisani-Ferry (2020) propose that governments could accept lower recovery in restructurings that allow firms to continue operating. Greenwood and Thesmar (2020) propose a tax credit for landlords and lenders that agree to such restructurings. Our model (Proposition 2) implies that such policies are socially optimal if banks are not financially constrained and fire-sale externalities are nontrivial.\(^\text{32}\) Given the health of the banking sector during the COVID-19 pan-

\(^{32}\)Indeed, the government accepting a larger haircut on distressed debt is analogous to the implementation that we describe in Proposition 3.
demic (Greenwood, Iverson, and Thesmar, 2020), it is likely that the social planner in our model would enact a similar policy in a crisis resembling the COVID-19 pandemic. More broadly, firms in the US have relied less on bank lending (Crouzet, 2018) in recent years and banks have been well capitalized (Corbae, D’Erasmo et al., Forthcoming). According to our model, this bolsters the case for subsidizing continuation in crises. Nonetheless, future crises or crises in different countries could call for the optimal subsidization of liquidation.

DeMarzo, Krishnamurthy, and Rauh (2020) propose that the Federal Reserve and the Treasury create a special purpose vehicle (SPV) to provide DIP loans to bankrupt firms. The SPV would provide highly subsidized loans to bankrupt firms, which would be fully collateralized, priming existing liens if necessary (11 U.S.C. §364(d)). While we do not formally model such a policy, our analysis nonetheless illustrates the potential benefits of the policy proposed by DeMarzo, Krishnamurthy, and Rauh (2020). Recall that in states in which banks are financially constrained, the social planner faces a tradeoff between reorganizing viable firms and preserving funding to solvent firms at reasonable credit spreads. If the government were to exogenously increase the supply of date-one loans to bankrupt firms at subsidized rates, then more reorganizations could be achieved without exacerbating the loan-price externality, in which privately supplied DIP loans crowd out funding to solvent firms. As such, our model suggests that this proposal may be especially effective when banks are also in financial distress and hence loan price externalities are strong. Outside of our model, it is possible that creditors may force marginally solvent firms into bankruptcy, increasing deadweight losses, in order to take advantage of subsidized government funding. However, this incentive for inefficient behavior would be mitigated by the fact that government DIP loans would be senior to all unsecured debt, and even potentially secured debt through priming liens.

Finally, other academics have proposed policies aimed at paying the debt of all firms to prevent deadweight losses associated with liquidation (Saez and Zucman, 2020; Hanson et al., 2020b). Specifically, Saez and Zucman (2020) recommend expanding unemployment insurance and paying a fraction of the maintenance costs of businesses in sectors that are affected by pandemic-induced shutdowns. Hanson et al. (2020b) propose an intervention in which the government would
“provide payment assistance to enable impacted businesses to meet their recurring fixed obligations—including interest, rent, lease, and utility payments.” We do not formally model interventions like these that entail paying the expenses of all operating firms in troubled sectors. Such an intervention is analogous to the implementation we describe in Proposition 3, in which the government subsidizes the continuation of insolvent businesses. Unlike the policy described in Proposition 3, these proposed interventions also subsidize the continuation of solvent businesses, making them potentially expensive. However, a policy like ours that makes aid contingent on demonstrated insolvency could incentivize firms to file for bankruptcy, incurring deadweight losses, just to take advantage of the government subsidy. In deciding how to allocate assistance, the government thus faces a tradeoff between the expense of helping solvent firms and the greater incentive for firms to demonstrate their insolvency through value-destroying bankruptcy filings.

5 Extensions

In this section, we present three main extensions of our model. First, we study the joint design of the insolvency rule and the seniority structure of debt when there are heterogeneous banks. Second, we study the possibility that insolvent firms can be acquired as an alternative to liquidation. Third, we study the possibility that other economic actors, such as arbitrageurs and firm employees, may appear in the social planner’s welfare criteria.

5.1 Heterogeneous banks

In Appendix C.1, we extend our model to consider the effect of heterogeneous bank creditors. In this extension, multiple creditors indexed by $b = 1, 2, ..., B$ face different state-contingent collateral constraints $\phi^b(s)$. We study the design of the liquidation rule as well as the optimal seniority structure among creditors. We show that a social planner can improve welfare by subordinating the claims of banks that, because of their idiosyncratic collateral constraints, struggle to lend against securities of distressed firms. Moreover, planner intervention in the seniority structure can serve
as a partial substitute for intervention in liquidation decisions.

We first show that in the presence of heterogeneous creditors, privately optimizing agents continue to choose the creditor-recovery-maximizing liquidation rule. Additionally, under the equilibrium seniority structure, the only banks that receive a positive recovery are those with the highest value of $\beta_{private}^b(s)$, where

$$\beta_{private}^b(s) \equiv \Lambda^b \frac{1 - \phi^b(s)q_1(s)}{(1 - \phi^b(s))q_1(s)}$$

and $\Lambda^b$ is the Lagrange multiplier on bank $b$’s participation constraint. Intuitively, firms allocate date-one recovery to banks that are most willing to lend against that dollar at date zero. In contrast, we show that the social planner assigns positive recovery to banks with the highest value of $\beta_{social}^b(s)$, where

$$\beta^b(s) \equiv \beta_{private}^b(s) + \mu(s) \frac{\phi^b(s)}{1 - \phi^b(s)}.$$  (27)

If $\mu(s) > 0$, the social planner thus assigns lower priority to banks that are constrained at date one (low $\phi^b(s)$) than firms would. Intuitively, the social planner internalizes the ability of banks with high $\phi^b(s)$ values to supply additional funds using distressed securities as collateral. If $\mu(s) > 0$, then the planner benefits from a higher supply of loanable funds, increasing the price at which banks lend and boosting firm solvency. The loan-price-externality motivation for altering the seniority structure among banks is therefore precisely the same loan-price-externality motivation for intervening in liquidation decisions. Because of this, we show that interventions in the seniority structure actually make it less necessary for a planner to intervene in liquidation decisions.

Interestingly, both private agents and the social planner choose a seniority structure with the potential for allocating all recovery to one bank. Such a seniority structure could potentially create a too-big-to-fail dilemma in future crises.

DeMarzo, Krishnamurthy, and Rauh (2020) propose that the government could provide DIP loans to bankrupt firms. Our heterogeneous banks extension (Appendix C.1) suggests a related
but different policy intervention into the market for DIP loans. The social planner of Appendix C.1 may wish to impose a bankruptcy seniority structure in which higher priority is given to those banks that are less constrained in their lending in a crisis. Such a seniority structure could be achieved by encouraging courts to approve more DIP loans and grant more priming liens. Such a policy would increase the seniority of DIP lenders, who by revealed preference are sufficiently unconstrained in their lending to provide financing to bankrupt firms. Even outside of a crisis, this socially optimal seniority structure provides a positive explanation for why DIP lenders enjoy high priority in bankruptcy.

5.2 Acquisitions

We assume that insolvent firms face two potential outcomes: liquidation or continuation. In practice, some insolvent firms are acquired. In Appendix C.2, we consider an extension in which arbitrageurs acquire insolvent firms to continue their operations, meaning that the value of a firm to arbitrageurs depends partly on its long-run value \( v \). We prove analogs of Propositions 1 and 2 in this modified setting, demonstrating that our key results are robust to allowing for acquisitions. Interestingly, the possibility of acquisitions can amplify the loan price externality and promote fewer continuations.

5.3 Arbitrageurs and workers in the social welfare function

We assume that the social planner places zero weight on arbitrageurs, and further that there are no spillovers from firm activities to other agents in the economy. However, if arbitrageurs use the purchased assets to start new businesses, it is reasonable to believe a social planner should consider arbitrageur welfare when determining the efficient intervention. Additionally, if firm employees are displaced by liquidations, a planner may also care about their employment losses, which might not be internalized by firms. In Appendix C.3, we extend the model first to allow for a positive welfare weight on arbitrageurs, and second to allow for a positive welfare weight on firm employee surplus (“wages”). We prove analogs of Proposition 2, showing that our main results are robust to
these additional welfare considerations.

6 Conclusion

We study policies that mitigate crises by altering the process for resolving insolvent firms. For example, in response to the COVID-19 pandemic, Section 4013 of the CARES Act suspended regulatory costs previously incurred by banks when negotiating with distressed borrowers to avoid liquidation. Solving a general equilibrium model, we show that policies like these can improve welfare by avoiding negative externalities associated with liquidations. Crisis interventions can thus improve welfare relative to existing rules like the best-interest-of-the-creditors test (11 U.S.C. §1129(a)7), which prohibits a Chapter 11 reorganization whenever a liquidation would improve creditor recovery. However, our model reveals that liquidation-preventing policies are not always beneficial. We show that a social planner will sometimes optimally intervene in a competitive equilibrium to liquidate more firms. If crisis conditions constrain bank lending, such an intervention improves welfare by reallocating scarce capital to stronger firms, helping them avoid financial distress. Surprisingly, an optimal policy response to extreme liquidation externalities sometimes calls for more liquidations, since banks harmed by these externalities must conserve capital for strong firms. Our results demonstrate that policymakers should jointly consider the strength of the banking sector and externalities in nonfinancial sectors when responding to crises.

References


Figure 1: Determinants of optimal liquidation thresholds

We solve our model under the assumptions outlined in Appendix B. We assume that there is a unique state $s$, so we suppress $s$ from our notation in this exercise. Holding other parameters fixed, we vary the collateral haircut $1 - \phi$ (panel a) and the magnitude of fire sales $\|\partial \gamma(L)/\partial L\|$ (panel b). For each set of parameters, we calculate the socially optimal liquidation threshold $v^L(\bar{e})$ for the highest realization $\bar{e}$ of $e$. We also calculate the creditor-recovery-maximizing threshold $v^{private}(\bar{e}) = (\gamma + \bar{e})/q_1$ from equation (19). The blue line plots the difference $v^L(\bar{e}) - v^{private}(\bar{e})$. Positive values of the blue line thus correspond to more liquidations under the socially optimal policy than the creditor-maximizing rule. The x-axis shows the assumed value of the parameter being varied. The social planner’s loan market-clearing constraint binds ($\mu \neq 0$) to the right of the vertical red line.

(a) Optimal liquidation thresholds and collateral constraints

- Difference between efficient and private liquidation thresholds
- Loan market-clearing constraint binds

(b) Optimal liquidation thresholds and fire-sale magnitudes

- Difference between efficient and private liquidation thresholds
- Loan market-clearing constraint binds
A Proofs

This appendix provides the proofs of our main results.

A.1 Proof of Proposition 1

Conjecturing a threshold rule $\rho(v, \varepsilon, s) = 1(v < v^L(\varepsilon, s))$, for any function $f(v, \varepsilon)$ such that $f(v, \varepsilon)g(v, \varepsilon|s)$ is integrable, we write

$$\frac{\partial}{\partial \rho(v, \varepsilon, s)} \int_{v \leq v^*(\varepsilon, s)} \rho(v, \varepsilon, s)f(v, \varepsilon)dG(s) = \frac{\partial}{\partial v^L(\varepsilon, s)} \int_{\varepsilon \geq 0} f(v, \varepsilon)g(v, \varepsilon|s)dvd\varepsilon$$

$$= \int_{\varepsilon \geq 0} \frac{\partial}{\partial v^L(\varepsilon, s)} \int_{0}^{v^L(\varepsilon, s)} f(v, \varepsilon)g(v, \varepsilon|s)dvd\varepsilon$$

$$= \int_{\varepsilon \geq 0} f(v^L(\varepsilon, s), \varepsilon)g(v^L(\varepsilon, s), \varepsilon|s)d\varepsilon,$$  \hspace{1cm} (28)

where the first equality is the definition of $g(v, \varepsilon|s)$, the second equality applies dominated convergence and the third equality is an application of the Leibniz rule.

The private firm Lagrangian is given by

$$\mathcal{L} = u^f_0(c^f_0) + \mathbb{E}^S \left[ \int_{v \geq v^*(\varepsilon, s)} vI - \frac{1}{q_1(s)} [\varepsilon I + D_0] dG(s) \right]$$

$$+ \lambda \left[ A_0^f + q_0D_0 - c^f_0 - \Phi(I) \right] + \Lambda \left[ \mathbb{E}^S \left[ 1 - \phi(s)q_1(s) \right] \left( Q^S(s) + \varepsilon(s) \right) \right] - A_0^b$$

Differentiating in the bankruptcy code $\rho(v, \varepsilon, s)$, we obtain

$$\frac{\partial \mathcal{L}}{\partial \rho(v, \varepsilon, s)} = \Lambda \frac{1 - \phi(s)q_1(s)}{q_1(s)} \frac{\partial}{\partial \rho(v, \varepsilon, s)} \left( Q^S(s) + \varepsilon(s) \right) f(s).$$

From equation (28) and equation (10),
\[
\frac{\partial Q^S(s) + \mathcal{E}(s)}{\partial \rho(v, \varepsilon, s)} = \frac{1}{1 - \phi(s)} \left[ \int_{\varepsilon \geq 0} \left[ -q_1(s)v^L(\varepsilon, s) + \gamma(s) + \varepsilon \right] g \left( v^L(\varepsilon, s), \varepsilon | s \right) \tilde{d} \varepsilon \right]
\]

from which the threshold rule \(-\varepsilon + q_1(s)v^L(\varepsilon, s) = \gamma(s)\) follows immediately.

### A.2 Proof of Proposition 2

The Lagrangian of the social planning problem is given by

\[
\mathcal{L} = u(c^f_0) + \mathbb{E}^S \left[ \int_{v \leq v^*} \left[ vI - \frac{1}{q_1(s)} \varepsilon I + D_0 \right] dG(s) \right] + \lambda \left[ A^f_0 + q_0 D_0 - c^f_0 - \Phi(I) \right] + \Lambda \mathbb{E}^S \left[ \frac{1 - \phi(s)q_1(s)}{q_1(s)} \left( Q^S(s) + \mathcal{E}(s) \right) \right] - A^b_0
\]

\[
+ \mathbb{E}^S \left[ \mu(s) \left( Q^S(s) - Q^D(s) \right) \right] + \mathbb{E}^S \left[ \xi(s) \left( \gamma(s) - \frac{\partial \mathcal{F}(L(s), s)}{\partial L(s)} \right) \right]
\]

where \(L(s) = \int_{v \leq v^*} \rho(v, \varepsilon, s) \tilde{d} G(s)\). Differentiating in \(\rho(v, \varepsilon, s)\) at an interior point, we have

\[
\frac{\partial \mathcal{L}}{\partial \rho(v, \varepsilon, s)} = \lambda \frac{1 - \phi(s)q_1(s)}{(1 - \phi(s))q_1(s)} \int_{\varepsilon \geq 0} \left[ -q_1(s)v^L(\varepsilon, s) + \gamma(s) + \varepsilon \right] g \left( v^L(\varepsilon, s), \varepsilon | s \right) \tilde{d} \varepsilon f(s)
\]

\[
+ \mu(s) \frac{\partial \left( Q^S(s) - Q^D(s) \right)}{\partial \rho(v, \varepsilon, s)} f(s) - \xi(s) \frac{\partial \gamma(s)}{\partial L(s)} \frac{\partial L(s)}{\partial \rho(v, \varepsilon, s)} f(s)
\]

From here, we have

\[
\frac{\partial \left( Q^S(s) - Q^D(s) \right)}{\partial \rho(v, \varepsilon, s)} = \frac{\partial Q^S(s)}{\partial \rho(v, \varepsilon, s)}
\]

\[
= \frac{1}{1 - \phi(s)} \left[ \int_{\varepsilon \geq 0} \left[ -\phi(s)q_1(s)v^L(\varepsilon, s) + \gamma(s) + \varepsilon \right] g \left( v^L(\varepsilon, s), \varepsilon | s \right) \tilde{d} \varepsilon \right]
\]
and

$$\frac{\partial L(v,s)}{\partial \rho(v,\varepsilon,s)} = \int_{\varepsilon \geq 0} \tilde{I} g(v^L(\varepsilon,s),\varepsilon|s) \, d\varepsilon,$$

so that we have

$$\frac{1}{f(s)} \frac{\partial \mathcal{L}}{\partial \rho(v,\varepsilon,s)} = \int_{\varepsilon \geq 0} \left( \Lambda \frac{1-\phi(s)q_1(s)}{(1-\phi(s))q_1(s)} + \mu(s) \frac{\phi(s)}{1-\phi(s)} \right) \left( q_1(s)v^L(\varepsilon,s) + \gamma(s) + \varepsilon \right) + \mu(s) \frac{1}{1-\phi(s)} \left( -\phi(s)q_1(s)v^L(\varepsilon,s) + \gamma(s) + \varepsilon \right) + \zeta(s) \left( \frac{\partial \gamma(s)}{\partial L(s)} \right) g(v^L(\varepsilon,s),\varepsilon|s) \, d\varepsilon.$$

From this, we obtain a threshold rule, given by

$$q_1(s)v^L(\varepsilon,s) = \left( 1 + \frac{\mu(s)}{\Lambda \frac{1-\phi(s)q_1(s)}{(1-\phi(s))q_1(s)} + \mu(s) \frac{\phi(s)}{1-\phi(s)} \right) \left( \varepsilon + \gamma(s) \right) + \mu(s) \frac{1}{1-\phi(s)} \left( -\phi(s)q_1(s)v^L(\varepsilon,s) + \gamma(s) + \varepsilon \right) + \zeta(s) \left( \frac{\partial \gamma(s)}{\partial L(s)} \right)$$

From here, we need to characterize the multipliers. Beginning with $\zeta(s)$, we have

$$0 = \frac{1}{f(s)} \frac{\partial \mathcal{L}}{\partial \gamma(s)} = \left( \Lambda \frac{1-\phi(s)q_1(s)}{(1-\phi(s))q_1(s)} + \mu(s) \frac{\phi(s)}{1-\phi(s)} \right) \frac{\partial Q^s(s)}{\partial \gamma(s)} + \zeta(s)$$

and using from here $\frac{\partial Q^s(s)}{\partial \gamma(s)} = \frac{1}{1-\phi(s)} \int_{v \leq v^*(\varepsilon,s)} \rho(v,\varepsilon,s) \tilde{I}dG(s) = \frac{1}{1-\phi(s)} L(s)$, we obtain

$$\zeta(s) = -\left( \Lambda \frac{1-\phi(s)q_1(s)}{(1-\phi(s))q_1(s)} + \mu(s) \frac{\phi(s)}{1-\phi(s)} \right) L(s)$$

which substituting into the threshold rule yields

$$q_1(s)v^L(\varepsilon,s) = \left( 1 + \frac{\mu(s)}{\Lambda \frac{1-\phi(s)q_1(s)}{(1-\phi(s))q_1(s)} + \mu(s) \frac{\phi(s)}{1-\phi(s)} \right) \left( \varepsilon + \gamma(s) - \left| \frac{\partial \gamma(s)}{\partial L(s)} \right| L(s) \right).$$

44
Finally, we obtain $\mu(s)$ from

$$
0 = \frac{1}{f(s)} \frac{\partial L}{\partial q_1(s)}
= \int_{v \geq v^*(\epsilon, s)} \frac{1}{q_1(s)} [\epsilon I + D_0] dG(s)
+ \Lambda \frac{\partial}{\partial q_1(s)} \left[ \frac{1 - \phi(s) q_1(s)}{q_1(s)} \left( Q^S(s) + \epsilon^*(s) \right) \right] + \mu(s) \frac{\partial \left( Q^S(s) - Q^D(s) \right)}{\partial q_1(s)}
$$

where we have used the threshold rule property to drop the term involving liquidation market clearing. From here, note that we have

$$
\frac{\partial \epsilon^*(s)}{q_1(s)} = \int_{\epsilon \geq 0} \frac{\partial v^*(\epsilon, s)}{q_1(s)} q_1(s) v^*(\epsilon, s) \tilde{I} g(v^*(\epsilon, s)|s) d\epsilon + \int_{v \leq v^*(\epsilon, s)} (1 - \rho(v, \epsilon, s)) v^* \tilde{I} dG(s)
$$

$$
\frac{\partial Q^S(s)}{\partial q_1(s)} = \frac{1}{1 - \phi(s)} \left[ \phi(s) \frac{\partial \epsilon^*(s)}{q_1(s)} + \int_{\epsilon \geq 0} \frac{\partial v^*(\epsilon, s)}{q_1(s)} \left( -D_0 - \epsilon \tilde{I} \right) d\epsilon \right]
$$

$$
\frac{\partial Q^D(s)}{\partial q_1(s)} = \int - \frac{\partial v^*(\epsilon, s)}{q_1(s)} \left( D_0 + \epsilon \tilde{I} \right) g(v^*(\epsilon, s)|s) d\epsilon.
$$

from which we obtain, noting that by definition $D_0 = \left( -\epsilon + q_1(s) v^*(\epsilon, s) \right) I$ and that $\frac{\partial v^*(\epsilon, s)}{q_1(s)} < 0$,

$$
\frac{\partial \left( Q^S(s) + \epsilon^*(s) \right)}{\partial q_1(s)} = \frac{1}{1 - \phi(s)} \left[ \int_{\epsilon \geq 0} \left| \frac{\partial v^*(\epsilon, s)}{q_1(s)} \right| \left( I - \tilde{I} \right) \frac{D_0}{I} g(v^*(\epsilon, s)|s) d\epsilon + \frac{1}{q_1(s)} \epsilon^*(s) \right].
$$
Now, substituting back in above, and rearranging, we have

\[
\mu(s) \frac{\partial}{\partial q_1(s)} \left( Q^D(s) - Q^S(s) \right) = \int_{v \geq v^*(\epsilon, s)} \frac{1}{q_1(s)^2} [\epsilon I + D_0] dG(s) - \Lambda \frac{1}{q_1(s)^2} \left( Q^S(s) + \epsilon(s) \right) \\
+ \Lambda \frac{1 - \phi(s)q_1(s)}{q_1(s)} \frac{1}{1 - \phi(s)} \left[ \int_{\epsilon \geq 0} \left| \frac{\partial v^*(\epsilon, s)}{\partial q_1(s)} \right| (I - \tilde{I}) \frac{D_0}{I} g(v^*(\epsilon, s)|s) d\epsilon \right] + \frac{1}{q_1(s)} \epsilon(s)
\]

and using that \( Q^S(s) = Q^D(s) \) when \( \mu(s) \neq 0 \), we obtain

\[
\mu(s) \frac{\partial}{\partial q_1(s)} \left( Q^D(s) - Q^S(s) \right) = \frac{1 - \Lambda}{q_1(s)^2} Q^D(s) + \frac{\Lambda}{q_1(s)^2} \frac{\phi(s)(1 - q_1(s))}{1 - \phi(s)} \epsilon(s) \\
+ \Lambda \frac{1 - \phi(s)q_1(s)}{(1 - \phi(s))q_1(s)} \left[ \int_{\epsilon \geq 0} \left| \frac{\partial v^*(\epsilon, s)}{\partial q_1(s)} \right| (I - \tilde{I}) \frac{D_0}{I} g(v^*(\epsilon, s)|s) d\epsilon \right] + \Lambda \frac{1 - \phi(s)q_1(s)}{(1 - \phi(s))q_1(s)} \left[ \int_{\epsilon \geq 0} \left| \frac{\partial v^*(\epsilon, s)}{\partial q_1(s)} \right| \chi^{v^*(\epsilon, s)} d\epsilon \right].
\]

Finally, it is helpful to define

\[
\chi^*_{v^*}(\epsilon, s) \equiv (v^*(\epsilon, s) + \epsilon) I g (v^*(\epsilon, s), \epsilon|s) = D_0 g (v^*(\epsilon, s), \epsilon|s)
\]
as the total value of threshold solvent firms. Then, we have

\[
\mu(s) \frac{\partial}{\partial q_1(s)} \left( Q^D(s) - Q^S(s) \right) = \frac{1 - \Lambda}{q_1(s)^2} Q^D(s) + \frac{\Lambda}{q_1(s)^2} \frac{\phi(s)(1 - q_1(s))}{1 - \phi(s)} \epsilon(s) \\
+ \Lambda \frac{1 - \phi(s)q_1(s)}{(1 - \phi(s))q_1(s)} \left[ \int_{\epsilon \geq 0} \left| \frac{\partial v^*(\epsilon, s)}{\partial q_1(s)} \right| \chi^{v^*(\epsilon, s)} d\epsilon \right].
\]
Finally, note that we have

$$\frac{\partial}{\partial q_1(s)} \left( Q^D(s) - Q^S(s) \right) = \frac{1}{1 - \phi(s)} \times \left( \int_{\varepsilon \geq 0} \left| \frac{\partial v^*(\varepsilon, s)}{\partial q_1(s)} \right| \left( \varepsilon - \phi(s) q_1(s) v^*(\varepsilon, s) \right) (I - I) g(v^*(\varepsilon, s)|s) \, d\varepsilon \right)$$

which is positive for $\phi(s)$ close to zero.

### A.3 Proof of Proposition 3

The proof follows from the proof of Proposition 1. In the case where liquidation is subsidized, the firm optimization problem including subsidies is equivalent to one with $\hat{\gamma} + \varepsilon = \gamma + \tau + \varepsilon$. In the case where continuation is subsidized, the problem including subsidies is equivalent to one with $\hat{\gamma} = \gamma - \tau + \varepsilon$. In both cases, the privately optimal rule is the creditor recovery maximizing rule under $\hat{\gamma}$, that is $q_1 v^L = \hat{\gamma} + \varepsilon$ which substituting in yields the social optimum.

### A.4 Proof of Proposition 4

Using the Lagrangian in the proof of Proposition 2, we have

$$\frac{dL}{dD_0} = \frac{\partial L}{\partial D_0} + \mathbb{E}^S \left[ \mu(s) \frac{\partial [Q^S(s) - Q^D(s)]}{\partial D_0} \right]$$

$$\frac{dL}{dB_0} = \frac{\partial L}{\partial B_0} + \mathbb{E}^S \left[ \mu(s) \frac{\partial [Q^S(s) - Q^D(s)]}{\partial B_0} \right]$$

so that the optimal wedges are

$$\tau^B_0 = -\frac{1}{\lambda} \mathbb{E}^S \left[ \mu(s) \frac{\partial [Q^S(s) - Q^D(s)]}{\partial B_0} \right]$$
\[ \tau_0^D = -\frac{1}{\lambda} \mathbb{E}^S \left[ \mu(s) \frac{\partial [Q^S(s) - Q^D(s)]}{\partial D_0} \right] \]

It remains only to characterize these derivatives. Starting with the derivative in \( B_0 \), we have

\[ \frac{\partial [Q^S(s) - Q^D(s)]}{\partial B_0} = -\frac{1}{1 - \phi(s)} \]

which yields the first result. Next evaluating the derivative in \( D_0 \), we have

\[
\frac{\partial [Q^S(s) - Q^D(s)]}{\partial D_0} = \frac{\phi(s)}{1 - \phi(s)} \int_{v \geq v^*(\epsilon, s)} dG(s) \\
+ \frac{\phi(s)}{1 - \phi(s)} \left[ \int_{\epsilon} \frac{\partial v^*}{\partial D_0} \left[ q_1(s)v^*I - (D_0 + \epsilon I) \right] g(v^*) d\epsilon \right] \\
= \frac{\phi(s)}{1 - \phi(s)} \left[ \int_{v \geq v^*(\epsilon, s)} dG(s) - \int_{\epsilon} \frac{\partial v^*}{\partial D_0} q_1(s)v^* \chi g(v^*) d\epsilon \right]
\]

which yields the second result.

## B Numerical Setting

To provide illustrative examples, we parameterize and solve our model numerically. This appendix describes our assumptions for this numerical exercise. The propositions in the main text do not rely on any of these assumptions.

We assume there is one state. We assume that \( u_0^f(\cdot) = 0 \). This implies that at time zero, firms simply borrow at rate \( q_0 \) to fund investment \( I \). This leads to debt burden \( D_0 = (\Phi(I) - A_0^f) / q_0 \), where \( A_0^f \) is a parameter and \( \Phi \) is an exogenous function given in Table 1. Banks borrow \( B_0 = (q_0D_0 - A_0^b) / q \) at time zero, where \( q \) and \( A_0^b \) are parameters.

At time one, firms realize viability shocks \( v \sim \text{Uniform}[0, \bar{v}] \) for a parameter \( \bar{v} \) and cost shocks \( \epsilon \sim \text{Uniform}[\underline{\epsilon}, \overline{\epsilon}] \) for parameters \( \underline{\epsilon}, \overline{\epsilon} \). Firms must pay \( \epsilon I \) and an endogenous interest rate \( q_1 \) determines a solvency threshold (equation (2)). Firms that are insolvent enter bankruptcy. In bankruptcy, a fraction \( \chi \) of assets are destroyed, where \( \chi \) is a parameter. If the firm is liquidated, creditors receive a liquidation payoff \( \gamma(L) \equiv \gamma_0 + \gamma_1L \) per unit of liquidated asset, where \( \gamma_0 \geq \gamma_1 \).
0, $\gamma_1 \leq 0$ are parameters and $L$ is the volume of liquidated assets. If the firm is reorganized, the bank receives equity that grants a continuation payoff at time two. At time one, banks can borrow an endogenous quantity $B_1$ that is limited by the product of $\phi$, a parameter, and the bank’s time-two cashflows (equation (8)). Table 1 shows our parameter-value assumptions for this numerical setting.

Table 1: Parameter values for numerical illustrative example

This table shows our parameter-value assumptions for our numerical setting. The results in the main text do not rely on these assumptions.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distribution of $\nu$</td>
<td>Uniform[0,1]</td>
</tr>
<tr>
<td>Distribution of $\epsilon$</td>
<td>Uniform[0.099,0.1001]</td>
</tr>
<tr>
<td>$\Phi(I)$</td>
<td>$.2I + 1(I \neq 1)\infty$</td>
</tr>
<tr>
<td>$\gamma(L)$</td>
<td>$0.2 - 0.5L$</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.15</td>
</tr>
<tr>
<td>$A_0^f$</td>
<td>0</td>
</tr>
<tr>
<td>$u_0^f(c_0^f)$</td>
<td>0</td>
</tr>
<tr>
<td>$A_0^p$</td>
<td>0.25</td>
</tr>
<tr>
<td>$q$</td>
<td>0.9</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.0001</td>
</tr>
</tbody>
</table>
B.1 Solution of the numerical setting

Firms must pay $\varepsilon I$ and an endogenous interest rate $q_1$ determines a solvency threshold $v^*$:

$$v^*(\varepsilon) \equiv \frac{1}{q_1} \left( \frac{D_0}{I} + \varepsilon \right). \quad (31)$$

Define $\xi \equiv D_0/(Iq_1)$. Define $\Delta \equiv \overline{\varepsilon} - \varepsilon$ and define $\hat{\varepsilon}$ to be the largest $\varepsilon$ value such that some firm is solvent:

$$\hat{\varepsilon} \equiv \min \left( \overline{\varepsilon}, q_1 \overline{v} - \xi \right).$$

Then calculations reveal that:

$$\int_{v \geq v^*(\varepsilon)} \varepsilon dG(s) = \frac{1}{v \Delta} \int \hat{\varepsilon} \int_{v^*(\varepsilon)}^{\overline{\varepsilon}} \varepsilon dv d\varepsilon \quad = \frac{1}{v \Delta} \int \hat{\varepsilon} \int_{\overline{\varepsilon}}^{v^*(\varepsilon)} \frac{v^2 - (\xi^2 + \frac{2\varepsilon^2}{q_1} + \frac{\varepsilon^2}{q_1})}{2} d\varepsilon \quad = \frac{1}{2v \Delta} \left( (\overline{v}^2 - \xi^2)(\hat{\varepsilon} - \varepsilon) - \frac{\xi (\hat{\varepsilon}^2 - \xi^2)}{q_1} - \frac{\xi (\hat{\varepsilon}^3 - \xi^3)}{3q_1^2} \right),$$

$$\int_{v \geq v^*(\varepsilon)} \varepsilon dG(s) = \frac{1}{v \Delta} \int \hat{\varepsilon} \int_{v^*(\varepsilon)}^{\overline{\varepsilon}} \varepsilon dv d\varepsilon \quad = \frac{1}{v \Delta} \int \hat{\varepsilon} \int_{\overline{\varepsilon}}^{v^*(\varepsilon)} \frac{\varepsilon(\overline{v} - \xi - \frac{\varepsilon}{q_1})}{d\varepsilon} \quad = \frac{1}{v \Delta} \left( (\overline{v} - \xi) \frac{\hat{\varepsilon}^2 - \xi^2}{2} - \frac{\hat{\varepsilon}^3 - \xi^3}{3q_1} \right).$$
\[
\int_{v \geq v^*} dG(s) = \frac{1}{\nu\Delta_e} \int_{\xi}^{\hat{\xi}} (v - \xi - \frac{\epsilon}{q_1}) d\epsilon \\
= \frac{1}{\nu\Delta_e} \left( (v - \xi)(\hat{\epsilon} - \xi) - \frac{\hat{\epsilon}^2 - \xi^2}{2q_1} \right).
\]

Solvent firms borrow \(D_0 + \epsilon I\) to rollover debt. The social planner maximizes the final value of solvent firms:

\[
\int_{v \geq v^*} vI - \frac{1}{q_1} \left[ \epsilon I + D_0 \right] dG(s) = \frac{I}{2\nu\Delta_e} \left( (v^2 - \xi^2)(\hat{\epsilon} - \xi) - \frac{\xi(\hat{\epsilon}^2 - \xi^2)}{q_1} - \frac{(\hat{\epsilon}^3 - \xi^3)}{3q_1^2} \right) \\
- \frac{I}{q_1\nu\Delta_e} \left( (v - \xi)\frac{\hat{\epsilon}^2 - \xi^2}{2} - \frac{\hat{\epsilon}^3 - \xi^3}{3q_1} \right) \\
- \frac{D_0}{q_1\nu\Delta_e} \left( (v - \xi)(\hat{\epsilon} - \xi) - \frac{\hat{\epsilon}^2 - \xi^2}{2q_1} \right).
\]

Solvent loan demand is

\[
Q^D \equiv \int_{v \geq v^*} D_0 + \epsilon I dG(s) = \frac{D_0}{\nu\Delta_e} \left( (v - \xi)(\hat{\epsilon} - \xi) - \frac{\hat{\epsilon}^2 - \xi^2}{2q_1} \right) \\
+ \frac{I}{\nu\Delta_e} \left( (v - \xi)\frac{\hat{\epsilon}^2 - \xi^2}{2} - \frac{\hat{\epsilon}^3 - \xi^3}{3q_1} \right).
\]

Motivated by Proposition 2, we assume that the optimal liquidation threshold is affine: \(\rho(\epsilon) = \rho_0 + \rho_1 \epsilon\). We conjecture and numerically verify that that \(0 \leq \rho_0 + \rho_1 \epsilon\) and that \(\rho_0 + \rho_1 \epsilon \leq \tilde{\xi} + \epsilon / q_1\) for all \(\epsilon\). Then

\[
L = \frac{I(1 - \chi)}{\nu\Delta_e} \int_{\xi}^{\hat{\xi}} \int_{0}^{\rho_0 + \rho_1 \epsilon} dv d\epsilon = \frac{I(1 - \chi)}{\nu\Delta_e} \int_{\xi}^{\hat{\xi}} (\rho_0 + \rho_1 \epsilon) d\epsilon \\
= \frac{I(1 - \chi)}{\nu\Delta_e} \left( \rho_0(\hat{\epsilon} - \xi) + \rho_1 \frac{\hat{\epsilon}^2 - \xi^2}{2} \right).
\]
\[ \mathcal{E} = I(1 - \chi)q_1 \int_{\tilde{\varepsilon}}^{\xi + \varepsilon / q_1} \int_{\rho_0 + \rho_1 \varepsilon}^{\tilde{\varepsilon}} v d\varepsilon d\tilde{\varepsilon} \]

\[ = I(1 - \chi)q_1 \int_{\tilde{\varepsilon}}^{\xi} \left( \frac{\xi^2 - \rho_0^2 + 2(\xi / q_1 - \rho_0 \rho_1) \varepsilon + (1/q_1^2 - \rho_1^2) \varepsilon^2}{2} \right) d\varepsilon \]

\[ = I(1 - \chi)q_1 \int_{\tilde{\varepsilon}}^{\xi} \left( [\xi^2 - \rho_0^2](\tilde{\varepsilon} - \varepsilon) + \left[ \frac{\xi}{q_1} - \rho_1 \rho_0 \right](\varepsilon^2 - \tilde{\varepsilon}^2) + \frac{1}{q_1^2 - \rho_1^2} (\varepsilon^3 - \tilde{\varepsilon}^3) \right). \]

\[ \frac{I(1 - \chi)}{\bar{\varepsilon} \Delta} \int_{\xi}^{\xi + \varepsilon / q_1} \int_{\rho_0 + \rho_1 \varepsilon}^{\tilde{\varepsilon}} v d\varepsilon d\tilde{\varepsilon} = I(1 - \chi) \int_{\xi}^{\xi} \int_{\rho_0 + \rho_1 \varepsilon}^{\tilde{\varepsilon}} e d\varepsilon d\tilde{\varepsilon} \]

\[ = I(1 - \chi) \left( [\xi - \rho_0] \frac{\varepsilon^2 - \tilde{\varepsilon}^2}{2} + [1/q_1 - \rho_1] \frac{\varepsilon^3 - \tilde{\varepsilon}^3}{3} \right). \]

It follows that

\[ Q^S = \frac{1}{1 - \phi} \left[ \phi \mathcal{E} - B_0 + \gamma_0 L + \gamma_1 L^2 + \frac{D_0}{\bar{\varepsilon} \Delta} \left( (\bar{v} - \tilde{\varepsilon})(\tilde{\varepsilon} - \varepsilon) - \frac{\varepsilon^2 - \tilde{\varepsilon}^2}{2q_1} \right) \right. \]

\[ - I(1 - \chi) \left. \left( [\xi - \rho_0] \frac{\varepsilon^2 - \tilde{\varepsilon}^2}{2} + [1/q_1 - \rho_1] \frac{\varepsilon^3 - \tilde{\varepsilon}^3}{3} \right) \right]. \]

In our numerical solution, we numerically optimize the social planner’s objective subject to the constraint that banks must prefer consuming their time two cashflow to consuming their endowment:

\[ A_0^b \leq \frac{1}{q_1} \left( Q^S + \mathcal{E} \right), \] (32)

and the constraint that loan markets must clear:

\[ Q^S \geq Q^D. \] (33)
C Extensions

C.1 Heterogeneous banks

In this section, we study the effect of multiple (heterogeneous) bank creditors facing different collateral constraints in the design of the liquidation rule as well as the optimal seniority structure among creditors. We show that a social planner can improve welfare by subordinating the claims of banks that, because of their idiosyncratic collateral constraints, struggle to lend against securities of bankrupt firms. Moreover, planner intervention in the seniority structure can serve as a partial substitute for intervention in liquidation decisions.

Because we explicitly model seniority, our leading application for this section is to a traditional bankruptcy process, such as Chapter 11, with changes in the liquidation rule reflecting different outcomes of the bankruptcy process. Nevertheless, we preserve the notation and terminology of previous sections for consistency.

C.1.1 Novel assumptions in the heterogeneous-banks extension

We assume that there are $N^B$ distinct banks, each of equal measure, indexed by $b = 1, 2, ..., N^b$. Bank $b$ has initial wealth $A^b_0$. At date zero, bank $b$ raises debt $B^b_0$ from households at an exogenous price $q^b_0$. For simplicity, we assume that each dollar that bank $b$ lends is divided equally among all firms, ensuring there are multiple creditors for each firm. Bank $b$ lends $q^b_0D^b_0$ to firms at date zero, where $q^b_0$ is the endogenous price of debt owned by bank $b$. Although different firms will offer the same debt price to the same bank $b$, different bank types $b$ may face different debt prices. Different prices arise because of potentially different bank participation constraints as well potentially distinct endogenous claim seniority, and are determined by the firm in competitive equilibrium as in our baseline model.

At date one, firms realize idiosyncratic shocks $(v, \varepsilon)$ as in Section 2. Firms roll over their debt,

---

33 For example, accomplished by implementations similar to those of Proposition 3.
34 Notice that even if firms were not given full bargaining power in contracting with banks at date zero, differences in seniority would still lead to differences in debt prices.
leading to a total date-one demand for bank funds equal to

\[
Q^D(s) = \int_{v \geq v^*(\epsilon,s)} \left( \epsilon I + \sum_{b=1}^{N^B} D^b_0 \right) dG(s),
\]

(34)

so that total firm demand is determined by total debt, \( D_0 = \sum_{b=1}^{N^B} D^b_0 \). As in Section 2.2, firms with idiosyncratic shocks \((v, \epsilon)\) are liquidated with probability \( \rho(v, \epsilon, s) \) in state \( s \). The cashflows associated with liquidation and continuation are identical to those described in Section 2.2. However, we now assume that the cashflow in insolvency is divided among banks according to an endogenous seniority structure. Specifically, if a firm with idiosyncratic shocks \((v, \epsilon)\) is insolvent in state \( s \), we assume that bank \( b \) receives a fraction \( S^b(v, \epsilon, s) \in [0, 1] \) of the cashflows associated with outcome of the liquidation rule. It follows that the date-one value of all of bank \( b \)'s equity in all continued firms is equal to

\[
\mathcal{E}^b(s) \equiv q_1(s) \int_{v \leq v^*(\epsilon,s)} S^b(v, \epsilon, s) \left( 1 - \rho(v, \epsilon, s) \right) \tilde{\nu} dG(s).
\]

(35)

In addition to this equity associated with continuation, bank \( b \) receives total date-one insolvency cashflows equal to

\[
\int_{v \leq v^*(\epsilon,s)} S^b(v, \epsilon, s) \left( - \left( 1 - \rho(v, \epsilon, s) \right) \epsilon + \rho(v, \epsilon, s) \gamma(s) \right) \tilde{\nu} dG(s).
\]

(36)

At date one, bank \( b \) borrows \( B^b_1 \) from households, so that total lending from bank \( b \) to firms is given by

\[
Q^{S,b}(s) = B^b_1 - B^b_0 + \int_{v \geq v^*(\epsilon,s)} D^b_0 dG(s)
\]

(37)

\[
+ \int_{v \leq v^*(\epsilon,s)} S^b(v, \epsilon, s) \left( - \left( 1 - \rho(v, \epsilon, s) \right) \epsilon + \rho(v, \epsilon, s) \gamma(s) \right) \tilde{\nu} dG(s).
\]

Bank \( b \)'s date-one borrowing from households is constrained by a collateral constraint
\[ B^b_1 \leq \phi^b(s) \left( Q^{S,b}(s) + \phi^b(s) \right), \]  

where \( 1 - \phi^b(s) \) is a state-dependent collateral haircut specific to bank \( b \). As in the baseline model, without loss of generality, we can assume that equation (38) holds with equality. Combining this with equation (37), the total date-one supply of bank funds by bank \( b \) is

\[
Q^{S,b}(s) = \frac{1}{1 - \phi^b(s)} \left[ \phi^b(s) \phi^b(s) - B^b_0 + \int_{v \geq v^*(\varepsilon, s)} D^b_0 dG(s) \right. \\
+ \left. \int_{v \leq v^*(\varepsilon, s)} S^b(v, \varepsilon, s) \left( - (1 - \rho(v, \varepsilon, s)) + \rho(v, \varepsilon, s) \gamma(s) \right) \right) dG(s). \]  

Notice that this equation is of the same form as in the baseline model, except that it is now the bank-specific loan supply of bank \( b \).

Finally, the bank-specific participation constraint is defined and derived analogously to Section 2.3, and is given by

\[
A^b_0 = \mathbb{E}^S \left[ \frac{1 - \phi^b(s)q_1(s)}{q_1(s)} \left( Q^{S,b}(s) + \phi^b(s) \right) \right]. \]  

In particular, there is now a participation constraint for each bank \( b \).

### C.1.2 Privately optimal liquidation and seniority

We begin by studying the liquidation rule \( \rho \) and seniority structure \( \{S^b(v, \varepsilon, s)\}^N_{b=1} \) that arises in a competitive equilibrium. Formally, firms solve

\[
\sup_{L, c_0^f, D_0^b, q_0^b, D_0, \rho, \{S^b \}} \mathbb{E}^S \left[ \int_{v \geq v^*(\varepsilon, s)} \left[ v I - \frac{1}{q_1(s)} \left( \rho + \sum_{b=1}^{N^b} D^b_0 \right) \right] dG(s) \right] \]  

subject to the date-zero budget constraint (1) and bank-specific participation constraints (41). As in Section 3.1, firms take equilibrium prices \( q_1 \) and \( \gamma \) as given as they choose a liquidation rule and seniority structure at date zero. The remaining features of a competitive equilibrium are defined analogously to Section 2.6.3.
The following result characterizes the privately optimal liquidation rule $\rho$ and the optimal seniority structure $S$ in the presence of heterogeneous banks.

**Proposition 5.** The privately optimal liquidation rule is the creditor-recovery-maximizing liquidation rule. Moreover, define

$$\beta^b_{\text{private}}(s) \equiv \Lambda^b \frac{1 - \phi^b(s)q_1(s)}{(1 - \phi^b(s))q_1(s)}.$$

Bank $b$ receives positive recovery in state $s$ only if $b \in \argmax_b \beta^b_{\text{private}}(s)$, that is

$$\beta^b_{\text{private}}(s) < \max \beta^b_{\text{private}}(s) \Rightarrow \sup_{v, \varepsilon} S^b(v, \varepsilon, s) = 0. \quad (43)$$

As in Proposition 1, the optimal liquidation rule $\rho$ chosen privately by firms is the rule that maximizes total creditor recovery. Just as before, such a liquidation rule is optimal because it maximizes each firm’s borrowing potential at date zero.

Additionally, in choosing the seniority structure of claims, Proposition 5 reveals that firms prioritize banks with high values of $\beta^b_{\text{private}}(s)$. In particular, $\beta^b_{\text{private}}(s)$ is the marginal value of a unit of repayment to bank $b$ at date one from the perspective of the firm at date zero. When bank $b$ receives a unit of repayment at date one, it uses it to borrow from households and lend, receiving total surplus $\left( 1 - \phi^b(s)q_1(s) \right) / \left( (1 - \phi^b(s))q_1(s) \right)$. This surplus is weighted by $\Lambda^b$, the shadow value to the firm of relaxing the bank $b$ participation constraint. As a result, $\beta^b_{\text{private}}$ provides a stochastic discount factor (SDF) used to price payoffs to bank $b$ from the firm’s perspective, with the seniority structure allocating claims to banks with the highest SDF.

The optimal seniority structure for firms prioritizes seniority for banks with a high shadow value of relaxing the participation constraint (high $\Lambda^b$) to encourage such banks to lend more at date zero. Moreover, we have:

$$\frac{\partial \beta^b_{\text{private}}(s)}{\partial \phi^b(s)} = \Lambda^b \frac{1 - q_1(s)}{(1 - \phi^b(s))^2 q_1(s)} > 0, \quad (44)$$

56
so the privately optimal seniority structure also prioritizes banks that are best able to capitalize on funds at date one (high pledgeability $\phi^b(s)$, or equivalently low haircuts $1 - \phi^b(s)$). Intuitively, banks facing the lowest collateral haircuts are best able to engage in lending at date one by borrowing against collateral, and so gain the most value from receiving payoffs in that state.

C.1.3 Proof of Proposition 5

Define $D_0 \equiv \sum_{b=1}^{N_B} D_0^b$. The Lagrangian of the private problem is given by

$$
L = u(c_f^0) + \mathbb{E}^S \left[ \int_{v \geq v^*(\epsilon, s)} [vI - \frac{1}{q_1(s)}[\epsilon I + D_0]] dG(s) \right] + \lambda \left[ A_0^f + q_0D_0 - c_f^0 - \Phi(I) \right] + \sum_b \Lambda^b \left[ \mathbb{E}^S \left[ \frac{1 - \phi^b(s)q_1(s)}{q_1(s)} \left( Q^{S,b}(s) + \epsilon^b(s) \right) \right] - A_0^b \right].
$$

**Optimal seniority:** The components of $L$ that depend on $S^b$ may be isolated as

$$
\sum_b \Lambda^b \left[ \mathbb{E}^S \left[ \frac{1 - \phi^b(s)q_1(s)}{q_1(s)} \left( Q^{S,b}(s) + \epsilon^b(s) \right) \right] \right].
$$

Further dropping terms not involving $S^b$ in the definition of $Q^{S,b}(s)$,

$$
\sum_b \mathbb{E}^S \left[ \beta^b_{private}(s) \int_{v \leq v^*(\epsilon, s)} S^b(v, \epsilon, s) \right. \times \left. \left( (1 - \rho(v, \epsilon, s))(q_1(s)v - \epsilon) + \rho(v, \epsilon, s)\gamma(s) \right) \right] dG(s) \right].
$$

Fixing $\epsilon, s, v$, we see that an optimal seniority structure must set $S^b(v, \epsilon, s) = 0$ whenever $\beta^b_{private}(s) \neq J(v, \epsilon, s)$, where
\[
J(v, \varepsilon, s) \equiv \begin{cases} 
\max_b \beta^b_{\text{private}}(s) & \text{if } q_1(s)v \geq \varepsilon \text{ or } \rho(v, \varepsilon, s) = 1 \\
\min_b \beta^b_{\text{private}}(s) & \text{if } q_1(s)v < \varepsilon \text{ and } \rho(v, \varepsilon, s) = 0.
\end{cases}
\]

Optimal bankruptcy rule: Differentiating \( \mathcal{L} \) in \( \rho(v, \varepsilon, s) \) at an interior point, we have

\[
\frac{\partial \mathcal{L}}{\partial \rho(v, \varepsilon, s)} = \sum_b \Lambda^b \frac{1 - \phi^b(s)q_1(s)}{(1 - \phi^b(s))q_1(s)} \int_{\varepsilon \geq 0} S^b(v_L(\varepsilon, s), \varepsilon, s) \\
\times \left[ -q_1(s)v_L(\varepsilon, s) + \gamma(s) + \varepsilon \right] g(\varepsilon) \tilde{d} \tilde{f}(s).
\]

It is immediate that this derivative is equal to zero for the creditor-maximizing rule

\[ q_1(s)v_L(\varepsilon, s) = \gamma(s) + \varepsilon. \]

Moreover, if \( q_1(s)v < \varepsilon \) for some \( v, \varepsilon, s \), then

\[ q_1(s)v < \varepsilon < \varepsilon + \gamma = q_1(s)v_L(\varepsilon, s), \]

implying a liquidation under the creditor-maximizing rule. It follows from equation (47) that for any \( v, \varepsilon, s \), we have \( J(v, \varepsilon, s) = \max_b \beta^b_{\text{private}}(s) \), completing the proof.

C.1.4 Socially optimal bankruptcy and seniority

We now consider the problem of a social planner that chooses a liquidation rule \( \rho(v, \varepsilon, s) \) and a seniority structure \( \{S^b(v, \varepsilon, s)\}_{b=1}^{N_b} \), along with a liquidation price, debt prices, debt levels, and firm consumption and investment policies, to maximize firm welfare (42) subject to the firm date-zero budget constraint (1), bank participation constraints (41), the liquidation-market-clearing condition
(14), and a loan-market-clearing condition

$$\sum_{b=1}^{N_B} Q^{S,b}(s) \geq Q^D(s),$$

with inequality only if \( q_1(s) = 1 \). Thus as before, the planner is subject to the same constraint as firms and banks, but internalizes the effects of her decisions on equilibrium loan and liquidation prices.

The following proposition characterizes the socially optimal liquidation rule and seniority structure in the presence of heterogeneous banks.

**Proposition 6.** Let \( \Lambda^b \) denote the Lagrange multiplier in the social planner’s problem associated with bank b’s participation constraint. Let \( \mu(s) \) denote the Lagrange multiplier on the loan-market-clearing condition in state \( s \) and define

$$\beta^b(s) \equiv \Lambda^b \frac{1 - \phi^b(s)q_1(s)}{(1 - \phi^b(s))q_1(s)} + \mu(s) \frac{\phi^b(s)}{1 - \phi^b(s)}.$$  \hspace{1cm} (49)

Suppose that in some state \( \hat{s} \), the social planner never reorganizes firms with a negative recovery value, that is

$$q_1(\hat{s})v < \varepsilon \Rightarrow \rho(v,\varepsilon,\hat{s}) = 0.$$ \hspace{1cm} (50)

Then the following statements are true in state \( \hat{s} \):

1. The socially optimal liquidation rule is a threshold rule, given by

$$q_1(\hat{s})v^L(\varepsilon,\hat{s}) = \left(1 + \frac{\mu(\hat{s})}{\max_b \beta^b(\hat{s})} \right) \left( \gamma(\hat{s}) + \varepsilon - \left. \frac{\partial \gamma(L(\hat{s},\hat{s}))}{\partial L(\hat{s})} \right|_{L(\hat{s})} \right).$$ \hspace{1cm} (51)
2. Bank $b$ receives positive recovery in state $\hat{s}$ only if $b \in \arg\max \hat{b} \beta^b(\hat{s})$, that is

$$\beta^b(\hat{s}) < \max_b \beta^b(\hat{s}) \Rightarrow \sup_{v, \varepsilon} S^b(v, \varepsilon, \hat{s}) = 0. \quad (52)$$

Condition (50) states that, in state $\hat{s}$, the social planner finds it optimal to choose liquidation whenever a continuation implies a negative bank recovery. In principle, it is conceivable that a social planner might implement a negative-payoff continuation if an additional liquidation would dramatically lower liquidation payoffs through the liquidation-price externality. However, equation (50) follows naturally in a setting in which bankruptcy participants enjoy limited liability.\[35]\n
Comparing Proposition 6 to Proposition 5, we see that the social planner differs from privately optimizing agents in its choice of both the liquidation rule and the seniority structure. Under condition (50), the social planner assigns positive recovery to banks with the highest value of $\beta^b(s)$, where:

$$\beta^b(s) - \beta^b_{private}(s) = \mu(s) \frac{\phi^b(s)}{1 - \phi^b(s)}. \quad (53)$$

If $\mu(s) > 0$, the social planner thus assigns lower priority to banks that are constrained at date one (low $\phi^b(s)$) than firms would. Intuitively, the social planner internalizes the ability of banks with high $\phi^b(s)$ values to supply additional funds using equity in reorganized firms as collateral. If $\mu(s) > 0$, then the planner benefits from a higher supply of loanable funds, increasing the price at which banks lend and boosting firm solvency. The loan-price-externality motivation for altering the seniority structure among banks is therefore precisely the same loan-price-externality motivation for intervening in liquidation decisions.

Notably, in a given aggregate state, it can be optimal to assign all payoffs from all bankruptcies to one creditor. This is true both in the private problem and the social planner’s problem (under

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\[35\] A continuation plan that assigns a creditor a negative payoff would almost certainly fail the “best-interest-of-the-creditors” test (11 U.S.C. §1129(a)7), which requires that each creditor receive at least as much as it would in a Chapter 7 liquidation. However, a creditor could nonetheless receive a negative payoff in a bankruptcy if the judge refuses to reimburse its legal expenses, or if it is sued by the trustee for a fraudulent transfer or preference.
condition (50)). Thus, given complete contracts, all proceeds might optimally be allocated to a unique creditor in each state. Absent complete contracts, the initial contract may have to average across aggregate states.

Recall that whenever $\mu(s) > 0$, the social planner has an incentive to avoid continuations in order to save funds for solvent firms. Formally, Proposition 6 shows that the liquidation threshold is inflated by a factor of $\mu(s)/\max_b \beta^b(s)$, where $\beta^b(s) > 0$ for all $b$ if $\mu(s) > 0$. Thus, holding everything else fixed in a state $s$ such that $\mu(s) > 0$, increases in $\max_b \beta^b(s)$ imply more continuations. In particular, to the extent that greater heterogeneity in $\beta^b(s)$ values leads to a larger maximum value $\max_b \beta^b(s)$, bank heterogeneity promotes continuation. Intuitively, the social planner can exploit bank heterogeneity by prioritizing those banks that can most easily redirect bankruptcy payoffs to solvent firms, so the social planner can assign more continuations without crowding out funding to solvent firms. This suggests that an efficient seniority structure can bring the socially efficient liquidation rule closer to the privately optimal rule, providing a partial substitute for direct intervention in the liquidation rule.

As discussed in Section 4.3, DeMarzo, Krishnamurthy, and Rauh (2020) propose that the government could provide DIP loans to bankrupt firms. Our heterogeneous banks extension (Section 5.1) suggests a related but different policy intervention into the market for DIP loans. The social planner of Section 5.1 may wish to impose a bankruptcy seniority structure in which higher priority is given to those banks that are less constrained in their lending in a crisis. Such a seniority structure could be achieved by encouraging courts to approve more DIP loans and grant more priming liens. Such a policy would increase the seniority of DIP lenders, who by revealed preference are sufficiently unconstrained in their lending to provide financing to bankrupt firms. Even outside of a crisis, this socially optimal seniority structure provides a positive explanation for why DIP lenders enjoy high priority in bankruptcy.

\[36\]Without condition (50), the social planner would assign all positive payoffs to one creditor and all negative payoffs to another creditor.
C.1.5 Proof of Proposition 6

Define $D_0 \equiv \sum_{b=1}^{N_d} D_0^b$. The Lagrangian of the social planning problem is given by

$$
\mathcal{L} = u(c_0^f) + \mathbb{E}^s \left[ \int_{v \geq v^*(\varepsilon, s)} \left[ vI - \frac{1}{q_1(s)}[\varepsilon I + D_0] \right] dG(s) \right] \\
+ \lambda \left[ A_0^f q_0 - c_0^f - \Phi(I) \right] \\
+ \sum_b \Lambda^b \left[ \mathbb{E}^s \left[ \frac{1 - \phi^b(s)}{q_1(s)} \left( Q^{S,b}(s) + \varepsilon^b(s) \right) \right] - A_0^b \right] \\
+ \mathbb{E}^s \left[ \mu(s) \left( -Q^D(s) + \sum_b Q^{S,b}(s) \right) \right] + \mathbb{E}^s \left[ \xi(s) \left( \gamma(s) - \frac{\partial F(L(s), s)}{\partial L(s)} \right) \right].
$$

Optimal seniority

The components of $\mathcal{L}$ that depend on $S^b$ may be isolated as

$$
\sum_b \Lambda^b \left[ \mathbb{E}^s \left[ \frac{1 - \phi^b(s)}{q_1(s)} \left( Q^{S,b}(s) + \varepsilon^b(s) \right) \right] \right] + \sum_b \mathbb{E}^s \left[ \mu(s) \left( Q^{S,b}(s) \right) \right].
$$

Further dropping terms not involving $S^b$ in the definition of $Q^{S,b}(s)$,

$$
\sum_b \Lambda^b \mathbb{E}^s \left[ \frac{1 - \phi^b(s)}{(1 - \phi^b(s)) q_1(s)} \int_{v \leq v^*(\varepsilon, s)} S^b(v, \varepsilon, s) \right] (1 - \rho(v, \varepsilon, s))(q_1(s)v - \varepsilon) + \rho(v, \varepsilon, s) \gamma(s) \right] \right] IdG(s) \\
+ \sum_b \mathbb{E}^s \left[ \frac{\mu(s)}{1 - \phi^b(s)} \int_{v \leq v^*(\varepsilon, s)} S^b(v, \varepsilon, s) \right] (1 - \rho(v, \varepsilon, s))(q_1(s)\phi^b(s)v - \varepsilon) + \rho(v, \varepsilon, s) \gamma(s) \right] \right] IdG(s). \tag{54}
$$

Rearranging the second line,
\[ \frac{\mu(s)}{1 - \phi^b(s)} \int_{v \leq v^*(s)} S^b(v, \varepsilon, s) \times \left( (1 - \rho(v, \varepsilon, s))(q_1(s)\phi^b(s)v - \varepsilon) + \rho(v, \varepsilon, s)\gamma(s) \right) \tilde{I}dG(s) \]

\[ = \frac{\mu(s)\phi^b(s)}{1 - \phi^b(s)} \int_{v \leq v^*(s)} S^b(v, \varepsilon, s) \times \left( (1 - \rho(v, \varepsilon, s))(q_1(s)v - \varepsilon) + \rho(v, \varepsilon, s)\gamma(s) \right) \tilde{I}dG(s) + \frac{\mu(s)}{1 - \phi^b(s)}(1 - \phi^b(s)) \int_{v \leq v^*(s)} S^b(v, \varepsilon, s) \times \left( (1 - \rho(v, \varepsilon, s))(-\varepsilon) + \rho(v, \varepsilon, s)\gamma(s) \right) \tilde{I}dG(s). \]

Applying the definition of \( \beta^b(s) \), we can thus rewrite equation (56) as

\[ \sum_b E^S \left[ \beta^b(s) \int_{v \leq v^*(s)} S^b(v, \varepsilon, s) \times \left( (1 - \rho(v, \varepsilon, s))(q_1(s)v - \varepsilon) + \rho(v, \varepsilon, s)\gamma(s) \right) \tilde{I}dG(s) \right] \]

\[ + \sum_b E^S \left[ \mu(s) \int_{v \leq v^*(s)} S^b(v, \varepsilon, s) \times \left( (1 - \rho(v, \varepsilon, s))(-\varepsilon) + \rho(v, \varepsilon, s)\gamma(s) \right) \tilde{I}dG(s) \right]. \]

Pulling the finite sum inside the expectation, the second line simplifies to

\[ E^S \left[ \mu(s) \int_{v \leq v^*(s)} (1 - \rho(v, \varepsilon, s))(-\varepsilon) + \rho(v, \varepsilon, s)\gamma(s) \right] \tilde{I}dG(s) \times \sum_b S^b(v, \varepsilon, s)dG(s) \]

\[ = E^S \left[ \mu(s) \int_{v \leq v^*(s)} (1 - \rho(v, \varepsilon, s))(-\varepsilon) + \rho(v, \varepsilon, s)\gamma(s) \right] \tilde{I}dG(s), \]

where we used the fact that \( \sum_b S^b = 1 \) by definition. The second line thus doesn’t depend on the seniority structure. The optimal seniority structure thus maximizes
\[
\sum_b \mathbb{E}^S \left[ \beta^b(s) \int_{v \leq v^*(\varepsilon, s)} S^b(v, \varepsilon, s) \right. \\
\times \left. \left( (1 - \rho(v, \varepsilon, s))(q_1(s)v - \varepsilon) + \rho(v, \varepsilon, s) \gamma(s) \right) \tilde{I} dG(s) \right].
\]

For any \(\varepsilon, s, v\), we see that any optimal seniority structure must set \(S^b(v, \varepsilon, s) = 0\) whenever \(\beta^b(s) \neq J(v, \varepsilon, s)\), where

\[
J(v, \varepsilon, s) \equiv \begin{cases} 
\max_b \beta^b(s) & \text{if } q_1(s)v \geq \varepsilon \text{ or } \rho(v, \varepsilon, s) = 1 \\
\min_b \beta^b(s) & \text{if } q_1(s)v < \varepsilon \text{ and } \rho(v, \varepsilon, s) = 0.
\end{cases}
\]

Optimal bankruptcy rule: Differentiating \(f\) in \(\rho(v, \varepsilon, s)\) at an interior point, we have

\[
\frac{\partial f}{\partial \rho(v, \varepsilon, s)} = \sum_b \Lambda^b \frac{1 - \phi^b(s)q_1(s)}{(1 - \phi^b(s))q_1(s)} \int_{\varepsilon \geq 0} S^b(v^f(\varepsilon, s), \varepsilon, s) \left[ -q_1(s)v^f(\varepsilon, s) + \gamma(s) + \varepsilon \right] \\
\times g\left( v^f(\varepsilon, s), \varepsilon | s \right) \tilde{I} d\varepsilon f(s) + \mu(s) \frac{\partial \left( Q^S(s) - Q^D(s) \right)}{\partial \rho(v, \varepsilon, s)} f(s) \\
- \zeta(s) \frac{\partial \gamma(s)}{\partial L(s)} \frac{\partial L(s)}{\partial \rho(v, \varepsilon, s)} f(s),
\]

where \(Q^S(s) \equiv \sum_b Q^{S,b}(s)\). From here, we have

\[
\frac{\partial \left( Q^S(s) - Q^D(s) \right)}{\partial \rho(v, \varepsilon, s)} = \frac{\partial Q^S(s)}{\partial \rho(v, \varepsilon, s)} \\
= \sum_b \frac{1}{1 - \phi^b(s)} \left[ \int_{\varepsilon \geq 0} S^b(v^f(\varepsilon, s), \varepsilon, s) \left[ -\phi^b(s)q_1(s)v^f(\varepsilon, s) + \gamma(s) + \varepsilon \right] \\
\times g\left( v^f(\varepsilon, s), \varepsilon | s \right) \tilde{I} d\varepsilon \right].
\]
and

\[ \frac{\partial L(v, s)}{\partial \rho(v, \varepsilon, s)} = \int_{\varepsilon \geq 0} \tilde{I} g(L(v, \varepsilon, s)) \, d\varepsilon, \]

so that we have

\[ \frac{1}{f(s)} \frac{\partial L}{\partial \rho(v, \varepsilon, s)} = \int_{\varepsilon \geq 0} g(L(v, \varepsilon, s)) \left[ \zeta(s) \frac{\partial \gamma(s)}{\partial L(s)} + \sum_b S^b(v, \varepsilon, s) \right] + \frac{1}{1 - \phi^b(s)} \int_{v \leq v^*} \rho(v, \varepsilon, s) \tilde{I} dG(s), \]

To characterize \( \zeta(s) \), we differentiate with respect to \( \gamma(s) \):

\[ 0 = \frac{1}{f(s)} \frac{\partial L}{\partial \gamma(s)} = \sum_b \left( \Lambda^b \frac{1 - \phi^b(s)q_1(s)}{q_1(s)} + \mu(s) \right) \frac{\partial Q^b(s)}{\partial \gamma(s)} + \zeta(s) \]

and using \( \frac{\partial Q^b(s)}{\partial \gamma(s)} = \frac{1}{1 - \phi^b(s)} \int_{v \leq v^*} S^b(v, \varepsilon, s) \rho(v, \varepsilon, s) \tilde{I} dG(s) \), from here we obtain

\[ \zeta(s) = - \sum_b \left( \Lambda^b \frac{1 - \phi^b(s)q_1(s)}{(1 - \phi^b(s))q_1(s)} + \mu(s) \right) L^b(s) \]

\[ = - \sum_b \left( \beta^b(s) + \mu(s) \right) L^b(s), \]

where

\[ L^b(s) \equiv \int_{v \leq v^*} S^b(v, \varepsilon, s) \rho(v, \varepsilon, s) \tilde{I} dG(s) \]

is the fraction of liquidated assets with proceeds accruing to bank \( b \).

Now, fix some \( s \) such that the limited liability condition holds. From equation (57), we see that \( S^b(v, \varepsilon, s) = 0 \) unless \( \beta^b(s) = \max_j \beta^j(s) \). Thus, for any \( b \), either \( S^b(v, \varepsilon, s) = 0 \) for all \( v, \varepsilon \) or \( \beta^b(s) = \max_j \beta^j(s) \). Plugging this in,
\[ \zeta(s) = - \sum_b \left( \max_j \beta^j(s) + \mu(s) \right) L^b(s) \]
\[ = - \left( \max_j \beta^j(s) + \mu(s) \right) \sum_b L^b(s) \]
\[ = - \left( \max_j \beta^j(s) + \mu(s) \right) L(s), \]

where the final equality follows from the fact that \( \sum_b S^b(v, \xi, s) = 1 \). Setting \( \frac{\partial \varphi}{\partial \rho(v, \xi, s)} = 0 \) and plugging this expression in for \( \zeta(s) \),

\[
0 = \int_{\xi \geq 0} g(v^L(\xi, s), \xi|s) \left[ - \left( \max_j \beta^j(s) + \mu(s) \right) L(s) \left| \frac{\partial \gamma(s)}{\partial L(s)} \right| + \sum_b S^b(v^L(\xi, s), \xi, s) \right. \\
\times \left. \left( \Lambda^b \frac{1 - \phi^b(s)}{(1 - \phi^b(s))q_1(s)} \left[ - q_1(s)v^L(\xi, s) + \gamma(s) + \xi \right] \right. \\
+ \mu(s) \frac{1}{1 - \phi^b(s)} \left[ - \phi^b(s)q_1(s)v^L(\xi, s) + \gamma(s) + \xi \right] \right) \right] d\xi.
\]

Noting that

\[
\frac{\mu(s)}{1 - \phi^b(s)} = \mu(s) + \phi^b(s) \mu(s) \frac{1}{1 - \phi^b(s)},
\]

and applying the definition of \( \beta^b(s) \), this is

\[
0 = \int_{\xi \geq 0} g(v^L(\xi, s), \xi|s) \left[ - \left( \max_j \beta^j(s) + \mu(s) \right) L(s) \left| \frac{\partial \gamma(s)}{\partial L(s)} \right| + \sum_b S^b(v^L(\xi, s), \xi, s) \right. \\
\times \left. \left( \beta^b(s) \left[ - q_1(s)v^L(\xi, s) + \gamma(s) + \xi \right] + \mu(s) \left[ \gamma(s) + \xi \right] \right) \right] d\xi.
\]

Recalling that either \( S^b(v, \xi, s) = 0 \) for all \( v, \xi \) or \( \beta^b(s) = \max_j \beta^j(s) \),
\[ 0 = \int_{\varepsilon \geq 0} g(v^L(\varepsilon, s), \varepsilon | s) \left[ - \left( \max_j \beta^j(s) + \mu(s) \right) L(s) \frac{\partial \gamma(s)}{\partial L(s)} \right. \\
+ \left( \max_j \beta^j(s) \left[ - q_1(s) v^L(\varepsilon, s) + \gamma(s) + \varepsilon \right] + \mu(s) \left[ \gamma(s) + \varepsilon \right] \right) \\
\times \sum_b S^b(v^L(\varepsilon, s), \varepsilon, s) d\varepsilon. \]

Noting that \( \sum_b S^b(v, \varepsilon, s) = 1 \), this implies the threshold rule

\[ q_1(s) v^L(\varepsilon, s) = \left( 1 + \frac{\mu(s)}{\max_j \beta^j(s)} \right) \left[ \gamma(s) + \varepsilon - L(s) \frac{\partial \gamma(s)}{\partial L(s)} \right], \]

completing the proof.

### C.2 Acquisitions by Arbitrageurs

In our baseline model, we assumed that an insolvent firm faces two potential outcomes: reorganization or liquidation. In practice, an insolvent firm might be acquired. Like a liquidation, an acquisition entails an insolvent firm selling all of its assets. Like a continuation, the value of the purchased assets to the acquirer depends on the long-run viability of the insolvent firm, since the acquirer will continue to operate that firm in some form. In this appendix, we consider an extension in which arbitrageurs acquire insolvent firms to continue operating them.

In our baseline model, arbitrageurs purchase a quantity

\[ L(s) = \int_{v \leq v^L(\varepsilon, s)} \tilde{d}G(s) \quad (58) \]

of liquidated assets for a price \( \gamma(s)L(s) \). In this extension, we assume instead that arbitrageurs acquire insolvent firms with potential enterprise value
\[
L(s) = \int_{v \leq v^L(\varepsilon, s)} (1 + b(s)v) \tilde{I} dG(s)
\]  
(59)

for a price \( \gamma(s)L(s) \). As before, arbitrageurs have a technology that allows them to operate the acquired firms, producing \( \mathcal{F}(L(s), s) \) units of consumption, but the viability of the arbitrageur’s business model depends on the parameter \( b(s) \geq 0 \). The case \( b(s) = 0 \) is that of the baseline model.\(^{37}\)

Since the arbitrageurs will pay \( \gamma(s)(1 + b(s)v)\tilde{I} \) for a project of scale \( v \), the creditor-recovery-maximizing liquidation rule is now to reorganize the firm if and only if

\[
q_1(s)v - \varepsilon \geq \gamma(s)(1 + b(s)v) \iff \left( q_1(s) - b(s)\gamma(s) \right)v \geq \varepsilon + \gamma(s).
\]  
(60)

This continues to be a threshold rule \( v \geq v^L(\varepsilon, s) \) provided that \( b(s) \) is not too large. In any case, the following proposition holds.

**Proposition 7.** The privately optimal liquidation rule is the creditor-recovery-maximizing liquidation rule.

**Proof.** The private firm’s Lagrangian is identical to that of Proposition 1. The only difference is that the term \( \gamma(s) \) in \( Q^S(s) \) is replaced with \( (1 + b(s)v)\gamma(s) \). Following the exact same steps, we arrive at

\[
\frac{\partial Q^S(s) + \varepsilon^S(s)}{\partial \rho(v, \varepsilon, s)} = \frac{1}{1 - \phi(s)} \left[ \int_{\varepsilon \geq 0} \left[ -q_1(s)v^L(\varepsilon, s) + (1 + b(s)v^L(\varepsilon, s))\gamma(s) + \varepsilon \right] d\varepsilon \right] 
\times g \left( v^L(\varepsilon, s), \varepsilon | s \right) \tilde{I} d\varepsilon
\]  
(61)

\(^{37}\)Notice that we could without loss of generality instead write \( (1 - \hat{b}(s) + \hat{b}(s)v)\tilde{I} \) with \( b(s) = \frac{\hat{b}(s)}{1 - \hat{b}(s)} \), where \( \hat{b} = 1 \) is the baseline model. This is true since we could always factor out \( 1 - \hat{b}(s) \) and incorporate it into the technology \( \mathcal{F} \).
from which the threshold rule \( q_1(s) - b(s)\gamma(s) \) \( v^L(\varepsilon, s) = \varepsilon + \gamma(s) \) follows.

From here, we can also study the social optimum.

**Proposition 8.** Define

\[
\delta(s) \equiv \left( 1 + \frac{\mu(s)}{\beta(s)} \right) \left( 1 - \tilde{\xi}(s) \right),
\]

where \( \tilde{\xi}(s) = \left| \frac{\partial \gamma(s)}{\partial L(s)} \right| \frac{L(s)}{\gamma(s)} \) is liquidation price elasticity (i.e. inverse demand elasticity). Provided \( \delta(s)b(s) < q_1(s) \), then the socially optimal liquidation rule is a threshold rule, with

\[
\left( q_1(s) - \delta(s)b(s)\gamma(s) \right) v^L(\varepsilon, s) = \left( 1 + \frac{\mu(s)}{\beta(s)} \right) \left( \varepsilon + \gamma(s) - \left| \frac{\partial \gamma(s)}{\partial L(s)} \right| L(s) \right) \]

**Proof.** Following the exact same steps as in the proof of Proposition 2, we arrive at a slightly different form of equation (30):

\[
q_1(s)v^L(\varepsilon, s) = \left( 1 + \frac{\mu(s)}{\beta(s)} \right) \left( \varepsilon + (1 + b(s)v^L(\varepsilon, s))\gamma(s) \right)
+ \frac{\zeta(s)[1 + b(s)v^L(\varepsilon, s)]}{\beta(s)} \left| \frac{\partial \gamma(s)}{\partial L(s)} \right|, \]

where the coefficient on \( \zeta(s) \) comes from the derivative of \( L(s) \) with respect to \( v^L(\varepsilon, s) \). Since we still have \( \frac{\partial Q^L(s)}{\partial \gamma(s)} = \frac{1}{1 - \phi(s)} L(s) \), differentiating with respect to \( \gamma(s) \) gives

\[
\zeta(s) = -\left( \beta(s) + \mu(s) \right) L(s)
\]

which substituting into the threshold rule yields

\[
q_1(s)v^L(\varepsilon, s) = \left( 1 + \frac{\mu(s)}{\beta(s)} \right) \left( \varepsilon + \gamma(s) - \left| \frac{\partial \gamma(s)}{\partial L(s)} \right| L(s) \right)
+ \left( 1 + \frac{\mu(s)}{\beta(s)} \right) \left[ b(s)\gamma(s)v^L(\varepsilon, s) - b(s)v^L(\varepsilon, s)L(s) \left| \frac{\partial \gamma(s)}{\partial L(s)} \right| \right].
\]
Rearranging delivers the result.

Notice that the right side in equation (63) is the same as in the baseline model. The left side is the same as the left side under the creditor recovery maximizing rule when \( \delta(s) = 1 \). If \( \delta(s) > 1 \), then all else equal \( v^L \) rises further. Thus provided that \( \xi(s) < 1 \), then \( \delta(s) \) increases in \( \mu(s) \), and the loan price externality is further amplified.

**Example 1.** Consider the case where \( F(L(s), s) = A(s)L^\alpha(s) \) for \( \alpha(s) < 1 \). Then, we have \( \xi(s) = 1 - \alpha(s) < 1 \), leading to further amplification of the loan price externality.

### C.3 Real Economy Spillovers and Positive Arbitrageur Welfare Weights

In our baseline model, social welfare was summarized by the equity value of firms, given that banks and households broke even in expectation. In doing so, we assigned welfare weights of zero to arbitrageurs. Moreover, in practice a social planner may be concerned about the broader economic impact of firm closures, for example on the welfare of firm workers. We now study both of these extensions.

#### C.3.1 Real Economy Spillovers

We model spillovers to other agents, such as workers, in the economy in reduced form as follows. A firm of initial scale \( I \) provides benefits \( (1 - \chi)w(s)I \) to the economy (”wages to workers”), where the scaling \( (1 - \chi) \) is a convenient normalization. This benefit is lost in liquidation, and is not internalized by firms. Thus, the social welfare function is the sum of firm equity, as in the baseline model, and economy benefits (we can interpret \( w \) as including any relevant social welfare weight). The model thus augmented admits a simple variant of Proposition 2.

**Proposition 9.** With real economy spillovers, the socially optimal liquidation rule is a threshold
rule, with

\[ q_1(s) v^L(\epsilon, s) = \left( 1 + \frac{\mu(s)}{\beta(s)} \right) \left( \gamma(s) + \epsilon - \frac{\partial \gamma(L(s), s)}{\partial L(s)} \bigg|_{L(s)} \right) \]

\[ \text{Loan Price Externalities} \]

\[ - \frac{1}{\beta(s)} w(s) \]

\[ \text{Liquidation Price Externalities} \]

\[ \text{Real Spillovers} \]

(68)

The presence of other beneficiaries of firm activities leads the social planner to prefer fewer liquidations, in order to preserve these benefits. In this sense, it has a similar effect to the liquidation price externality of the baseline model.

**Proof.** The planner optimizes the same Lagrangian (29) with an additional term

\[ E^S \left[ \int_{v \geq v^L(\epsilon, s)} (1 - \chi) w(s) I dG(s) \right]. \]

(69)

The derivative of this term with respect to \( v^L(\epsilon, s) \) is

\[ - \int_{\epsilon \geq 0} (1 - \chi) w(s) I g \left( v^L(\epsilon, s), \epsilon | s \right) d \epsilon f(s). \]

(70)

Following the exact same steps as in the proof of Proposition 2 through equation (30), it follows that the threshold rule must be identical except for the additional term \( -w(s)/\beta(s) \) on the right side, completing the proof.

**C.3.2 Positive Arbitrageur Welfare Weights**

We now allow for the possibility of positive arbitrageur welfare weights. As usual, to guarantee a Pareto improvement, we may need to combine interventions in the liquidation rule with ex ante lump sum transfers from banks to arbitrageurs. Thus, we allow for such transfers in the social planning problem.
Proposition 10. With positive arbitrageur welfare weights, the socially optimal liquidation rule is a threshold rule, with

\[
q_1(s) v^L(\epsilon, s) = \left(1 + \frac{\mu(s)}{\beta(s)}\right) \left(\gamma(s) + \epsilon - \left[\frac{\partial \gamma(s)}{\partial L(s)}\right] L(s)\right)
\]

Loan Price Externalities

\[
\left[\frac{\partial \gamma(s)}{\partial L(s)}\right] L(s),
\]

Liquidation Price Externalities

\[
\text{Arbitrageur Surplus}
\]

\[
+ \frac{1}{u^d(A_0^a) \beta(s)} \left[\frac{\partial \gamma(s)}{\partial L(s)}\right] L(s),
\]

Intuitively, a positive welfare weight assigned to arbitrageurs encourages liquidations, since it mitigates the direct impact of the fire sale externality. Absent both a binding collateral constraint for banks and a binding borrowing constraint from arbitrageurs, we have \(u^d = 1\) and \(\beta(s) = \Lambda\) (since \(q_1(s) = 1\)), and hence the decentralized economy is Pareto efficient. The baseline model is the limiting case where \(\omega^A \to 0\) and hence \(u^d(A_0^a) \to +\infty\), meaning the final term drops out.

Proof. Assigning a positive welfare weight \(\omega^A\), then we have \(\lambda = \omega^A u^d(A_0^a)\) (equalization of date 0 marginal utilities through lump sum transfers). As a result, we have

\[
q_1(s) v^L(\epsilon, s) = \left(1 + \frac{\mu(s)}{\beta(s)}\right) \left(\gamma(s) + \epsilon - \left[\frac{\partial \gamma(L(s), s)}{\partial L(s)}\right] L(s)\right)
\]

Loan Price Externalities

\[
\left[\frac{\partial \gamma(L(s), s)}{\partial L(s)}\right] L(s),
\]

Liquidation Price Externalities

\[
\left[\frac{\partial \gamma(L(s), s)}{\partial L(s)}\right] L(s),
\]

Arbitrageur Surplus

\[
\left[\frac{\partial \gamma(L(s), s)}{\partial L(s)}\right] L(s)
\]
\( q_1(s) v^L_1(\epsilon, s) = \left( 1 + \frac{\mu(s)}{\beta(s)} \right) \left( \gamma(s) + \epsilon - \left| \frac{\partial \gamma(L(s), s)}{\partial L(s)} \right| L(s) \right) \) \hfill (74)

Loan Price Externalities

\[ + \frac{1}{\mu^d(A^0_0)} \beta(s) \left| \frac{\partial \gamma(L(s), s)}{\partial L(s)} \right| L(s). \] \hfill (75)

Liquidation Price Externalities

Aribtrageur Surplus