Crisis Interventions in Corporate Insolvency

Samuel Antill*  Christopher Clayton†

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Abstract

We model the optimal resolution of insolvent firms in general equilibrium. Privately optimal insolvency rules maximize creditor recovery. A social planner optimally intervenes during a crisis because of two pecuniary externalities: (i) a fire sale externality promotes more reorganizations; (ii) a loan-price externality arises when constrained banks allocate scarce capital to reorganizing distressed firms rather than lending to healthier firms, promoting more liquidations. Both externalities amplify one another. Interventions that encourage liquidation are desirable when the corporate sector is in greater distress, measured by larger operating losses, larger debt, and lower long-term profitability. Surprisingly, large fire sales promote interventions encouraging even more liquidations. Our flexible framework allows us to study synergies between insolvency interventions and other interventions, including macroprudential regulation, bailouts, and debt seniority structures.

Keywords: Corporate Insolvency, Bankruptcy, Crisis Intervention, Fire Sales, Zombie Lending

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*Antill: Harvard Business School. Email: santill@hbs.edu
†Clayton: Yale School of Management. Email: christopher.clayton@yale.edu.

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1 Introduction

The COVID-19 pandemic motivated a wave of proposals for government interventions into existing processes for resolving insolvent firms. Most proposals aimed to preserve firms that would otherwise be liquidated under existing insolvency laws.\(^1\) Insolvency interventions, however, are not COVID-specific and have not always promoted reorganization. For example, in an attempt to end the problem of nonperforming loans impairing bank balance sheets, Japan in the 2000s implemented policies more closely resembling subsidies for liquidation under the “Takenaka Plan.”\(^2\) This raises an important policy question of whether a planner should intervene in the corporate insolvency process and, if so, under what conditions should policy promote reorganization or promote liquidation. Moreover, it is important to understand how insolvency interventions complement other interventions, such as macroprudential regulation or bailouts.

This paper provides a general equilibrium framework to study optimal interventions in existing corporate insolvency systems. Our model embeds both a fire sale externality – larger liquidations reduce liquidation prices – and a loan-price (“zombie loan”) externality – distressed loans congest bank balance sheets and constrain financing for healthy firms. We show that greater corporate distress – measured by high operating losses, low profitability, and high debt – leads a social planner to intervene to promote liquidation over reorganization, due to increased costs of balance

\(^1\)To paraphrase Greenwood, Iverson, and Thesmar (2020); (i) Hanson, Stein, Sunderam, and Zwick (2020b) recommend keeping firms solvent by funding fixed obligations like rent; (ii) Saez and Zucman (2020) recommend keeping firms solvent by funding all expenses; (iii) Brunnermeier and Krishnamurthy (2020) recommend subsidizing refinancing for small firms; (iv) Greenwood and Thesmar (2020) recommend extending a tax credit to claimants (i.e., landlords) that accept a haircut on loan obligations; (v) Iverson, Ellias, and Roe (2020) recommend hiring additional bankruptcy judges; (vi) Skeel (2020) recommend creating a standard “prepacked” restructuring process; (vii) Blanchard, Philippon, and Pisani-Ferry (2020) recommend extending the government accept larger losses than creditors in reorganizations; (viii) DeMarzo, Krishnamurthy, and Rauh (2020) recommend a government-funded vehicle extend debtor-in-possession financing; (ix) the Bankruptcy and COVID-19 Working Group recommends extending deadlines for small businesses in Chapter 11.

\(^2\)The Takenaka Plan (referring to the minister of the Japanese Financial Services Agency) entailed a forceful effort to end the non-performing loans problem. Takenaka forced banks to “make more rigorous evaluation of assets using discounted expected cash flows or market prices of non-performing loans... This stopped the process of ever-growing non-performing loans” (Hoshi and Kashyap, 2010). Indeed, the Takenaka plan led to an overall reduction in nonperforming loans on bank balance sheets by 50%. See also Hoshi and Kashyap (2011).

Japan has also implemented policies encouraging reorganizations. In response to slow economic growth in the 1990’s and a proliferation of nonperforming loans on bank balance sheets, “The Resolution and Collection Corporation (RCC), a government asset management company that already existed, also shifted their activities to put much more emphasis on reorganizing troubled borrowers.”
sheet congestion relative to fire sales. Most surprisingly, larger liquidation discounts also lead the planner to promote liquidation. Intuitively, larger discounts raise marginal congestion costs relative to marginal fire sale costs. Thus paradoxically, large fire sales can promote even more liquidations. We use our framework to study interactions between insolvency interventions and other policies.

In our two-period baseline model, firms enter a crisis with assets in place and debt owed to banks. Firms can be healthy, in which case they have a high long-run value and a new investment opportunity, or distressed, in which case they have a stochastic long-run value and a current operating loss that must be covered to maintain viability. Distressed firms are insolvent and must be either reorganized or liquidated by their creditors (banks). Banks are collateral constrained and use loans as collateral. Banks allocate their funding between new lending to healthy firms, and reorganizing distressed firms while covering their operating losses. Distressed firms that are liquidated are sold to arbitrageurs at a fire sale price, resulting in a fire sale problem. At the same time, liquidating a distressed firm frees up funding to lend to healthy firms, and so reduces bank balance sheet congestion.

We begin in Section 3 by characterizing the privately optimal insolvency rule chosen by banks for distressed firms. Formally, an insolvency rule is a probability of liquidating a distressed firm of a given long-run value. We show that the privately optimal liquidation rule is a threshold rule for liquidation: firms with long-run value above a threshold are reorganized, while firms with long-run value below the threshold are liquidated. The optimal threshold for liquidation equalizes on the margin the value to the bank of reorganizing or liquidating the firm. Crucially, the optimal threshold depends on the bank’s private value of loanable funds, which depends on the excess return the bank can earn from lending to healthy firms that arises due to the collateral constraint. A larger private value of loanable funds leads to more liquidations because it increases the opportunity cost of reorganizing a given firm.

Section 4 provides the main results of the paper: characterizing socially optimal insolvency rules. We study the problem of a social planner who chooses an insolvency rule to maximize social welfare, internalizing equilibrium price impacts but otherwise respecting constraints faced
by private agents. We show that the socially optimal insolvency rule is also a threshold rule for liquidation, but that the socially optimal threshold differs from the privately optimal threshold for two reasons. The first is the fire sale externality: additional liquidations depress the fire sale price, which reduces recovery and limits the ability of liquidations to alleviate banks’ loanable funds problem. The second is the loan price externality: the social value of loanable funds is higher than the private value of loanable funds, because the planner internalizes the fact that a higher loan price increases the collateral value of loans, relaxing banks’ collateral constraints and increasing borrowing capacity. In contrast to the fire sale, the loan price externality leads the planner to prefer more liquidations. More liquidations increase lending to healthy firms, which in turn boosts the loan price and the value of collateral, generating a positive externality for banks.

The model features significant interactions between the two externalities. On the one hand, larger loan price externalities increase the marginal cost of a contraction in loanable funds, and so increases the cost of the fire sale externality that contracts loanable funds. On the other hand, larger fire sales tighten bank collateral constraints, and so contract loanable funds. This increases excess returns (lower loan price) and so exacerbates the loan price externality. This significant interaction makes the direction of intervention – more or fewer liquidations – a priori uncertain.

Our model delivers concrete insights on whether optimal policy promotes more or fewer liquidations. We show that the social optimum can be decentralized with a simple tax or subsidy on liquidations. The tax/subsidy is not contingent on the long-term value of a firm, meaning the planner does not need specific knowledge of a specific insolvent firm’s characteristics to implement the social optimum. We show that optimal policy (ceteris paribus) favors liquidation subsidies for various measures of greater firm distress, including when firm operating losses are higher, when firm productivity is lower, and when firm debt is larger.

Most surprisingly, we show that lower liquidation prices – that is, larger fire sales – can also lead to liquidation subsidies being optimal. Although at first surprising, the result is intuitive: under constant elasticities, already-low liquidation prices make the marginal impact of further liquidations on total recovery relatively lower. Thus the marginal impact of fire sales on total recovery falls
even as its absolute magnitude rises. Although the larger fire sale reduces loanable funds obtained from liquidation, liquidation also saves banks the cost of covering the firms’ operating losses. This leads the loan price externality to dominate when fire sales are large, making liquidation subsidies desirable.

In Section 5, we extend our analysis to consider several policies that can potentially complement or substitute for insolvency interventions. In Section 5.1, we incorporate an initial (date 0) firm-bank borrowing-lending problem, and studying macroprudential regulation of bank balance sheets. Interestingly, optimal macroprudential regulation is tailored entirely to the loanable funds problem and not the fire sale problem. Macroprudential regulation is largest when the social value of loanable funds is largest, that is when liquidation subsidies are desirable. In Section 5.2, we study the optimal sector for a planner to target bailouts. We show that bailouts to banks always dominate bailouts to healthy firms because banks can capitalize on a multiplier effect of collateralizing loans. Bailouts are more valuable as the social value of loanable funds rises. Interestingly, this suggests a role for non-revenue-neutral liquidation (or reorganization) subsidies, which serve a dual role of both recapitalizing banks and correcting their insolvency rule, over unconditional bailouts. In Section 5.3, we study the possibility that banks can boost loanable funds by avoiding recognizing losses on loans to insolvent firms: banks can pledge “zombie loans” at full collateral value. While this leads both the bank and the planner to prefer fewer liquidations, we show that the optimal intervention is qualitatively similar to the baseline model. Finally in Section 5.4, we study intervention when banks are heterogeneous in their lending capabilities. We show that the planner bifurcates banks into two separate functions: banks with small haircuts are “secured” creditors that receive payoffs from liquidations, while banks with large haircuts are distressed lenders that reorganize distressed firms. Interestingly, this promotes more reorganizations, suggesting that seniority structure interventions can partially substitute for interventions promoting liquidation.

Finally, Section 6 provides a brief policy-oriented discussion of how to implement liquidation

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3 This “double dividend” of Pigouvian subsidies – Pigouvian subsidies both correct externalities and generate bailouts for banks – is analogous to the double dividend that Pigouvian taxes both correct externalities and generate revenues for the government (Tullock 1967, Clayton and Schaab 2022b).
Related literature. This paper contributes to the theoretical literature studying optimal resolution of insolvent firms. In early seminal work, Shleifer and Vishny (1992) show that fire-sale externalities create a motive for social planners to avoid liquidations. Donaldson, Morrison, Piacentino, and Yu (2020) study complementarities between bankruptcy and out-of-court restructurings, and use their model to analyze recent proposed interventions in firm insolvency. Hanson, Stein, Sunderman, and Zwick (2020a) model an economy in which extending credit to otherwise insolvent firms helps to mitigate aggregate demand externalities, promoting reorganization. Philippon (2020) models a mechanism-design problem in which the government seeks to prevent inefficient liquidations without resorting to an indiscriminate bailout. Chari and Kehoe (2016) study optimal policy with time inconsistent bailouts in a costly state verification framework, and show that restrictions on debt and size constitute optimal policy and that intervention in resolution is not required. Clayton and Schaab (2022a) show that an orderly bank resolution (bail-in) regime promoting reorganization is a socially optimal policy in a optimal contracting model with an incentive problem and fire sales. Colliard and Gromb (2018) and Keister and Mitkov (2021) study how the prospect of bailouts distorts incentives of banks to privately bail-in or renegotiate with their creditors. Glode and Opp (2021) study complementarities in debt renegotiation when businesses are connected in a debt chain, and study how government interventions can prevent waves of defaults. Corbae and D’Erasmo (2021) estimate a general-equilibrium model with both reorganization and liquidation in bankruptcy, but do not consider fire-sale externalities or collateral constraints. Li and Li (2021) show that public liquidity interventions during crises preserve low-quality firms and mitigate the cleansing effect of crises. Our paper contributes to this literature by showing in a problem of optimal insolvency rule design that optimal intervention can favor either liquidation or continuation, depending on the strength of externalities in the banking sector versus the nonfinancial sector.

A substantial literature has studied optimal macroprudential and bailout interventions for financial intermediaries in the presence of pecuniary externalities (Bianchi 2016, Bianchi and Mendoza...
2018, Caballero and Krishnamurthy 2001, Dávila and Korinek 2018, Lorenzoni 2008, Stein 2012). We complement this literature by studying insolvency interventions in a model with two-sided pecuniary externalities. Lanteri and Rampini (2021) provide a related argument for new investment subsidies that lower future capital prices. In their heterogeneous firm model, a lower capital price generates a positive distributive externality from reallocating resources from low marginal product capital sellers to high marginal product capital buyers that dominates the usual collateral externality. In our model, liquidating distressed firms provides loanable funds to constrained banks, resulting in loans to healthy firms that support the lending price and induces a positive collateral externality.

Finally, this paper relates to the literature on zombie loans: subsidized bank loans to insolvent firms. Caballero, Hoshi, and Kashyap (2008) and Acharya, Crosignani, Eisert, and Eufinger (2020) show theoretically and empirically that, by keeping insolvent firms alive to compete in product markets, zombie loans lead to lower product prices and markups, reducing entry and productivity. We contribute to this literature by studying optimal insolvency intervention with both a fire sale externality from inefficient liquidation, and an opposing “zombie lending” externality from misallocation of credit due to socially suboptimal firm preservation. Our results provide guidance on the interaction between the externalities, and under which conditions each externality dominates the other.

2 Model

There are two dates, $t = 1, 2$. The economy consists of three types of agents: firms, banks, and arbitrageurs. There is a unit continuum of each type of agent. All agents are risk neutral and do not discount the future.

\footnote{For empirical evidence of zombie lending and the economic impact of zombie loans, see Caballero, Hoshi, and Kashyap (2008); Acharya, Eisert, Eufinger, and Hirsch (2019); Blattner, Farinha, and Rebelo (2019); Acharya, Borchert, Jager, and Steffen (2021) and Acharya, Crosignani, Eisert, and Eufinger (2020).}

\footnote{See Antill (2020) and Ayotte and Morrison (2009) for empirical evidence of inefficient liquidations and a review of the empirical literature on inefficient liquidations.}
2.1 Firms

Firms enter date 1 with an inherited project scale $I_0 > 0$ and long-term debt $D_0$ due at date two. The inherited project and long-term debt come from an ex ante (date 0) financing problem that we study in Section 5.1. At date 1, firms are either healthy with probability $p$ or distressed with probability $1 - p$.

Healthy firms have a constant exogenous high return $v_h$ on their outstanding project, paying off $v_h I_0$ at date 2. They can also invest in a new technology, whereby $I_1 \geq 0$ of the consumption good at date 1 produces $g_h(I_1)$ units of the consumption good at date 2. Letting $q_1$ denote the endogenous price of funding at date 1 – that is, $1/q_1$ is the gross interest rate – then healthy firms choose $I_1$ to maximize date-two cashflows (net of debt repayment),

$$\sup_{I_1} g_h(I_1) - \frac{I_1}{q_1} + v_h I_0 - D_0,$$

where $I_1/q_1$ is the required debt repayment to raise $I_1$. This means the healthy firm investment choice $I_1(q_1)$ is given by

$$q_1 g_h'(I_1) = 1.$$

Distressed firms have an idiosyncratic return $v \in [\underline{v}, \bar{v}]$ on their outstanding project, with density $f$ and corresponding CDF $F$. Distressed firms do not have a new investment opportunity. However, they experience a deterministic current operating loss $C \geq 0$ per unit of initial scale $I_0$ that must be paid at date 1 in order to maintain viability. If it is not paid, the distressed firm can be liquidated at an endogenous price $\gamma \geq 0$ per unit of scale $I_0$. Distressed firms are insolvent. Distressed firms are resolved by their creditors, which we describe below, and have no residual equity value.

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$^6$The arbitrageur may be interpreted as a strategic buyer that will use the liquidated assets in its business operations. Because the arbitrageur uses the assets in its own business, the liquidation payoff does not depend on the insolvent firm’s viability or expenses.
2.2 Banks

Banks at date 1 have an outstanding stock $B_0$ of short-term debt owed to households, and own the debt claims of healthy and distressed firms. Banks at date 1 choose new borrowing $B_1$ from households (at a constant price of 1), new loans $D_1$ to healthy firms (at price $q_1$), and how to resolve distressed firms.

The insolvency-resolution process is a rule for sorting firms into one of two outcomes: liquidation or continuation. Formally, it is a probability $\rho(v) \in [0, 1]$ that an insolvent (distressed) firm with viability $v$ is liquidated, with probability $1 - \rho(v)$ of continuation. Liquidation entails a sale of firm assets at the market price $\gamma I_0$. Continuation entails the bank agreeing to pay the firm’s date-one expenses in exchange for the firm’s future cashflows. This means the bank pays $CI_0$ at date 1 to get a total claim worth $vI_0$ at date 2. For technical reasons, we assume that a small fraction of the very worst firms ($v \leq \hat{v}$) are nonviable and must be liquidated.

In our baseline model, we omit the possibility of acquisitions for ease of exposition. In Appendix C, we consider an extension in which arbitrageurs acquire insolvent firms to continue their operations. We find results that are qualitatively similar to our baseline-model results.

The bank’s date 1 budget constraint is given by

$$pq_1D_1 + (1 - p) \int (1 - \rho(v))CI_0dF(v) \leq B_1 - B_0 + (1 - p) \int \rho(v)\gamma I_0dF(v)$$

(3)

so that total funds lent out to healthy and distressed firms equals resources obtained from liquidating distressed firms and (net) borrowing from households. New bank borrowing from households

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7 This could be achieved through either an out-of-court foreclosure and sale or through a bankruptcy filing under Chapter 7 or Chapter 11 of the US bankruptcy code.

8 Continuation could represent a “zombie loan” in which a bank lends to a firm at a subsidized rate, an out-of-court restructuring, or a reorganization under Chapter 11 of the US bankruptcy code. Since all uncertainty in our model is resolved at date one, it is irrelevant in our model whether the bank receives debt or equity from the insolvent firm in a continuation.

9 This bounds the liquidation price even with CRRA arbitrageur preferences, as in Assumption 1. Formally, we assume there is an exogenous cutoff $\hat{v} \in (\underline{v}, \overline{v})$ such that firms with $v < \hat{v}$ must be liquidated. The cutoff $\hat{v}$ can be arbitrarily close to $\underline{v}$. 
is subject to a collateral constraint, given by

\[ B_1 \leq \phi_h p q_1 (D_0 + D_1) + \phi_d (1 - p) q_1 \int (1 - \rho(v)) v I_0 dF(v), \]  

(4)

which allows for potentially different haircuts on loans to healthy firms versus distressed securities. We assume \( \phi_h \geq \phi_d \). Collateral is pledged based on the date 1 market value \( q_1 \) of financial securities.

Banks choose \((B_1, D_1, \rho)\) in order to maximize their final value,

\[
\sup_{B_1, D_1, \rho} \left\{ p \left( D_1 + D_0 \right) + (1 - p) \int (1 - \rho(v)) v I_0 dF(v) - B_1 \right\},
\]

(5)

subject to their budget constraint (3) and their collateral constraint (4). Recall that \( \rho \) is a function of \( v \).

### 2.3 Arbitrageurs

Arbitrageurs are the second-best users of firms’ liquidated investment projects. We interpret arbitrageurs as strategic buyers, such as new entrants, who repurpose firms’ liquidated assets for another business. Arbitrageurs have a technology that converts \( L \) dollars of liquidated assets into \( g_a(L) \) units of consumption. Given an equilibrium price \( \gamma \), arbitrageurs solve

\[ \max_L g_a(L) - \gamma L \]

resulting in demand for liquidated assets given implicitly by

\[ g_a'(L) = \gamma. \]  

(6)
2.4 Market clearing and competitive equilibrium

Our model has two equilibrium prices at date 1: the price $q_1$ of financial securities, and the price $\gamma$ of real liquidated assets. Both markets must clear at date 1, that is

$$q_1 D_1 = I_1$$  \hspace{1cm} (7)

$$\left(1 - p\right) \int \rho(v) I_0 dF(v) = L.$$  \hspace{1cm} (8)

A competitive equilibrium of the model is prices $(q_1, \gamma)$ and allocations $(D_1, B_1, \rho, I_1, L)$ such that: (i) healthy firms choose $I_1$ to maximize utility; (ii) banks choose $(B_1, D_1, \rho)$ to maximize utility; (iii) arbitrageurs choose $L$ to maximize utility; (iv) markets clear.

3 Privately optimal liquidation rules

We begin by characterizing the privately optimal liquidation rule chosen by banks in a competitive equilibrium. The policies $(B_1, D_1)$ can then be determined using the liquidation rule, constraints, and market prices.

Proposition 1. In any competitive equilibrium, the bank’s privately optimal liquidation rule is a threshold rule $\rho(v) = 1$ ($v \leq v_L^p$), where

$$v_L^p = \frac{1 + \theta^p}{1 + \theta^p \phi_d q_1} (\gamma + C),$$  \hspace{1cm} (9)

and $\theta^p$ is the private value of loanable funds,\footnote{Formally, $\theta^p$ is the value of the Lagrange multiplier on the constraint (4) in the Lagrangian for the bank’s problem, evaluated at the optimum.} defined as

$$\theta^p = \frac{1}{1 - \phi_h} \times \left(\frac{1}{q_1} - 1\right) \geq 0.$$  \hspace{1cm} (10)

\footnote{Unless explicitly mentioned, we focus attention on cases with interior solutions $v_L^p \in (\psi, \tilde{\psi})$ and $v_L^p > \tilde{\psi}$.}
All proofs appear in Appendix A. Intuitively, Proposition 1 says that banks choose the liquidation rule that maximizes their ex post recovery. If bank collateral constraints do not bind in competitive equilibrium, then $q_1 = 1$ and hence $\theta^p = 0$. Thus there is no (excess) private value to loanable funds, and $v_L^p = \gamma + C$. The threshold firm that is liquidated is precisely the one where the value from liquidation, $\gamma$, is equated with the value from reorganization, $v_L^p - C$. Thus the rule in this case maximizes creditor recovery. Although in principle banks could choose arbitrary rules $\rho$, in equilibrium banks always select a threshold rule.

When $q_1 < 1$ (binding collateral constraint), then we have $\theta^p > 0$. In this case, equation (9) still reflects creditor recovery maximization, but accounts for the fact that part of the value of loans comes from the private value of loanable funds. Intuitively, the total effective value of a reorganized firm is its direct final value, $v_L^p$, plus the collateral value of the distressed security, $\theta^p \phi_d q_1 v_L^p$. If $\phi_d = 0$ and distressed securities cannot be used as collateral, then this value is just $v_L^p$. The bank trades off this total value against the value that could be obtained by liquidating and redeploying the funds into loans to healthy firms. This accounts not only for the direct value $\gamma + C$, but also for the value of loanable funds, $\theta^p(\gamma + C)$.

The private value of loanable funds represents the product of the amount of new lending times the excess return on new lending. Intuitively, a unit of loanable funds allows the bank to create a loan with excess return $\frac{1}{q_1} - 1$, which it can then pledge as collateral. By pledging it as collateral, the bank gains an additional $\phi_h$ units of loanable funds, which then creates a loan that earns an excess return and can be pledged as collateral. Iterating this process, total new loan creation is $\sum_{n=0}^{\infty} \phi_h^n = \frac{1}{1 - \phi_h}$, which is then multiplied by the excess return to get the private value of loanable funds. The private value $\theta^p$ of collateral therefore increases in the excess return $\frac{1}{q_1} - 1$ (i.e., value of new loans), and increases in the ability to collateralize new loans $\phi_h$ (i.e., quantity of new loans).

In sum, Proposition 1 represents a form of creditor-recovery-maximizing liquidation decisions, whereby the bank privately trades off the total (direct + shadow) value of a reorganized firm against the value of having funds to redeploy to new loans. The optimal liquidation rule sets equal on the margin these two values, and liquidates firms when the opportunity cost of reorganization is larger.
than the total value of reorganization.

4 Socially optimal liquidation rules

We next study the socially optimal liquidation rule that is designed by a social planner who internalizes the determination of equilibrium prices, but must otherwise respect the same constraints faced by private agents. The planner has a complete set of Pigouvian wedges $\tau$ on the decisions of banks, but must take as given the borrowing decisions of healthy firms and the purchasing decisions of arbitrageurs. Given complete wedges, we can adopt the primal approach whereby the planner directly chooses $(B_0, D_1, \rho)$ for banks, but must take as given the constraints faced by private banks (equations 3 and 4), the equilibrium pricing conditions of firms and arbitrageurs (equations 2 and 6), and the market clearing conditions (equations 7 and 8).

The social planner has a utilitarian objective function, where as usual we can think of the planner achieving Pareto efficiency via lump sum transfers (either at date 0 (Section 5.1) or at date 2). The utilitarian objective function, summed across all agents (see the proof of Proposition 2 for a derivation), is given by

$$
\sup_{D_1, B_1, \rho, I_1, L, q_1, \gamma} \left[ p \left( g_h(I_1) + v_hI_0 \right) + (1 - p) \int (1 - \rho(v))vI_0 dF - B_1 + g_a(L) - \gamma L \right].
$$

(11)

The utilitarian objective nets out transfers between arbitrageurs, banks, firms, and households. As a result, the objective captures the two sources of surplus in the economy: surplus from firm production and surplus from arbitrageur purchases. Note $B_1$ and $\gamma L$ appear in surplus because they also represent diversion of consumption resources towards production: $B_1$ represents funds lent to firms intermediated through banks, while $\gamma L$ represents funds from arbitrageurs intermediated through banks via liquidation sales. Loans between banks and firms drop out because they reflect intermediated resources from households/arbitrageurs.
In the results to come, it will be helpful to define the elasticities $\xi_q$ and $\xi_\gamma$ of equilibrium loan and liquidation prices,

$$\xi_q \equiv \frac{I_1}{q_1} \frac{\partial q_1}{\partial I_1}, \quad \xi_\gamma \equiv -\frac{L}{\gamma} \frac{\partial \gamma}{\partial L}. \tag{12}$$

These elasticities are not required to be constant. These elasticities are characterized using the demand functions of healthy firms and arbitrageurs. Note that we define these elasticities such that $\xi_q, \xi_\gamma \geq 0$. It will also be helpful to define the total values of "old" collateral (date 0 loans) and "new" collateral (date 1 loans),

$$C^{\text{old}} = \phi_h p q_1 D_0 + q_1 \phi_d (1 - p) \int (1 - \rho(v)) v I_0 dF(v) \tag{13}$$

$$C^{\text{new}} = \phi_h p q_1 D_1. \tag{14}$$

We are now ready to solve the social planner’s problem. The following proposition characterizes the socially optimal liquidation rule.

**Proposition 2.** The socially optimal liquidation rule is a threshold rule $\rho(v) = 1 \ (v \leq v^s_L)$, where

$$v^s_L = \frac{1 + \theta^s}{1 + \theta^s \phi_d q_1} (\gamma + C) - \frac{\theta^s}{1 + \theta^s \phi_d q_1} \gamma \xi_\gamma \geq v^s_L \tag{15}$$

and where $\theta^s = M^s \theta^p$ is the social value of loanable funds,\footnote{Formally, $\theta^s$ is the value of the Lagrange multiplier on the constraint (4) in the Lagrangian for the planner’s problem, evaluated at the social optimum.} with

$$M^s \equiv \frac{1}{1 - \frac{\phi_h}{1 - \phi_h} \xi_q} \geq 1. \tag{16}$$

Proposition 2 shows that like the privately optimal rule, the socially optimal rule is a threshold rule for liquidation: firms with $v \leq v^s_L$ are liquidated, while firms with $v \geq v^s_L$ are reorganized. If
bank collateral constraints do not bind in equilibrium, then there is no constraint on loanable funds and hence $q_1 = 1$. Given $q_1 = 1$, we have $\theta^s = \theta^p = 0$, and there is no private or social value to loanable funds (i.e., the Lagrange multiplier on the collateral constraint is zero). As a result, equation (15) collapses to

$$v_L^s = \gamma + C,$$

that is to the simple creditor recovery maximizing rule. In this case, the private and social optima coincide, and there is no need for intervention. Observe that in this case in absence of binding constraints, a depressed liquidation price $\gamma$ is simply redistribution between buyers and sellers (a welfare-irrelevant pecuniary externality).

Proposition 2 shows that when the collateral constraint binds, the socially optimal rule differs from the privately optimal rule in two key manners. The first force driving the planner’s solution is the fire sale of real assets. Banks do not internalize that liquidating a firm pushes down the price of liquidated assets, lowering liquidation recoveries for all banks. This lower liquidation price leads to a transfer from banks to arbitrageurs. However in the presence of a binding collateral constraints, the marginal value of wealth at date 1 is higher for banks than for arbitrageurs, giving rise to a welfare-relevant pecuniary externality. In particular, fire sales reduce surplus because lower liquidation prices reduce all banks’ loanable funds. This effect scales based on the social value of loanable funds, which is the source of the pecuniary externality. This force leads the planner to prefer fewer liquidations than private banks.

The second difference between the planner’s solution and the private solution arises due to the difference between the private and social value of loanable funds. In particular, the social value of loanable funds differs from the private value of loanable funds by a multiplier $M^s$, and is always larger. Intuitively, an optimizing bank does not internalize that its loans to healthy firms can change the price $q_1$. In particular, bank lending creates a positive externality. More loans lead to higher $q_1$. Higher $q_1$ values in turn increase collateral values on all loans for all banks, allowing banks to borrow more and lend more. While banks do not internalize this externality, the social planner does internalize this extra multiplier effect of lending. This multiplier effect leads the social value
of loanable funds to exceed the private value of loanable funds. Because liquidations free up funds for lending to healthy firms, this force leads the planner to prefer *more* liquidations than private banks.

Equation (16) displays the multiplier $M^s$. When $M^s$ is larger, the gap between the social and private value of loanable funds grows. Intuitively, $M^s$ reflects the extent to which an additional loan to a healthy firm helps other banks borrow. Holding all else equal, $M^s$ is larger when additional loans have a large impact on the loan price - when the elasticity $\xi_q$ is large. Likewise, $M^s$ is high when an increase in the loan price leads to a large improvement in collateral valuations. This occurs when loans to healthy firms can easily be used as collateral to finance additional loans - when $\phi_h$ is high. This also occurs when the pool of old collateral being revalued is large - when $\mathcal{C}^\text{old}$ is large. The pool of old collateral is large when the outstanding stock of debt $D_0$ is large, or when a large number of distressed firms are being reorganized (and distressed securities provide collateral value).

Proposition 2 nests both limiting cases where only one externality is present, and the direction of intervention is unambiguous. If $\xi_f = 0$ and there no fire sale of real assets (i.e., arbitrageurs have linear technology $g_a(I_1) = \gamma L$ and $\gamma = \overline{\gamma}$), then the fact that $M^s \geq 1$ means $\theta^s \geq \theta^p$. As a result, on the margin optimal policy *encourages* liquidations. On the other hand if $\xi_q = 0$ (i.e., healthy firms have linear technology $g_h(I_1) = A_h I_1$, with $q_1 = \frac{1}{A_h} < 1$), banks are borrowing constrained but the lending price is constant. A binding borrowing constraint still means banks have a higher marginal value of wealth than arbitrageurs ($\theta^p > 0$), but at the same time we have $M^s = 1$ and hence the private and social values of loanable funds coincide ($\theta^s = \theta^p$). As a result the fire sale is the relevant externality, resulting in optimal policy *discouraging* liquidations.

Proposition 2 additionally reveals significant interactions between the two externalities. The fire sale externality, all else equal, scales up in $\theta^s$, which in turn rises in the excess social value of loanable fund, $M^s$. This means that large excess social value of loanable funds amplifies the welfare costs of fire sales, which reduce recovery value and loanable funds. At the same time,

\footnote{Note that we can form a similar argument by setting $\phi_h = 0$ instead, in which case banks are borrowing constrained but constraints do not depend on $q_1$.}
larger fire sales also feed into larger loan price externalities: a larger fire sale tightens the bank collateral constraint, which all else equal contracts lending to healthy firms and so (from equation 2) reduces the loan price \( q_1 \). This drives up the excess return \( \frac{1}{q_1} - 1 \) and hence the baseline value of loanable funds, \( \theta^p \), amplifying the loan price externality.

### 4.1 Liquidation taxes or liquidation subsidies?

Propositions 1 and 2 provide the privately and socially optimal insolvency rules. We can now conduct the exercise of asking what is the optimal tax (or subsidy) on liquidations that would decentralize the socially optimal outcome.

Formally, we introduce a tax \( \tau \) (per unit of scale \( I_0 \)) on liquidations. Taxes are paid out of banks’ final payout at date 2. This means that if a bank liquidates a distressed firm, it gets the usual \( \gamma I_0 \) units of loanable funds at date 1, but must make a payment \( \tau I_0 \) to the government out of its final equity value at date 2. If \( \tau < 0 \), then the policy is a liquidation subsidy (the bank receives payment at date 2). The tax/subsidy proceeds are remitted lump sum at the end of date 2, with the lump sum rebate taken as given in decision rules. As a result, the privately optimal value the bank receives from liquidating a firm is \( (1 + \theta^p)(\gamma + C) - \tau \) per unit, while the value of continuation is \( (1 + \theta^p \phi_d q_1) v \). We thus obtain the following characterization.

**Proposition 3.** The liquidation tax/subsidy \( \tau \) that decentralizes the social optimum of Proposition 2 is given by

\[
\tau = \left[ -\frac{M^s - 1}{M^s} \left( 1 - \phi_d q_1 \right) \left( \gamma + C \right) + \left[ 1 + \theta^p \phi_d q_1 \right] \gamma \xi \gamma \right] \frac{\theta^s}{1 + \theta^s \phi_d q_1}. \tag{17}
\]

Proposition 3 provides a simple decentralization of the social optimum using a uniform tax/subsidy on liquidating a firm. The tax/subsidy is independent of firm long-term value, and incentivizes banks to move their optimal threshold \( v^p_L \) to coincide with the socially optimal threshold, that is to set \( v^p_L = v^s_L \). Intuitively, since both banks and the planner choose threshold rules, a con-
stant tax/subsidy suffices to align the private and social optimum, without the need to specify \( \nu \)-contingent taxes or subsidies. This means that implementing the efficient rule in this context requires no knowledge from the planner about the long-term value of any specific individual firm. This allows the planner to achieve the social optimum with a simple tool, without the need to tailor intervention towards specific firms.

Intuitively, the optimal tax/subsidy on liquidation trades off two effects. On the one hand, a liquidation increases loanable funds by \( \gamma + C \), which enhances the loan price. This promotes a liquidation subsidy. On the other hand, it exacerbates the fire sale, \( \gamma \xi \). This promotes a liquidation tax. The first effect is given a relative weight \( \frac{M^s - 1}{M^s} \geq 0 \), which functions akin to a relative Pareto weight on this effect.

Proposition 3 provides strong intuition on the direction of intervention, that is whether a liquidation tax or subsidy is optimal. All else held equal, higher operating losses \( C \) and higher social values of loanable funds \( M^s \) both promote liquidation subsidies, \( \tau < 0 \). This is intuitive, as both increase the desirability of loanable funds by either directly increasing demand for them or by increasing their social value. Also intuitively, a higher fire sale elasticity promotes a liquidation tax by increasing the cost of the fire sale. Interestingly, this relative importance of this effect is dampened as the value of loanable funds rises, reflecting the increased desire of the planner to obtain loanable funds even at the cost of a higher price impact.

These comparisons are made based on equilibrium objects. For example, \( M^s \) is itself endogenously determined, and for example rises with the security price elasticity \( \xi q^s \). To shed further light on the problem, we now make the following assumptions, which enable us to directly characterize the direction of intervention in terms of date 1 exogenous objects.

**Assumption 1.** Assume that \( \phi_d = 0 \), that \( g_h(I_1) = A_h \log(I_1) \), and that \( g_a(L) = \frac{1}{1 - \xi L} \log(1 - L) \). Finally, assume \( D_0 < \frac{1}{\phi_h} - 1 \).

We maintain Assumption 1 for the rest of this section and focus on the case where \( q_1 < 1 \).

\[ ^{14} \text{This final assumption is a technical assumption to guarantee that a loan does not generate more than one unit of loanable funds through collateral.} \]
Assumption 1 implies constant elasticities \( \xi_g = 1 \) and \( \xi_y = \xi_y \geq 0 \). It also implies that there is a highest feasible value \( \bar{\gamma} \) of the liquidation price \( \gamma \).\footnote{Recall that we assume a firm must be liquidated if \( v < \hat{v} \) for \( \hat{v} \approx v \). Together with Assumption 1, this implies \( \gamma \) must be less than \( \bar{\gamma} \equiv (I_0(1-p)F(\hat{v}))^{1/\gamma} \).} Under Assumption 1, we obtain the following characterization.

**Proposition 4.** Under Assumption 1, a sufficient condition for \( \tau < 0 \) is

\[
\bar{\xi}_y \leq \frac{\phi_h D_0}{1 - \phi_h A_h} \left( 1 + \frac{C}{\bar{\gamma}} \right).
\]

Proposition 4 provides a simple sufficient condition under which the optimal intervention takes the form of a liquidation subsidy. The condition depends on three objects taken as given at date 0. The intuition for this result mirrors the intuitions discussed above. The left side of (18) is the extent to which additional liquidations reduce liquidation prices. The right side of (18) represents the planner’s internalized multiplier effect of additional loans. Under Assumption 1, the loan-price elasticity \( \xi_q \) is equal to one. The multiplier is large when additional loans lead to a large improvement in collateral values - when there are a large number of outstanding loans, \( D_0 \), that can be employed as collateral, \( \phi_h \). The multiplier is also large when healthy firm profitability \( A_h \) is low. Proposition 4 thus shows that comparative statics with respect to (from the perspective of date 1) exogenous model parameters confirm the intuitions behind Propositions 2 and 3. The proof of Proposition 4 shows we can obtain an almost-identical sufficient condition with \( \phi_d > 0 \).

Proposition 4 also implies that higher operating losses \( C \) encourage liquidation subsidies. Intuitively, the planner values each unit of loanable funds more than a privately optimizing bank. As \( C \) grows, loanable funds become scarcer as resources are tied up in restructuring distressed companies. Since the planner values the lost units of loanable funds more than private banks, the planner must incentivize banks to liquidate more in response to this increase in \( C \) than they otherwise would. Higher \( C \) thus encourages liquidation subsidies.

To give a sufficient condition for \( \tau < 0 \) that does not rely on endogenous objects, we write (18)
in terms of \( \bar{\gamma} \), the highest possible realization of \( \gamma \). As a result, (18) is a sufficient but not necessary condition for \( \tau < 0 \). To present a sufficient and necessary condition, we now give a characterization based the endogenous liquidation price \( \gamma \). Since market clearing for liquidated assets implies that \( \gamma = L^{-\bar{\gamma}} \), we can equivalently state the condition in terms of the quantity of liquidated assets.

**Proposition 5.** \( \tau < 0 \) if and only if the following holds:

\[
L > C^{-\frac{1}{\bar{\gamma}}} \left( \frac{1 - \phi_h A_h \bar{\xi} \bar{\gamma}}{\phi_h D_0 \bar{\gamma} - 1} \right)^{\frac{1}{\bar{\gamma}}}. \tag{19}
\]

Moreover, equation (19) is equivalent to:

\[
\gamma < C \left( \frac{1 - \phi_h A_h \bar{\xi} \bar{\gamma}}{\phi_h D_0 \bar{\gamma} - 1} \right)^{-1}. \tag{20}
\]

Proposition 5 provides a surprising result that optimal intervention targets the less distressed market: it is only at relatively low liquidation values \( \gamma \) (high levels of liquidation \( L \)) that intervention promotes liquidation. Put another way, optimal intervention promotes reorganization unless the liquidation fire sale has become sufficiently severe. The intuition comes from the combination of liquidation discounts and sunk costs. When liquidation discounts \( 1 - \gamma \) are already large, there is little additional cost to exacerbating the fire sale and depressing liquidation values, but there is still a large cost to keeping afloat a loss-making firm. This results in liquidation subsidies that promote further liquidations being desirable when a large number of firms are already being liquidated.

In sum, the results of this section show that the efficient insolvency intervention takes a simple form, involving a uniform tax or subsidy on liquidation that does not depend on information about the long-term prospects of an individual firm. This allows the planner to intervene to promote efficiency without deep involvement in the insolvency process of an individual firm. We also showed that a number of measures of corporate distress – high operating losses, high debt levels, low future profitability, and high liquidation *discounts* (low liquidation values) – promote liquidation *subsidies* by increasing the amount or value of loanable funds obtained through liquidation.
5 Regulation and policy interventions

In this section, we consider how insolvency interventions (liquidation taxes and subsidies) interact with other regulations aimed at mitigating crises. We extend our model to study ex ante macroprudential regulation, ex post fiscal interventions, impaired loan recognition by banks, and interventions in creditor seniority structures.

5.1 Ex ante macroprudential regulation

We study the role for macroprudential regulation of banks in conjunction with ex post insolvency intervention. To do so, we introduce the ex ante borrowing/lending decision at date 0. We streamline the date 0 firm problem in order to sharpen focus on date 0 bank balance sheet regulation.\[16\]

5.1.1 Date 0 setup

**Firms.** At date 0, there are a large number (measure $N > 1$) of identical firm managers. Each firm manager is penniless and has the ability to undertake a project of fixed scale $I_0 > 0$. A firm manager faces an agency friction that limits her pledgeable income to $D_0 \leq \bar{D}_0$.\[17\] Each firm manager offers banks a contract $(Q_0, D_0)$, which promises repayment of $D_0$ in exchange for $Q_0$ in funding. Any contract offered must have $Q_0 = I_0$ and must promise repayment $D_0 \leq \bar{D}_0$. Any firm manager whose contract is accepted gets financed and forms a firm with initial investment scale $I_0$, then proceeds to date 1. These firm managers receive indirect utility as described in Section 2. Firm managers whose contracts are not accepted at date 0 exit the economy.

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\[16\] We abstract away from aggregate uncertainty for simplicity. Results of this extension are easily extended with aggregate uncertainty.

\[17\] One simple microfoundation is unobservable effort. Firm managers at date 0 can work (“high effort”) or shirk (“low effort”). Firms that work are healthy with probability $p$ and distressed with probability $1 - p$. Firms that shirk generate a private benefit $p(I_0 - \bar{D}_0)$ but no cashflow regardless of whether they are healthy or distressed. However, firms that shirk and are healthy can default on existing debt and subsequently undertake the date 1 project, meaning cashflows from the date 1 project cannot be pledged at date 0. Thus, the agency rent is $p(I_0 - \bar{D}_0)$ and the maximum pledgeable debt level is $\bar{D}_0$. 

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21
**Banks.** A representative bank has finite lending capability $I_0$. The bank can finance these loans either by raising debt $B_0$ from households at a price of 1 per unit, or by raising costly inside equity $A_0$ at separable utility cost $\Psi(A_0)$. Thus, the financing constraint of banks is $A_0 + B_0 = I_0$, and their utility is the final value specified in Section 2 net of utility costs $\Psi(A_0)$ at date 0 from costly equity.

It is immediate that the bank accepts contracts from measure 1 of the $N$ firm managers in descending order by offered debt level, with potentially random allocations among firms offering the same contract. It is easy to see that there is a symmetric equilibrium in which the bank allocates funding with uniform probability $\frac{1}{N}$ among managers that offer $D_0 = \overline{D}_0$, and all managers offer contracts $D_0 = \overline{D}_0$.\(^{19}\)

Thus if we denote $V^B(B_0, q_1, \gamma)$ to be bank’s indirect utility at date 1 given prices and its funding choice $B_0$, then banks solve

$$\max_{B_0, A_0} -\Psi_0(A_0) + V^B(B_0, q_1, \gamma) \quad \text{s.t.} \quad A_0 + B_0 = I_0.$$  

### 5.1.2 Private and social optima

In this environment, the planner’s optimum can be described as the optimal choice of ex ante borrowing $B_0$ and ex post liquidation rule $\rho$. It is easy to verify that the optimal liquidation rule $\rho$ is analogous to Section 4, conditional on a financing structure. Thus if we describe $V_1(B_0)$ to be the planner’s continuation value at date 1, then we know the date 0 planner’s objective is simply $-B_0 - \Psi_0(A_0) + V_1(B_0)$. Note the planner’s continuation value does not depend on equilibrium prices because the planner internalizes that equilibrium prices are a function of borrowing.

In this environment, we obtain the following privately and socially optimal financing rules.

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\(^{18}\)Formally, the bank’s marginal cost function is 1 for $I \leq I_0$ and $1 + \kappa$ for $I > I_0$, where $\kappa$ is large.

\(^{19}\)A manager that deviates from the conjectured equilibrium is rejected with probability 1 and earns no surplus, whereas a manager that offers $Q_0 = \overline{D}_0$ is accepted with probability $\frac{1}{N}$ and earns positive expected surplus.
Proposition 6. Banks’ privately optimal equity issuance satisfies

$$\Psi'(A_0) = 1 + \theta^p,$$

while the socially optimal equity issuance satisfies

$$\Psi'(A_0) = 1 + \theta^s.$$

Thus, the date 0 social optimum can be decentralized with a tax on debt\(^{20}\) given by

$$\tau^B_0 = (M^s - 1)\theta^p$$

Intuitively, banks’ privately optimal debt-equity decision trades off the higher cost of equity financing, $$\Psi'(A_0) - 1$$, against the indirect cost of debt financing through the contraction in loanable funds, $$\theta^p$$. Banks therefore adopt a balance sheet that uses more debt when the private value of loanable funds at date 1 is lower. The intuition is that banks recognize that more debt reduces their lending capacity at date 1, and so they prefer to incur greater equity costs when lending capacity is valuable.

Proposition 6 also shows that the social optimum features more equity issuance and less debt issuance than the private optimum. This is reflected by the excess social value of loanable funds relative to the private value of loanable funds, resulting in a tax on debt of $$\tau^B_0 = (M^s - 1)\theta^p > 0$$ being optimal. Intuitively, the planner internalizes a higher value of loanable funds at date 1 than the bank, as in Section 4, in proportion to the excess social value $$M^s - 1$$ of loanable funds. This leads to discouraging debt financing in proportion to the excess social value of loanable funds.

Under the special case of Assumption 1, we had $$M^s = \frac{1}{1 - \phi_h \frac{D_0}{\phi_h}}$$. Thus the subsidy is small when $$\phi_h \approx 0$$ and there is little excess value to loanable funds, but is larger when $$\phi_h$$ and $$D_0$$ are large.

\(^{20}\)It is immediate that the social optimum could also be implemented with an equity capital requirement $$A_0/I_0 \leq \phi^*$$, where $$\phi = A_0^*/I_0$$ and $$A_0^*$$ socially optimal equity, or with a leverage cap $$B_0/I_0 \leq 1 - \phi$$. 23
or when $A_h$ is small. Interestingly, this implies that macroprudential regulation of bank liabilities is particularly valuable when the nonfinancial corporate sector is more heavily indebted, or when nonfinancial corporates are less profitable in the future.

It is also interesting to observe that ex ante regulation is tied to the excess social value of loanable funds, and not directly to the liquidation fire sale. This is because in this model, an increase in bank debt reduces date 1 loanable funds but does not directly force the bank to increase liquidations, which are a choice variable in insolvency. Notably, both macroprudential interventions and liquidation subsidies become more attractive as $M^s$ increases. Intuitively, strong balance sheet regulation is desirable when the social value of loanable funds at date 1 is high. However, it is precisely in this case that intervention promoting liquidation, which bolsters loanable funds, is also attractive. Interestingly, this suggests that macroprudential interventions and liquidation subsidies can go hand in hand.

In principle, stronger balance sheet regulation can provide a substitute for insolvency interventions: choosing even higher values of $A_0$ would serve to increase loanable funds and reduce $\theta^s$. For example, stronger balance sheet regulation might thus reduce the need for liquidation subsidies. However in the lens of the model, this constitutes a third-best intervention. Insolvency interventions in our model provide the planner an additional tool for boosting loanable funds at date 1, which is separate from the macroprudential tool. Moreover, our model highlights that there is a natural synergy between the two tools: insolvency interventions and macroprudential interventions targeting loanable funds are desirable in tandem when the excess social value of loanable funds $M^s - 1$ is high.

### 5.2 Ex post fiscal interventions

Section 4 illustrates how interventions in the insolvency rule can be a desirable method of boosting loanable funds at the interim date (either by directly increasing loanable funds or by increasing the liquidation price). We now study a potentially complementary policy: fiscal interventions ("bailouts") that directly boost loanable funds. This question is interesting in particular because,
given there are multiple agents (banks, healthy firms, distressed firms), it is not a priori obvious to which agent a social planner would want to allocate bailout funds, or how it might interact with insolvency interventions. We briefly characterize how optimal bailouts and insolvency interventions go hand in hand.

We study the following exercise: suppose that the social planner was endowed with a marginal unit of bailout funds that it could allocate to any agent at date 1, in conjunction with its insolvency intervention. As our focus is on where the benefit is highest, we abstract away from the costs of raising bailout funds.\(^{21}\) The following proposition characterizes the marginal social welfare impact of allocating this unit of funds to different agents. These results also easily extend to characterizing optimal bailout rules, which we discuss briefly below at the end of the section.

**Proposition 7.** The marginal social welfare benefit of a bailout is:

1. \(1 + \theta^s\) when transferred to either banks or distressed firms
2. \(1 + \theta^s(1 - \phi_h)\) when transferred to healthy firms.

Proposition 7 yields a striking result that bailing out banks (weakly) dominates bailing out firms. Intuitively, bailing out a healthy firm has the effect of creating a unit of loans, whereas bailing out a bank has the effect of creating a unit of loanable funds. The latter dominates the former because the social value of loanable funds is amplified by the ability of the bank to leverage those funds as collateral to engage in further lending, giving rise to a multiplier effect. In this environment, it is thus always better for the planner to supply the bank with funds to lend, rather than to directly lend to nonfinancials.

On the other hand, in this environment it is interesting to observe that a bailout of a bank and a distressed firm are equivalent. This intuition arises most directly when studying a reorganized firm. If a firm is reorganized, the bank has to extend funding to cover the operating loss \(C\). The government bailout reduces the effective size of the operating loss that the bank needs to cover.

\(^{21}\)One might think of the marginal cost as \(1 + \kappa\) for some \(\kappa \geq 0\).
But given the decision to reorganize, this is equivalent to a direct bailout of the bank, which the bank then uses to cover part of the operating loss. Thus, bailouts of a distressed firm and bank are equivalent, conditional on an insolvency rule for that firm.

Proposition 7 reveals that the social value of bailouts is precisely the social value of loanable funds, $\theta^s = M^s \theta^p$. This means that the social value of bailouts increases with the excess social value of loanable funds, $M^s$. Interestingly, this suggests a synergy between bailouts and liquidation subsidies: both serve to increase loanable funds when loanable funds are particularly valuable. One particularly interesting manifestation of this idea is to use Pigouvian interventions that generate revenues for banks, and so simultaneously correct incentives and recapitalize banks. For example, a revenue-negative liquidation subsidy (i.e., that transfers resources to banks) achieves the dual benefit of encouraging banks to adopt the optimal insolvency rule (Proposition 3) and of providing bailouts to banks (Proposition 7). Given that liquidation subsidies are more attractive when bailouts are also attractive – that is, $M^s$ is large – this suggests that liquidation subsidies are actually more efficient than unconditional bailouts. In a similar fashion, a planner that wished to promote reorganization could offer subsidized DIP loans to distressed firms that were reorganized, achieving the dual benefit of a Pigouvian intervention in the insolvency decision along with an indirect bailout transfer to banks.

Finally, it is straightforward to characterize ex post optimal bank bailout rules in this environment. Suppose that bailouts $T_1$ are raised from households at total cost $T_1 + \kappa(T_1)$, where $\kappa$ is increasing and weakly convex. It follows immediately from the utilitarian objective that the optimal bank bailout rule is

$$\kappa'(T_1) = \theta^s.$$  

Therefore, the optimal bailout size increases in the equilibrium $\theta^s$. This provides another perspective on why large bailouts and large insolvency interventions can go hand in hand: if in equilibrium $\theta^s$ is small, not only are bailouts small but also the private and socially optimal liquidation rules more closely coincide. By contrast if in equilibrium $\theta^s$ is large, then not only are bailouts large but also the gap between private and optimal insolvency rules is large.
5.3 Bank writedown avoidance and zombie loans

The literature studying zombie loans notes that when a bank recognizes a nonperforming loan, it will likely have to write off existing capital, tightening minimum capital constraints (Caballero, Hoshi, and Kashyap, 2008). This creates an incentive for banks to offer credit to an insolvent firm, even if the firm’s liquidation value exceeds its going-concern value, to avoid recognizing a loss.

In Appendix B, we extend the model to consider delayed loss recognition. In particular, we assume that if a bank chooses continuation for a particular distressed firm, it can pledge the loan as collateral as if the firm were solvent. Formally, the collateral value of a distressed loan in this extension is $\phi_d q_1 D_0$ rather than $\phi_d q_1 v I_0 < \phi_d q_1 D_0$.

In this setting, we derive analogs of the results of Sections 3 and 4 (see Appendix B). All of the model forces in our baseline model play the same roles in this setting. In particular, banks consider both direct recovery and the value of loanable funds when choosing liquidations. The planner trades off the same externalities. However, this setting produces two noteworthy novel results. First, privately optimizing banks are more inclined to reorganize firms in this extension. In our baseline setting, a positive value to loanable funds always pushes for more liquidations relative to a rule maximizing direct creditor recovery. In this setting, the incentive to avoid writedowns pushes in the other direction, and as a result banks can find it optimal to choose more continuations or more liquidations, relative to a rule maximizing direct creditor recovery.

Second, comparing the planner solution to the private solution, we show that bank writedown avoidance amplifies the existing tradeoffs. Bank writedown avoidance increases the value of old collateral, which in turn increases the social multiplier $M^s$ and pushes for more liquidations. On the other hand, the greater collateralizability of distressed collateral also increases the loss in loanable funds from liquidation, which pushes for fewer liquidations. Delayed loss recognition thus serves to amplify the existing trade-offs. Appendix B provides further details.
5.4 Heterogeneous banks

In Appendix D, we extend our model to consider the effect of heterogeneous bank creditors. In this extension, multiple creditors indexed by $b = 1, 2, ..., B$ differ in the extent to which their collateral constraints bind: the parameter $\phi^b_h$ varies across banks. We study the design of the socially optimal liquidation rule, as well as the socially optimal seniority structure among creditors.

We show that a social planner can improve welfare by strategically subordinating the claims of some banks based on their idiosyncratic collateral constraints. Specifically, the planner bifurcates banks into two groups. Banks that can easily collateralize loans to healthy firms (i.e., high $\phi^b_h$) become “secured lenders:” they receive seniority in liquidations, allowing them to lend more to healthy firms. Banks that have a hard time collateralizing loans (i.e., low $\phi^b_h$) become “distressed lenders:” they receive seniority in continuations and provide the necessary capital because their opportunity cost is lower. Interestingly, the social planner chooses a seniority structure with the potential for allocating all recovery to just two banks: the ones with the highest and lowest $\phi^b_h$ values in liquidations and continuations, respectively.\footnote{Although beyond our model, such a seniority structure could cause problems in practice such as a too-big-to-fail dilemma in future crises.}

In this extension, the socially optimal liquidation rule now depends on the extent to which secured and distressed lenders are constrained. For expositional purposes, suppose there are two banks, $b = 1, 2$, with $\phi^1_h < \phi^2_h$, and let $\phi_d = 0$ (with the full case in the appendix). We show that the socially optimal liquidation rule is

$$v^L_L = (1 + \theta^{1,s})C + (1 + \theta^{2,s})\gamma - \theta^{2,s}\gamma\xi\gamma$$

where $\theta^{b,s}$ is the social value of loanable funds to bank $b$, with $\theta^{1,s} < \theta^{2,s}$. Given the bifurcation of seniority, the planner internalizes the difference in social value of loanable funds based on outcome. When the planner chooses reorganization, it ties up resources $C$ of bank 1 (distressed lender), who has a lower value of loanable funds. Conversely if the planner chooses liquidation, frees up resources for bank 2 (secured lender), who has a higher value of loanable funds. Taken
together, this suggests a greater tendency towards reorganization when seniority interventions allow the planner to allocate the proceeds/costs of liquidation/reorganization towards the bank with a comparative advantage in handling that bankruptcy outcome. Note, however, that these statements are all comparative statements on the planner’s solution, and not the difference between the planner and private solutions.

While we do not explicitly model private equilibrium in this setting (as an initial financing decision is not specified under which to obtain it), heterogeneous collateral constraints have interesting implications for banks’ private incentives. In particular, some banks might find it optimal to restructure a distressed firm while other banks would find it optimal to liquidate the same firm. This conflict arises because banks with tighter funding constraints place a higher premium on the immediate availability of liquidation proceeds. This suggests an interesting friction in the bankruptcy process even between two banks in the same creditor class, as the two banks have different relative values of reorganization and liquidation.

6 Implementing liquidation subsidies/taxes in practice

Section 4 shows how a social planner can use liquidation taxes or subsidies to achieve the social optimum, depending on the direction of intervention. The planner does not need to observe the long-run viability of individual firms to calculate the optimal subsidy. In this section, we briefly discuss practical methods of implementing such policies. Readers primarily interested in the paper’s formal economic analysis may skip this section.23

To effectively mitigate the crisis externalities that we model, policymakers need tools that can be quickly implemented. Ideally, a crisis response should not require a lengthy legislative process or new government fundraising, which could create delays. Further, our results imply that policy tools must be able to subsidize liquidation or continuation, depending on the nature of the crisis. We argue below that conditional tax forgiveness for bankrupt firms could feasibly and quickly

23In Appendix E, we provide some overview of policies implemented in practice and of related policy proposals.
implement the optimal policy in our model without immediate fundraising.\textsuperscript{24}

In response to the COVID-19 pandemic, Blanchard, Philippon, and Pisani-Ferry (2020) proposed that governments could deter liquidations by subsidizing restructurings that allow insolvent firms to continue operating.\textsuperscript{25} Specifically, they propose that governments could accept larger write downs or “haircuts” on tax claims than the haircuts accepted by private creditors. Such a policy amounts to subsidizing creditors in any restructuring that results in the continuation of an insolvent firm. However, this form of government subsidy could just as easily be applied to incentivize liquidations: governments accept a haircut on their tax claims in liquidation but not in reorganization. This policy tool is thus an attractive means of implementing the optimal policy of Proposition 3, in which either liquidation or continuation is subsidized. Indeed, the government accepting a larger haircut on distressed debt is analogous to the implementation that we describe in Proposition 3. Interestingly, such a policy also provides the Pigouvian bailout suggested by Section 5.2 and Proposition 7.

Implementing the optimal policy of Proposition 3 through the approach proposed by Blanchard, Philippon, and Pisani-Ferry (2020) would be especially feasible in the US Chapter 11 bankruptcy system. In the US, bankrupt firms frequently owe money to the Internal Revenue Services (IRS) for unpaid taxes.\textsuperscript{26} According to 11 U.S.C. §507(a)\textsuperscript{8}, IRS claims receive priority over general unsecured claims. The government could thus increase unsecured creditor recovery by announcing that it would accept plans in which IRS claims receive zero recovery.\textsuperscript{27} The government could subsidize a specific bankruptcy outcome, such as continuation or liquidation, by announcing that the subsidy only applies to plans implementing that outcome. In each bankruptcy, the U.S. Trustee

\begin{footnotesize}
\textsuperscript{24}Of course, forgiven taxes would eventually need to be offset for the government to balance its budget.
\textsuperscript{25}Similarly, Greenwood and Thesmar (2020) proposed that the government could create a tax credit for lenders and landlords that agree to a firm-preserving restructuring.
\textsuperscript{26}For example, the IRS held a $9.5 million claim in Guitar Center’s bankruptcy, a $22 million claim in J. Crew’s bankruptcy, and a $9.5 million claim in GNC’s bankruptcy. See https://cases.primeclerk.com/GNC/Home-ClaimDetails?id=NDQzNDU1MQ==; and https://cases.primeclerk.com/guitarcenter/Home-ClaimDetails?id=NDk2OTYyMQ==; and https://casedocs.omniagentsolutions.com/pocvol1/JCrew/Claim%20Scan/Claims/20-32181/7095000322.pdf.
\textsuperscript{27}The government could theoretically do this without hindering plan confirmation because the fair and equitable standard only applies to creditors that do not accept a plan (11 U.S.C. §1129(b)(1)).
\end{footnotesize}
could determine whether a plan meets the desired criteria.\textsuperscript{28}

The strength of this implementation approach is its flexibility. The government could subsidize liquidation or continuation without novel legislation or fundraising. In the context of our model, these subsidies can improve social welfare in a crisis. However, several caveats are in order. First, targeted tax forgiveness could incentivize nonbankrupt firms to distort their behavior in anticipation of future tax forgiveness. This distortion could lead to suboptimal firm investment and affect government tax revenues. Second, to the extent that tax forgiveness for bankrupt firms must be offset by the government forgoing some future spending, the welfare benefit of the bankruptcy subsidy could be more than offset by the welfare cost of the future forgone policy. Third and perhaps most importantly, implementable subsidy amounts would be limited by the size of the government’s claim in a particular bankruptcy. Thus, while conditional tax forgiveness could feasibly implement the optimal policy in our model, further study is warranted to determine whether the benefits of such a policy outweigh the potential costs.

7 Conclusion

We study policies that mitigate crises by altering the process for resolving insolvent firms. In a general equilibrium environment, we show that crisis interventions can improve welfare relative to existing rules like the best-interest-of-the-creditors test (11 U.S.C. §1129(a)\textsuperscript{7}), which prohibits a Chapter 11 reorganization whenever a liquidation would improve creditor recovery. However, our model reveals that liquidation-preventing policies are not always beneficial: a social planner can find it optimal to \textit{encourage} liquidations. If crisis conditions constrain bank lending, such an intervention improves welfare by reallocating scarce capital to stronger firms, helping them avoid financial distress. We show that various measures of corporate distress – low profitability, high operating loss, and high debt – tend to promote liquidation subsidies. Surprisingly, an optimal policy response to extreme fire sale externalities sometimes calls for even more liquidations, since

\textsuperscript{28}The U.S. Trustee is already tasked with reviewing reorganization plans, see 28 U.S.C. §586(a)\textsuperscript{3}. 

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banks harmed by these externalities must conserve capital for strong firms. Our results demonstrate that policymakers should jointly consider externalities in the banking and nonfinancial sectors when responding to crises.

References


A Proofs

A.1 Proof of Proposition 1

The private bank problem is:

\[\sup_{B_1,D_1,\rho} p\left(D_1 + D_0\right) + (1 - p) \int (1 - \rho(v))vI_0dF(v) - B_1, \quad (A.1)\]

subject to

\[pq_1D_1 + (1 - p) \int (1 - \rho(v))CI_0dF(v) \leq B_1 - B_0 + (1 - p) \int \rho(v)\gamma I_0dF(v) \quad (A.2)\]

and

\[B_1 \leq \phi_h pq_1(D_0 + D_1) + \phi_d(1 - p)q_1 \int (1 - \rho(v))vI_0dF(v). \quad (A.3)\]

Finally, recall the constraints \(0 \leq \rho(v) \leq 1\).

We begin with a proof by contradiction that the optimum involves a threshold rule. Suppose that the optimum did not involve a threshold rule, so that there are two points \(v_1 < v_2\) such that \(\rho(v_1) < 1\) and \(\rho(v_2) > 0\).\(^{29}\) Consider a perturbation whereby the bank increases \(\rho(v_1)\) by \(\frac{p}{f(v_1)}\)

---

\(^{29}\)To see that violation of this condition implies a threshold rule, suppose no two such points exist. If \(\rho(v) \in \{0, 1\}\) for all \(v\), a threshold rule follows. We need only to rule out that \(0 < \rho(v) < 1\) over a positive measure set. Suppose there is a \(\hat{v} < \check{v} < \tau\) such that \(0 < \rho(\check{v}) < 1\). Thus for any \(v_1 < \hat{v}\), \(\rho(v_1) = 1\), while for any \(v_2 > \check{v}\), \(\rho(v_2) = 0\). Thus we have a threshold rule.
and decreases $\rho(v_2)$ by $\frac{\varepsilon}{f(v_2)}$. This perturbation increases the objective function since $v_2 > v_1$. This perturbation has no impact on the budget constraint, which depends on $v$ only through $f(v)$. Finally, this perturbation relaxes the collateral constraint, since $v_2 > v_1$. Thus, this is a feasible perturbation that improves welfare and (weakly) relaxes constraints, a contradiction that the rule was optimal. We therefore have a threshold rule.

Given a threshold rule is optimal, we can redefine the bank’s problem over the threshold $v^p_L$, rather than over the entire rule $\rho$. The private bank Lagrangian (substituting in the threshold rule) is

$$
L = p(D_1 + D_0) + (1 - p) \int_{v \geq v^p_L} vI_0dF(v) - B_1
+ \theta \left( \phi_h pq_1(D_0 + D_1) + \phi_d (1 - p)q_1 \int_{v \geq v^p_L} vI_0dF(v) - B_1 \right)
+ \delta \left( B_1 - B_0 + (1 - p) \int_{v \leq v^p_L} yI_0dF(v) - [pq_1D_1 + (1 - p) \int_{v \geq v^p_L} CI_0dF(v)] \right).
$$

Differentiating with respect to $B_1$,

$$
0 = -1 - \theta + \delta. \quad (A.4)
$$

Differentiating with respect to $v^p_L$ and dividing by $I_0$:

$$
0 = -(1 - p)v^p_Lf(v^p_L) + \theta \left( -\phi_d (1 - p)q_1v^p_Lf(v^p_L) \right) + \delta \left( (1 - p)f(v^p_L)(\gamma + C) \right).
$$

Dividing by $(1 - p)f(v^p_L)$ and substituting in $\delta = 1 + \theta$,

$$
0 = -v^p_L + \gamma + \phi_d q_1 v^p_L + \theta \left( -\phi_d q_1 v^p_L + \gamma + C \right). \quad (A.5)
$$

Thus, letting $\theta^p$ denote the Lagrange multiplier in the private problem,

$$
v^p_L = \frac{1 + \theta^p}{1 + \theta^p \phi_d q_1} (\gamma + C). \quad (A.6)
$$
Finally, differentiate with respect to $D_1$.

$$0 = p + \theta^p \phi_h p q_1 - \delta p q_1$$
$$0 = \frac{p(1-q_1)}{q_1} + \theta^p p (\phi_h - 1)$$
$$\theta^p = \frac{1-q_1}{q_1(1-\phi_h)} = \frac{1}{1-\phi_h} \left( \frac{1}{q_1} - 1 \right),$$

which completes the proof.

### A.2 Proof of Proposition 2

We begin by deriving the social planner’s objective. The planner’s objective is equal to the sum of the utility of all agents. Mechanically, distressed firms receive zero utility because they are insolvent, so all their value goes to banks. Households lend out $B_1$ dollars then receive $B_1$ dollars from banks, so they receive zero utility. The planner’s objective is thus the sum of: (i) healthy firm utility, (ii) bank utility, and (iii) arbitrageur utility. Summing these and noting the constraint that $I_1 = q_1 D_1$,

$$p \left( v_h I_0 + g_h (q_1 D_1) - \frac{1}{q_1} (q_1 D_1 - D_0) \right)$$

**Healthy firms**

$$+ p \left( D_1 + D_0 \right) + (1-p) \int (1-\rho(v)) v I_0 dF(v) - B_1 + g_a \left( L \right) - \gamma L$$

**Banks**

$$= p \left( v_h I_0 + g_h (I_1) \right) + (1-p) \int (1-\rho(v)) v I_0 dF(v) - B_1 + g_a \left( L \right) - \gamma L.$$  

We now proceed to the main part of the proof. To simplify, note the market-clearing constraints imply that $q_1 D_1 = I_1$ and $L = \int (1-\rho(v)) I_0 dF(v)$. Substituting this in and ignoring the exogenous part of the planner’s objective, $p v_h I_0$, the planner’s optimization problem is
\[
\sup_{I_1, B_1, q_1, \gamma, \rho} \quad p g_h(I_1) + (1 - p) \int (1 - \rho(v)) v I_0 dF - B_1 + g_a \left( (1 - p) I_0 \int \rho(v) dF \right) - \gamma I_0 (1 - p) \int \rho(v) dF
\]

subject to

\[
B_1 \leq \phi_i p q_1 (D_0 + \frac{I_1}{q_1}) + \phi_d (1 - p) q_1 \int (1 - \rho(v)) v I_0 dF(v) \quad (A.7)
\]

\[
p I_1 + (1 - p) \int (1 - \rho(v)) \gamma I_0 dF(v) \leq B_1 - B_0 + (1 - p) \int \rho(v) \gamma I_0 dF(v) \quad (A.8)
\]

\[
1 = q_1 g'_h(I_1)
\]

\[
g'_a \left( (1 - p) I_0 \int \rho(v) dF \right) = \gamma. \quad (A.10)
\]

As in the proof of Proposition 1, we begin with a proof by contradiction that the social optimum involves a threshold rule for \( \rho \). As in that proof, if the social optimum did not involve a threshold rule, there are points \( v_1 < v_2 \) such that \( \rho(v_1) < 1 \) and \( \rho(v_2) > 0 \). Consider the same perturbation of increasing \( \rho(v_1) \) by \( \frac{\epsilon}{f(v_1)} \) and decreasing \( \rho(v_2) \) by \( \frac{\epsilon}{f(v_2)} \). Note that this perturbation has no impact on total liquidations \( L \equiv (1 - p) I_0 \int \rho(v) dF \) and hence no impact on equilibrium prices (the final two conditions). Therefore as in the private optimum, this perturbation increases the objective since \( v_2 > v_1 \), has no impact on the budget constraint, and relaxes the collateral constraint since \( v_2 > v_1 \). Thus, this is a feasible perturbation that improves welfare, a contradiction that the rule was optimal. We therefore have a threshold rule.

Denote the threshold rule \( v'_L \). Incorporating the threshold rule into the problem, the Lagrangian
\[ \mathcal{L} = pg_h(I_1) + (1 - p) \int_{v \geq v_L^*} v \text{d}F - B_1 + g_a \left( (1 - p)I_0F(v_L^*) \right) - \gamma I_0(1 - p)F(v_L^*) \\
+ \theta \left( \phi_h \left( D_0 + \frac{I_1}{q_1} \right) + \phi_d(1 - p)q_1 \int_{v \geq v_L^*} v \text{d}F(v) - B_1 \right) \\
+ \delta \left( B_1 - B_0 + (1 - p) \int_{v \leq v_L^*} \gamma I_0dF(v) - \left[ pI_1 + (1 - p) \int_{v \geq v_L^*} CI_0dF(v) \right] \right) \\
+ \kappa \left( 1 - q_1 g_h(I_1) \right) \\
+ \lambda \left( g_a' \left( (1 - p)F(v_L^*)I_0 \right) - \gamma \right). \]

where \( \theta, \delta, \kappa, \lambda \) are the Lagrange multipliers (with \( \theta, \delta \) unambiguously nonnegative).

Differentiating with respect to \( B_1 \),

\[ \delta = 1 + \theta. \tag{A.11} \]

Differentiating in \( v_L^* \), dividing through by \( (1 - p)I_0f(v_L^*) \), and substituting in \( \delta = 1 + \theta \) gives

\[ 0 = -v_L^* + \gamma + C + g_a' \left( (1 - p)F(v_L^*)I_0 \right) - \gamma + \theta \left( -\phi_d q_1 v_L^* + \gamma + C \right) + \lambda g_a'' \left( (1 - p)F(v_L^*)I_0 \right). \tag{A.12} \]

Noting that market clearing in the liquidation market implies \( \gamma = g_a' \left( (1 - p)F(v_L^*)I_0 \right) \),

\[ 0 = -v_L^* + \gamma + C + \theta \left( -\phi_d q_1 v_L^* + \gamma + C \right) + \lambda g_a'' \left( (1 - p)F(v_L^*)I_0 \right). \tag{A.13} \]

Next, differentiating the Lagrangian with respect to \( \gamma \) and rearranging,

\[ \lambda = -I_0(1 - p)F(v_L^*) + \delta I_0(1 - p)F(v_L^*) \tag{A.14} \]

\[ = \theta I_0(1 - p)F(v_L^*), \tag{A.15} \]
where the second line uses \( \delta = 1 + \theta \) (A.11). Substituting into equation (A.14) and rearranging,

\[
v^*_L = \gamma + C + \theta \left( -\phi_d q_1 v^*_L + \gamma + C + I_0 (1 - p) F(v^*_L) g''_a \left( (1 - p) F(v^*_L) I_0 \right) \right) . \tag{A.16}
\]

This equation reduces to

\[
v^*_L = \frac{1 + \theta}{1 + \theta \phi_d q_1} (\gamma + C) + \frac{\theta}{1 + \theta \phi_d q_1} L g''_a (L) . \tag{A.17}
\]

where we substituted back in \( L = (1 - p) I_0 F(v^*_L) \) (market clearing). Recalling that \( g'_a(L) = \gamma \), then we have \( \frac{\partial \gamma}{\partial L} = g''_a(L) \) is the required change in equilibrium price for an increase in liquidations. Recalling the definition of price elasticity, we can substitute in above to obtain

\[
v^*_L = \frac{1 + \theta}{1 + \theta \phi_d q_1} (\gamma + C) - \frac{\theta}{1 + \theta \phi_d q_1} \gamma g''_a(L). \tag{A.18}
\]

This gives the socially optimal liquidation rule.

Finally, we characterize \( \theta^* \) and derive \( M^s \). The price \( q_1 \) satisfies \( q_1 g'_a(I_1) = 1 \), so that \( \frac{\partial q_1}{\partial I_1} = \frac{-q_1 g''_a(I_1)}{g'_a(I_1)} = -g''_h(I_1) \).

Now, differentiate the Lagrangian with respect to \( I_1 \):

\[
0 = p g'_a(I_1) + \theta \phi_h p - \delta p - \kappa q_1 g''_h(I_1) \tag{A.19}
\]

\[
= p(g'_a(I_1) - 1) + \theta p(\phi_h - 1) - \kappa q_1 g''_h(I_1). \tag{A.20}
\]

Differentiating the Lagrangian with respect to \( q_1 \),

\[
0 = \theta (\phi_h p D_0 + \phi_d (1 - p) \int_{v \geq v^*_L} v I_0 dF(v)) - \kappa g'_a(I_1). \tag{A.21}
\]

Plugging this into the above:

\[
0 = p(g'_a(I_1) - 1) + \theta \left( p(\phi_h - 1) - \frac{q_1 g''_h(I_1)}{g'_a(I_1)} (\phi_h p D_0 + \phi_d (1 - p) \int_{v \geq v^*_L} v I_0 dF(F)) \right). \tag{A.22}
\]
Substituting in the definition of \( C_{\text{old}} \equiv \phi_h pq D_0 + q_1 \phi_d (1 - p) \int (1 - \rho(v)) v I_0 dF(v) \),

\[
0 = p(g'_h(I_1) - 1) + \theta \left( p(\phi_h - 1) - \frac{g''_h(I_1)}{g'_h(I_1)} C_{\text{old}} \right). \quad (A.23)
\]

Multiplying and dividing the last term by \( C_{\text{new}} \equiv \phi_h p I_1 \),

\[
0 = p(g'_h(I_1) - 1) + \theta \left( p(\phi_h - 1) - \frac{\phi_h p I_1 g''_h(I_1)}{g'_h(I_1)} C_{\text{old}} \right). \quad (A.24)
\]

Noting that market clearing implies \( g'_h(I_1) - 1 = (1/q_1) - 1 \), dividing by \(-p(1 - \phi_h)\) and adding \( \theta p \) to both sides,

\[
\theta p = \theta \left( 1 + \frac{\phi_h I_1 g''_h(I_1)}{g'_h(I_1) C_{\text{new}}} \right). \quad (A.25)
\]

Finally, applying the expression for \( \xi_q \),

\[
\theta p = \theta \left( 1 - \frac{\phi_h \xi_q}{C_{\text{new}}} \right). \quad (A.26)
\]

Dividing gives \( \theta^* = M^* \theta p \), where

\[
M^* = \left( 1 - \frac{\phi_h \xi_q}{C_{\text{new}}} \right)^{-1}. \quad (A.27)
\]

This completes the proof.

### A.3 Proof of Proposition 3

Given a tax \( \tau \), the private bank problem is:

\[
\sup_{B_1, D_1, \rho} p \left( D_1 + D_0 \right) + (1 - p) \int (1 - \rho(v)) v I_0 dF(v) - B_1 - \tau (1 - p) I_0 \int \rho(v) dF(v), \quad (A.28)
\]

subject to the same conditions as before. Thus following the same argument as the proof of Proposition 1 (and noting that the perturbation has no impact on the tax burden), a threshold rule is
privately optimal. Following the same steps as in the proof of Proposition 1, we obtain the analog of equation (A.5) that accounts for the tax burden,

\[ 0 = -(v_L^p + \tau) + \gamma + C + \theta \left( -\phi_d q_1 v_L^p + \gamma + C \right). \]

Note the only difference relative to equation (A.5) is that there is the additional penalty \( \tau \) for increasing the liquidation threshold. Thus rearranging, we obtain

\[ v_L^p = \frac{(1 + \theta^p)(\gamma + C) - \tau}{1 + \theta^p \phi_d}. \]

It now remains simply to find the tax \( \tau \) that leads the bank to privately implement \( v_L^p = v_L^s \) when facing the socially optimal prices \( (q_1, \gamma) \). From Proposition 2, we have:

\[ \frac{1 + \theta^p}{1 + \theta^p \phi_d q_1} (\gamma + C) - \frac{\tau}{1 + \theta^p \phi_d q_1} = \frac{1 + \theta^s}{1 + \theta^s \phi_d q_1} (\gamma + C) - \frac{\theta^s \gamma \xi}{1 + \theta^s \phi_d q_1}. \]

This reduces to

\[ \tau = \frac{1}{1 + \theta^s \phi_d q_1} \left[ (1 + \theta^p)(1 + \theta^s \phi_d q_1)(\gamma + C) - (1 + \theta^p \phi_d q_1) \left( (1 + M^s \theta^p)(\gamma + C) - M^s \theta^p \gamma \xi \right) \right] \]

(A.29)

\[ = \frac{1}{1 + \theta^s \phi_d q_1} \left[ (\gamma + C) \left( (1 + \theta^p + \theta^s \phi_d q_1 + \theta^p \theta^s \phi_d q_1 - 1 - \theta^p \phi_d q_1 - \theta^s - \theta^p \phi_d q_1 - \theta^s \phi_d q_1) + (1 + \theta^p \phi_d q_1)M^s \theta^p \gamma \xi \right) \right] \]

(A.30)

\[ + (1 + \theta^p \phi_d q_1)M^s \theta^p \gamma \xi \]

(A.31)

\[ = \frac{1}{1 + \theta^s \phi_d q_1} \left[ (\gamma + C) \left( \theta^p + \theta^s \phi_d q_1 - \theta^p \phi_d q_1 - \theta^s \right) + (1 + \theta^p \phi_d q_1)M^s \theta^p \gamma \xi \right] \]

(A.32)

\[ = \frac{1}{1 + \theta^s \phi_d q_1} \left[ (\gamma + C)(M^s - 1) \theta^p (\phi_d q_1 - 1) + \frac{M^s - 1}{M^s - 1}(1 + \theta^p \phi_d q_1)M^s \theta^p \gamma \xi \right] \]

(A.33)

\[ = \]

(A.34)

giving the result.
A.4 Proof of Proposition 4

Proposition 3 tells us that $\tau \leq 0$ when

$$(1 + \theta p \phi d q_1) \gamma \xi \gamma \leq \frac{M^s - 1}{M^s} (\gamma + C)(1 - \phi d q_1).$$

Given $\phi_d = 0$, this reduces to $\xi \gamma \leq \frac{M^s - 1}{M^s} (1 + \frac{C}{\gamma})$. Assumption 1 tells us $\xi \gamma$ is a constant, and further that $\xi q = 1$ (given log). Moreover, the first order condition for investment gives us $I_1 = A_h q_1$, which, combined with market clearing, gives us $D_1 = \frac{I_1}{q_1} = A_h$. From here, the values of old and new collateral are $C_{\text{old}} = \phi_h p q_1 D_0$ and $C_{\text{new}} = \phi_h p q_1 D_1 = \phi_h p q_1 A_h$. Thus we have

$$M^s = \left(1 - \frac{\phi_h}{1 - \phi_h A_h} D_0\right)^{-1} > 1. \quad (A.35)$$

Thus, substituting back in above and reducing gives

$$\xi \gamma \leq \frac{\phi_h}{1 - \phi_h A_h} D_0 (1 + \frac{C}{\gamma}).$$

Finally, the RHS is decreasing in $\gamma$, thus substituting in $\gamma$ provides a sufficient condition, completing the result.

A.4.1 Sufficient condition with $\phi_d > 0$

Suppose that we maintain Assumption 1 but allow for $\phi_d > 0$. In this case, we start from the same necessary condition,

$$(1 + \frac{1}{1 - \phi_d} (1 - q_1) \phi_d) \gamma \xi \gamma \leq \frac{M^s - 1}{M^s} (\gamma + C)(1 - \phi_d q_1).$$
We can provide an overly strong sufficient condition by setting $q_1 = 0$ on the LHS and $q_1 = 1$ on the RHS, and by using $M \geq \frac{1}{1 - \frac{\phi_d}{\phi_h}}$. Thus we get the sufficient condition

$$
\xi \gamma \leq \phi_h \frac{D_0}{1 - (\phi_h - \phi_d)} A_h (1 + \frac{C}{\gamma}).
$$

This sufficient condition is almost identical, but provides a discount on $\phi_h$ based on the added collateral value $\phi_d$.

A.5 Proof of Proposition 5

From the proof of Proposition 4, we have $\xi \gamma \leq \phi_h \frac{D_0}{1 - (\phi_h - \phi_d)} (1 + \frac{C}{\gamma})$. Rearranging to obtain a bound on $\gamma$, we have $\gamma \leq C \left( \frac{1 - \phi_d A_h}{\phi_h D_0} \xi \gamma - 1 \right)^{-1}$, while the condition on $L$ comes from substituting in $\gamma = L^{-\frac{\xi}{\gamma}}$.

A.6 Proof of Proposition 6

At date 0, banks choose $(B_0, D_1, \rho)$ (note banks’ problem is time consistent) to maximize

$$
\sup_{B_0, B_1, D_1, \rho} p \left( D_1 + D_0 \right) + (1 - p) \int (1 - \rho(v)) v I_0 dF(v) - B_1 - \Phi(I_0 - B_0), \tag{A.36}
$$

where we note we have internalized the constraint $A_0 = I_0 - B_0$, subject to

$$
pq_1 D_1 + (1 - p) \int (1 - \rho(v)) v I_0 dF(v) \leq B_1 - B_0 + (1 - p) \int \rho(v) \gamma I_0 dF(v) \tag{A.37}
$$

and

$$
B_1 \leq \phi_h pq_1 (D_0 + D_1) + \phi_d (1 - p) q_1 \int (1 - \rho(v)) v I_0 dF(v). \tag{A.38}
$$

Under the same Lagrange multiplier conventions as before defining the Lagrangian, the derivative in $B_0$ yields

$$
0 = \Phi'(I_0 - B_0) - \delta, \tag{A.39}
$$
and using that $\delta = 1 + \theta^s$ obtains the result.

The social objective follows the same logic, except that now $\delta = 1 + \theta^s$, giving the social condition. The tax follows immediately from here.

### A.7 Proof of Proposition 7

The proof follows from defining the Lagrangian of Proposition 2 to account for bailouts,

$$
\mathcal{L} = p g_h(I_1 + \frac{1}{p} T_0^b) + (1 - p) \int_{v \geq v_L^b} v I_0 dF - B_1 + g_a \left( (1 - p) I_0 F(v_L^b) \right) - \gamma I_0 (1 - p) F(v_L^b) \\
+ \theta \left( \phi_h p q_1 (D_0 + \frac{1}{q_1}) + \phi_d (1 - p) q_1 \int_{v \geq v_L^b} v I_0 dF - B_1 \right) \\
+ \delta \left( B_1 - (B_0 - T_0^b) + (1 - p) \int_{v \leq v_L^b} \gamma I_0 dF (v) \right) - \left( [p I_1 + (1 - p) \int_{v \geq v_L^b} C I_0 dF (v)] \right) \\
+ \kappa \left( 1 - q_1 g_h'(I_1 + \frac{1}{p} T_0^b) \right) \\
+ \lambda \left( g_a' \left( (1 - p) F(v_L^b) I_0 \right) - \gamma \right),
$$

where $T_0^b$ is a bank bailout and $\frac{1}{p} T_0^f$ is a healthy firm bailout of the same size (since it is distributed among $p$ healthy firms as opposed to 1 banks). By Envelope Theorem, the social value of a bank bailout is

$$
\frac{\partial \mathcal{L}}{\partial T_0^b} = \delta = 1 + \theta^s.
$$

The distressed firm bailout follows exactly the same.

For the healthy firm bailout, by Envelope Theorem,

$$
\frac{\partial \mathcal{L}}{\partial T_0^b} = g_h'(I_1) - \kappa q_1 g_h''(I_1).
$$

From the proof of Proposition 2, $\kappa q_1 g_h''(I_1) = p (g_h'(I_1) - 1) + \theta^s p (\phi_h - 1)$, which substituting in gives

$$
\frac{\partial \mathcal{L}}{\partial T_0^b} = 1 + \theta^s (1 - \phi_h)
$$
Online Appendix

B  Extension with Bank Writedown Avoidance

In this appendix, we consider a model extension in which banks have an incentive to avoid writing down loans on their balance sheet. Specifically, we assume that if a bank chooses continuation for a particular distressed firm, it can pledge the promised value $D_0$ of the loan as collateral rather than the true value $v$. Specifically, the bank’s budget constraint is unchanged

$$pq_1D_1 + (1 - p) \int (1 - \rho(v))CI_0 dF(v) \leq B_1 - B_0 + (1 - p) \int \rho(v)\gamma I_0 dF(v)$$

but the collateral constraint now reflects the ability of banks to fool households by not writing down distressed loans in continuations:

$$B_1 \leq \phi_h pq_1 (D_0 + D_1) + \phi_d (1 - p) q_1 D_0 \int (1 - \rho(v))dF(v).$$

Otherwise, the model is identical to the one in the main text. Note that under Assumption 1, $\phi_d = 0$ so this model is fully identical to the one in the text. Thus, Propositions 4 - 5 hold as stated in this setting.

All of the model forces in our baseline model play the same roles in this setting. In particular, banks consider both direct recovery and the value of loanable funds when choosing liquidations. The planner trades off the same externalities.

We now prove analogs of Propositions 1 - 3.

**Proposition B.1.** In any competitive equilibrium, the bank’s privately optimal liquidation rule is a threshold rule $\rho(v) = I\left(v \leq v^p_L\right)$, where

$$v^p_L = (1 + \theta^p)(\gamma + C) - \theta^p \phi_d q_1 D_0,$$

and $\theta^p$ is the same private value of loanable funds as in Proposition 1,

$$\theta^p = \frac{1}{1 - \phi_h} \times \left(\frac{1}{q_1} - 1\right) \geq 0.$$  

**Proof.** The proof is identical to that of Proposition 1 up to the defined value of collateral. \qed
The main differences between the private optimum in the baseline model and here is that privately optimizing banks are more inclined to liquidate firms. In our baseline setting, the incentive to liquidate and preserve loanable funds always pushes for excess liquidations relative to a rule maximizing direct creditor recovery. In this setting, the incentive to avoid writedowns pushes in the other direction, and as a result banks can find it optimal to choose excess continuation or excess liquidation, relative to a rule maximizing direct creditor recovery.

We next characterize the social planner’s solution. Note that the new definition of the value of old collateral is

\[ C_{\text{old}} \equiv \phi_h pq_1 D_0 + q_1 \phi_d (1 - p) D_0 \int (1 - \rho(v)) dF(v) \]  

(B.5)

Note that all else equal, the value of old collateral has risen from writedown avoidance.

Using this new definition, we obtain an analog of Proposition 2:

**Proposition B.2.** The socially optimal liquidation rule is a threshold rule \( \rho(v) = 1 \) \( (v \leq v_L^s) \), where

\[ v_L^s = (1 + \theta^s)(\gamma + C) - \theta^s \gamma \xi \gamma - \theta^s \phi d q_1 D_0 \]  

(B.6)

where \( \theta^s, M^s \) are defined as before for the new collateral value.

**Proof.** The proof is identical to that of Proposition 2 up to the defined value of collateral. \( \square \)

As with the private bank, the planner now has a tendency towards fewer liquidations owing to the larger ability to use distressed loans as collateral. Finally, we can compare the private and social optima as decentralized by \( \tau \).

**Proposition B.3.** The tax that decentralizes the social optimum is:

\[ \tau = \left[ -\frac{M^s - 1}{M^s} (\gamma + C - \phi d q_1 D_0) + \gamma \xi \gamma \right] \theta^s \]  

(B.7)

**Proof.** The proof follows analogous steps to that of Proposition 3, in particular:

\[ \gamma + C + \theta^s \left( -\phi d q_1 D_0 + \gamma + C - \gamma \xi \gamma \right) = -\tau + (1 + \theta^p) (\gamma + C) - \theta^p \phi d q_1 D_0 \]  

(B.8)

\[ \tau = (\theta^p - \theta^s)(\gamma + C - \phi d q_1 D_0) + \theta^s \gamma \xi \gamma \]  

(B.9)

\[ = \theta^p (1 - M^s)(\gamma + C - \phi d q_1 D_0) + \theta^s \gamma \xi \gamma \]  

(B.10)

\[ = \theta^p (M^s - 1) \left[ - (\gamma + C - \phi d q_1 D_0) + \frac{M^s}{M^s - 1} \gamma \xi \gamma \right]. \]  

(B.11)

\( \square \)
Comparing the planner solution to the private solution, we see that bank writedown avoidance amplifies the existing tradeoffs. Bank writedown avoidance increases the value of old collateral, which in turn increases the social multiplier $M^s$. This promotes liquidation. At the same time, the greater value of collateral also leads to a greater contraction in loanable funds from losing the distressed security as collateral, which amplifies the cost of liquidation. Thus writedown avoidance serves to amplify existing trade-offs.

C Acquisitions by Arbitrageurs

In our baseline model, we assume that an insolvent firm faces two potential outcomes: reorganization or liquidation. In practice, an insolvent firm might be acquired. Like a liquidation, an acquisition entails an insolvent firm selling all of its assets. Like a continuation, the value of the purchased assets to the acquirer depends on the long-run viability of the insolvent firm, since the acquirer will continue to operate that firm in some form. In this appendix, we consider an extension in which arbitrageurs acquire insolvent firms to continue operating them. The main result of this appendix is that acquisitions amplify the existing model tradeoffs: fire sale externalities versus the collateral externalities associated with allocating scarce funds to continuation.

In this extension, we assume that distressed firms can be acquired by arbitrageurs. If a distressed firm acquires a firm with viability $v$, it can be employed with enterprise value $1 + bv$, where $b \geq 0$ reflects capabilities of arbitrageurs. Arbitrageur technology is the same as before, but the total enterprise value purchased by arbitrageurs to manage is

$$L = I_0(1 - p) \int \rho(v)(1 + bv)dF(v). \quad (C.1)$$

Thus, $b = 0$ reduces to the baseline model.

We derive analogs of Propositions 1 - 3 in this setting. All of the model forces in our baseline model play the same roles in this setting. In particular, banks consider both direct recovery and the value of loanable funds when choosing liquidations. The planner trades off the same externalities.

The main differences between our main results and the results in this extension are as follows. First, holding all prices fixed, increasing $b$ leads privately optimizing banks to choose a higher liquidation threshold. This makes sense, because by assumption higher $b$ means liquidations produce more value as the arbitrageur can use the original firm’s business lines for profit. By the same logic, in the reasonable case where $\xi_\gamma \leq 1$, the planner liquidates more as $b$ increases.

Since higher $b$ increases both the private and social liquidation thresholds, it is useful to characterize the gap. We derive the decentralizing liquidation tax $\tau$ under the assumption that $\phi_d = 0$ and show it is the following:
\[ \tau = \frac{\theta^s}{1 - \gamma b[1 + \theta^s(1 - \xi \gamma)]} \left( -\frac{M^s - 1}{M^s}(\gamma + C)M^s - 1 + \gamma \xi \gamma(1 + bC(1 + \theta^p)) \right). \] \hfill (C.2)

Holding all else fixed, we see that increasing \( b \) has two effects. First, since \( b \) multiplies \( \xi \gamma \) in the term inside parentheses, we see that acquisitions exacerbate the firesale externality. Second, increasing \( b \) reduces the denominator in the fraction multiplying the terms in parentheses. This tends to increase the magnitude of the tax. In particular, when \( \xi \gamma = 0 \), we have \( \tau < 0 \) and increases in \( b \) make \( \tau \) more negative. In this sense, increases in \( b \) amplify the existing model tradeoffs - fire sales are worse and the opportunity cost of loanable funds tied up in continuations increases.

### C.1 Analog of Proposition 1

**Proposition C.1.** In any competitive equilibrium, the bank’s privately optimal liquidation rule is a threshold rule \( \rho(v) = 1 \) \( (v \leq v^p_L) \), where

\[ v^p_L = \frac{(1 + \theta^p)(\gamma + C)}{1 - \gamma b(1 + \theta^p) + \theta^p \phi_d q_1}, \] \hfill (C.3)

and \( \theta^p \) is the same private value of loanable funds as in Proposition 1,

\[ \theta^p = \frac{1}{1 - \phi_h} \times \left( \frac{1}{q_1} - 1 \right) \geq 0. \] \hfill (C.4)

**Proof:** The private bank problem is:

\[ \sup_{B_1, D_1, \rho} p \left( D_1 + D_0 \right) + (1 - p) \int (1 - \rho(v))vI_0dF(v) - B_1, \] \hfill (C.5)

subject to

\[ pq_1D_1 + (1 - p) \int (1 - \rho(v))CI_0dF(v) \leq B_1 - B_0 + (1 - p)\gamma I_0 \int \rho(v)(1 + bv)dF(v) \] \hfill (C.6)

and

\[ B_1 \leq \phi_h pq_1(D_0 + D_1) + \phi_d (1 - p)q_1 \int (1 - \rho(v))vI_0dF(v). \] \hfill (C.7)

An identical argument to the one in the proof of Proposition 1 shows we can replace \( \rho \) with a cutoff rule. The Lagrangian is then
\[ p \left(D_1 + D_0\right) + (1 - p) \int_{v \geq v^*_L} v I_0 dF(v) - B_1 \]
\[ + \theta \left( \phi_n p q_1 (D_0 + D_1) + \phi_d (1 - p) q_1 \int_{v \geq v^*_L} v I_0 dF(v) - B_1 \right) \]
\[ + \delta \left( B_1 - B_0 + (1 - p) \gamma I_0 \int_{v \leq v^*_L} (1 + bv) dF(v) - \left[pq_1 D_1 + (1 - p) \int_{v \geq v^*_L} C I_0 dF(v)\right]\right). \]

Differentiating with respect to \(B_1\),
\[-1 - \theta + \delta = 0. \tag{C.8}\]

Differentiating with respect to \(v^*_L\) and dividing by \(I_0\):
\[ 0 = -(1 - p)v^*_L f(v^*_L) + \theta \left( -\phi_d (1 - p) q_1 v^*_L f(v^*_L) \right) + \delta \left( (1 - p) f(v^*_L) (\gamma (1 + bv^*_L) + C) \right). \]

Dividing by \((1 - p) f(v^*_L)\) and plugging in \(\delta = 1 + \theta,\)
\[ 0 = -v^*_L + \gamma (1 + bv^*_L) + C + \theta \left( -\phi_d q_1 v^*_L + \gamma (1 + bv^*_L) + C \right). \]

Thus, letting \(\theta^p\) denote the Lagrange multiplier in the private problem,
\[ v^*_L = \frac{(1 + \theta^p)(\gamma + C)}{1 - \gamma b (1 + \theta^p) + \theta^p \phi_d q_1}. \tag{C.9} \]

Finally, differentiating with respect to \(D_1\) delivers the same derivative as in the proof of Proposition 1, delivering the same expression for \(\theta^p\).

### C.2 Analog of Proposition 2

We obtain an analog of Proposition 2:

**Proposition C.2.** The socially optimal liquidation rule is a threshold rule \(\rho(v) = 1 (v \leq v^*_L),\)

where
\[ v^*_L = \frac{1}{1 + \theta^s \phi_d q_1 - \gamma b (1 + \theta^s) + \theta^s \gamma^s \xi \gamma b} \left[(\gamma + C)(1 + \theta^s) - \theta^s \gamma^s \xi \gamma \right] \tag{C.10} \]

and the social value of loanable funds is \(\theta^s = M^s \theta^p,\) where
\[ M^s \equiv \frac{1}{1 - \frac{\phi_h}{L - \phi_h} g^{\text{old}} q} \geq 1. \] (C.11)

Notice that the only difference between this and Proposition 2 is the term \( b\gamma(\theta^s \xi \gamma - (1 + \theta^s)) \).

In the reasonable case where \( \xi \gamma \leq 1 \), this is negative. Thus, the possibility of acquisitions pushes \( v_L^f \) higher.

**Proof:** Introducing the new acquisition assumption, the planner’s Lagrangian is

\[
pg_h(I_1) + (1 - p) \int_{v \geq v_L^f} vI_0 dF - B_1 + g_a \left( (1 - p)I_0 \int_{v \leq v_L^f} (1 + bv)dF(v) \right) - \gamma I_0 (1 - p) \int_{v \leq v_L^f} (1 + bv)dF(v) \\
+ \theta \left( \phi_h pq_1(D_0 + \frac{I_1}{q_1}) + \phi_d (1 - p) q_1 \int_{v \geq v_L^f} vI_0 dF(v) - B_1 \right) \\
+ \delta \left( B_1 - B_0 + (1 - p) \gamma I_0 \int_{v \geq v_L^f} (1 + bv)dF(v) - \left[ pI_1 + (1 - p) I_0 CI_0 dF(v) \right] \right) \\
+ \kappa \left( 1 - q_1 g_h(I_1) \right) \\
+ \lambda \left( g_a' \left( (1 - p) F(v_L^f) I_0 \right) - \gamma \right).
\]

Differentiating with respect to \( B_1 \),

\[-1 - \theta + \delta = 0. \] (C.12)

Plugging this in, differentiating with respect to \( v_L^f \), and dividing by \( I_0 \),

\[-(1 - p) v_L^f f(v_L^f) + g_a' \left( (1 - p) I_0 \int_{v \leq v_L^f} (1 + bv)dF(v) \right) (1 - p) f(v_L^f)(1 + bv_L^f) - \gamma (1 - p) f(v_L^f)(1 + bv_L^f) \\
- \theta \left( \phi_d (1 - p) q_1 v_L^f f(v_L^f) \right) + (1 + \theta)(1 - p) f(v_L) \left( \gamma (1 + bv_L^f) + C \right) \\
+ \lambda g_a'' \left( (1 - p) I_0 \int_{v \leq v_L^f} (1 + bv)dF(v) \right) (1 - p) f(v_L^f)(1 + bv_L^f).
\]

Dividing by \( (1 - p) f(v_L^f) \) and setting this equal to 0,

\[ 0 = -v_L^f + \gamma (1 + bv_L^f) + C + g_a' \left( L \right) (1 + bv_L^f) - \gamma (1 + bv_L^f) \\
+ \theta \left( -q_1 v_L^f f(v_L^f) + \gamma (1 + bv_L^f) + C \right) + \lambda g_a'' \left( L \right) (1 + bv_L^f). \] (C.13)
Next, differentiating the Lagrangian with respect to $\gamma$, setting it equal to 0, and adding $\lambda$ to both sides,

$$\lambda = -I_0(1-p) \int_{v \leq v_L^*} (1+bv) dF(v) + \delta I_0(1-p) \int_{v \leq v_L^*} (1+bv) dF(v) = \theta I_0(1-p) \int_{v \leq v_L^*} (1+bv) dF(v),$$  

(C.15)

Noting that market clearing in the liquidation market implies $\gamma = g_a'(L)$,

$$0 = -v_L^* + \gamma(1+bv_L^*) + C + \theta \left(-\phi_d q_1 v_L^* + \gamma(1+bv_L^*) + C\right) + \lambda g_a'' \left(L\right) (1+bv_L^*).$$  

(C.16)

Applying the above equation and adding $v_L^*$ to both sides,

$$v_L^* = \gamma(1+bv_L^*) + C + \theta \left(-\phi_d q_1 v_L^* + \gamma(1+bv_L^*) + C + Lg_a'' \left(L\right) (1+bv_L^*)\right).$$  

(C.17)

Adding $\theta \phi_d q_1 v_L^* - \gamma bv_L^*(1+\theta) - \theta Lg_a''(L) bv_L^*$ and dividing,

$$v_L^* = \frac{1}{1 + \theta^s \phi_d q_1 - \gamma b(1+\theta^s) - \theta^s Lg_a''(L) b} \left[(\gamma + C)(1+\theta^s) + \theta^s Lg_a''(L)\right].$$  

(C.18)

Following the same steps to derive $\xi_q = -Lg_a''(L)/\gamma$,

$$v_L^* = \frac{1}{1 + \theta^s \phi_d q_1 - \gamma b(1+\theta^s) + \theta^s \xi_q b} \left[(\gamma + C)(1+\theta^s) - \theta^s \xi_q\right].$$  

(C.19)

Finally, we characterize $\theta^s$ and derive $M^s$. Following the same manipulations of $q_1 g_h'(I_1) = 1$,

$$\xi_q = -\frac{I_1 g_h''(I_1)}{g_h'(I_1)}.$$  

(C.20)

Noting that the derivatives of the Lagrangian with respect to $I_1$ and $q_1$ do not depend on $b$,

$$0 = p(g_h'(I_1) - 1) + \theta \left(p(\phi_h - 1) - \frac{q_1 g_h''(I_1)}{g_h'(I_1)} (\phi_h pD_0 + \phi_d (1-p) \int_{v \geq v_L^*} v I_0 dF(v))\right).$$  

(C.21)

Substituting in as in the proof of Proposition 2 and noting that $\theta^p$ does not depend on $b$, we arrive at the same expression for $M^s$. 

C-5
C.3 Analog of Proposition 3

To simplify calculations, we assume $\phi_d = 0$. Under this assumption, we can characterize the decentralizing liquidation tax.

**Proposition C.3.** If $\phi_d = 0$, the optimal tax is

$$\tau = \frac{\theta p}{1 - \gamma b[1 + \theta \xi(1 - \xi \gamma)]} \left( - (\gamma + C)(M^f - 1) + M^f \gamma \xi (1 + bC(1 + \theta p)) \right).$$  \hfill (C.22)

**Proof:** Given a tax $\tau$, which now is levied on acquisitions, the private bank problem is:

$$\sup_{B_1, D_1, \rho} p\left(D_1 + D_0 \right) + (1 - p) \int (1 - \rho(v)) vI_0 dF(v) - B_1 - \tau(1 - p)I_0 \int \rho(v) dF(v),$$  \hfill (C.23)

where taxes can be viewed as a tax on the fraction of firms liquidated, normalized by $(1 - p)I_0$. Since it is not a tax on the value of liquidations, there is no $1 + bv$ term in the integral. As before, this objective is optimized subject to

$$pq_1 D_1 + (1 - p) \int (1 - \rho(v)) CI_0 dF(v) \leq B_1 - B_0 + (1 - p) \gamma I_0 \int \rho(v)(1 + bv)dF(v)$$  \hfill (C.24)

and

$$B_1 \leq \phi_h pq_1 (D_0 + D_1) + \phi_d (1 - p) q_1 \int (1 - \rho(v)) vI_0 dF(v).$$  \hfill (C.25)

An identical argument to the one used in the proof of Proposition 1 shows that the optimal liquidation rule is a threshold rule. Introducing Lagrange multipliers $\theta, \delta$, the Lagrangian is

$$p\left(D_1 + D_0 \right) + (1 - p) \int_{v \geq v_L^p} vI_0 dF(v) - B_1 - \tau(1 - p)I_0 \int_{v \leq v_L^p} dF(v)$$

$$+ \theta \left( \phi_h pq_1 (D_0 + D_1) + \phi_d (1 - p) q_1 \int_{v \geq v_L^p} vI_0 dF(v) - B_1 \right)$$

$$+ \delta \left( B_1 - B_0 + (1 - p) \gamma I_0 \int_{v \leq v_L^p} (1 + bv)dF(v) - \left[ pq_1 D_1 + (1 - p) \int_{v \geq v_L^p} CI_0 dF(v) \right] \right).$$

Differentiating with respect to $B_1$,

$$-1 - \theta + \delta = 0.$$  \hfill (C.26)
Differentiating with respect to $v^p_L$ and dividing by $I_0$:

\[
0 = -(1 - p)(v^p_L + \tau)f(v^p_L) + \theta \left( -\phi_d(1 - p)q_1 v^p_L f(v^p_L) \right) + \delta \left( (1 - p)f(v^p_L)(\gamma(1 + b v^p_L) + C) \right).
\]

Dividing by $(1 - p)f(v^p_L)$ and plugging in $\delta = 1 + \theta$,

\[
0 = -v^p_L - \tau + \gamma(1 + b v^p_L) + C + \theta \left( -\phi_d q_1 v^p_L + \gamma(1 + b v^p_L) + C \right) v^p_L \left( 1 - \gamma b + \theta^p \phi_d q_1 - \theta^p \gamma b \right) = -\tau + (\gamma + C)(1 + \theta^p).
\]

As in the proof of the analog of Proposition 1, $\theta^p$ is unchanged. All that is left is to define $\tau$ such that the above equation yields $v^p_L$ from the analog of Proposition 2.

At this point, we impose $\phi_d = 0$. The above then simplifies to

\[
v^p_L = \frac{-\tau + (\gamma + C)(1 + \theta^p)}{1 - \gamma(1 + \theta^p)}
\]

The social threshold simplifies to

\[
v^s_L = \frac{1}{1 - \gamma b[1 + \theta^s(1 - \xi)]} \left[ (\gamma + C)(1 + \theta^s) - \theta^s \gamma \xi \right].
\]

Setting these equal and multiplying by denominators:

\[
[-\tau + (\gamma + C)(1 + \theta^p)] \left( 1 - \gamma b[1 + \theta^s(1 - \xi)] \right) = \left[ (\gamma + C)(1 + \theta^s) - \theta^s \gamma \xi \right] \left( 1 - \gamma b(1 + \theta^p) \right)
\]

Gathering $\tau$ terms on the left, we have

\[
-\tau \left( 1 - \gamma b[1 + \theta^s(1 - \xi)] \right).
\]

Moving other terms to the right, we have

\[
- \theta^s \gamma \xi \left( 1 - b \gamma(1 + \theta^p) \right) + (\gamma + C) \left[ (1 + \theta^p)(1 - b \gamma(1 + \theta^p)) - (1 + \theta^p) \left( 1 - \gamma b[1 + \theta^s(1 - \xi)] \right) \right].
\]
Rearranging,

\[-\theta^s\gamma \xi_y \left(1 - b\gamma(1 + \theta^p) + b(\gamma + C)(1 + \theta^p)\right)\]  \hspace{2cm} (C.33)

\[+ (\gamma + C)\left[(1 + \theta^s)(1 - b\gamma(1 + \theta^p)) - (1 + \theta^p)(1 - b\gamma(1 + \theta^s))\right]. \hspace{2cm} (C.34)\]

Simplifying,

\[-\theta^s\gamma \xi_y \left(1 + bC(1 + \theta^p)\right) \hspace{2cm} (C.35)\]

\[+ (\gamma + C)\left[1 + \theta^s - b\gamma(1 + \theta^p) - \theta^s b\gamma(1 + \theta^p) - 1 - \theta^p + b\gamma(1 + \theta^s) + \theta^p b\gamma(1 + \theta^s)\right]. \hspace{2cm} (C.36)\]

After cancelling terms in brackets, this is just

\[-\theta^s\gamma \xi_y \left(1 + bC(1 + \theta^p)\right) + (\gamma + C)(\theta^s - \theta^p). \hspace{2cm} (C.37)\]

In summary

\[
\tau = \frac{\theta^p}{1 - \gamma b[1 + \theta^s(1 - \xi_y)]} \left(- (\gamma + C)(M^s - 1) + M^s \gamma \xi_y (1 + bC(1 + \theta^p))\right). \hspace{2cm} (C.38)\]

### D Heterogeneous Banks

In this appendix, we study the effect of multiple (heterogeneous) bank creditors facing different collateral constraints in the design of the liquidation rule as well as the optimal seniority structure among creditors. We show that a social planner can improve welfare by subordinating the claims of banks that, because of their idiosyncratic collateral constraints, struggle to lend against securities of bankrupt firms. Moreover, planner intervention in the seniority structure can serve as a partial substitute for intervention in liquidation decisions.

Because we explicitly model seniority, our leading application for this section is to a traditional bankruptcy process, such as Chapter 11, with changes in the liquidation rule reflecting different outcomes of the bankruptcy process. Nevertheless, we preserve the notation and terminology of previous sections for consistency.
D.1 Novel assumptions in the heterogeneous-banks extension

We assume that there are \( N^B \) distinct banks, each of equal measure, indexed by \( b = 1, 2, \ldots, N^B \). For simplicity, banks are identical at date zero, so each bank has \( B_0 \) in deposits and makes an identical quantity of loans \( D_0/N^B \). For simplicity, we assume that each dollar that bank \( b \) lends is divided equally among all firms, ensuring there are multiple creditors for each firm.

However, banks differ in their ability to pledge collateral: each bank has its own \( \phi^b_h \). At date one, bank \( b \) raises debt \( B^b_1 \) from households to lend \( q^b_1D^b_1 \) dollars to firms. Additionally, banks differ in the seniority. For simplicity, we assume bank \( b \) receives a fraction \( S^b(v) \) of any cashflows and collateral associated with a bankruptcy for a firm with viability \( v \).

We maintain Assumption 1 throughout this section so \( \phi_d = 0 \). Given a liquidation rule \( \rho \), bank \( b \)'s budget constraint is

\[
pq^b_1D^b_1 + I_0(1-p) \int (1-\rho(v))S^b(v)CdF(v) \leq B^b_1 - B_0 + I_0(1-p) \int \rho(v)S^b(v)\gamma dF(v) \tag{D.1}
\]

In this equation, we require that \( \sum_b S^b(v) = 1 \) for all banks to avoid double counting project scale across banks. Since we maintain Assumption 1, bank \( b \)'s collateral constraint is:

\[
B^b_1 \leq \phi^b_h pq^b_1(D_0/N^B + D^b_1). \tag{D.2}
\]

Otherwise, the model is identical to the one in the main text.

We find the following main results:

D.2 Analog of Proposition 1

It is immediate that any bank will find it optimal to give itself 100% seniority. We thus abstract from seniority for the private bank’s problem, instead characterizing the optimal liquidation rule.

**Proposition D.1.** Bank \( b \)'s optimal strategy is a threshold rule \( \rho(v) = 1 \ ( v \leq \nu^{b,p}_L ) \), where

\[
\nu^{b,p}_L = (1 + \theta^b)(\gamma + C) \tag{D.3}
\]

and \( \theta^b \) is bank \( b \)'s private value of loanable funds,

\[
\theta^b = \frac{1}{1 - \phi^b_h} \left( \frac{1}{q_1} - 1 \right). \tag{D.4}
\]

**Proof:** The private bank problem is:
\[
\sup_{B_1, D_1, p} \left( D_1^b + \frac{D_0}{N^B} \right) + (1 - p) \int (1 - \rho(v)) v I_0 dF(v) - B_1^b, \tag{D.5}
\]

subject to

\[
pq_1 D_1^b + (1 - p) \int (1 - \rho(v)) CI_0 dF(v) \leq B_1^b - B_0 + (1 - p) \int \rho(v) \gamma I_0 dF(v) \tag{D.6}
\]

and

\[
B_1 \leq \phi^b_B pq_1 \left( \frac{D_0}{N^B} + D_1^b \right). \tag{D.7}
\]

The argument used in the proof of Proposition 2 shows the optimal liquidation rule is a threshold rule. Introducing Lagrange multipliers \(\theta, \delta\), the Lagrangian is thus

\[
p \left( D_1^b + \frac{D_0}{N^B} \right) + (1 - p) \int_{v \geq v_L^p} v I_0 dF(v) - B_1^b \\
+ \theta \left( \phi^b_B pq_1 \left( \frac{D_0}{N^B} + D_1^b \right) - B_1^b \right) \\
+ \delta \left( B_1^b - B_0 + (1 - p) \int_{v \leq v_L^p} \gamma I_0 dF(v) - \left[ pq_1 D_1^b + (1 - p) \int_{v \geq v_L^p} CI_0 dF(v) \right] \right).
\]

Differentiating with respect to \(B_1\),

\[-1 - \theta^b + \delta^b = 0. \tag{D.8}\]

Differentiating with respect to \(v_L^p\) and dividing by \(I_0\):

\[0 = -(1 - p)v_L^p f(v_L^p) + \delta^b \left( (1 - p) f(v_L^p)(\gamma + C) \right).\]

Dividing by \((1 - p) f(v_L^p)\) and plugging in \(\delta^b = 1 + \theta^b\),

\[v_L^p = (\gamma + C)(1 + \theta^b).
\]

Finally, differentiate with respect to \(D_1^b\),
\[0 = p + \theta^b \phi^b_p q_1 - \delta^b p q_1\]
\[0 = \frac{p(1-q_1)}{q_1} + \theta^b p(\phi^b_h - 1)\]
\[\theta^b = \frac{1-q_1}{q_1(1-\phi^b_h)} = \frac{1}{1-\phi^b_h} \left( \frac{1}{q_1} - 1 \right).\]

### D.3 Analog of Proposition 2

We now consider a planner that optimizes over bank specific borrowing, lending, and seniority, as well as the liquidation rule and prices: the planner chooses \(\{B^b_1, D^b_1, S^b\}, q_1, \gamma, \rho\).

Let
\[\hat{C}^{\text{old}} \equiv \sum_b \theta^b \phi^b_p q_1 D_0 \]  
\[\hat{C}^{\text{new},b} \equiv \phi^b_h p I_1\]  

denote the value of all old collateral, weighted across banks. Let \(\hat{C}^{\text{new},b} \equiv \phi^b_h p I_1\) denote the value of new collateral evaluated at bank \(b\)'s collateral parameter.

**Proposition D.2.** The social value \(\theta^{b,s}\) of bank \(b\)'s loanable funds is
\[\theta^b = \theta^{p,b} + \phi^b_h \frac{\hat{C}^{\text{old}}}{\hat{C}^{\text{new},b}} \xi_q.\]  

The socially optimal seniority policy is
\[S^b(v) = I(b = \text{argmax}_s \theta^{z,s}) I(v < v^*_L) + I(b = \text{argmin}_s \theta^{z,s}) I(v > v^*_L)\]  

The socially optimal seniority structure is liquidation rule is a threshold rule \(\rho(v) = 1 - \left( v \leq v^*_L \right)\), where
\[v^*_L = \gamma + C + \min_b \theta^{b,s} C + \max_b \theta^{b,s} \gamma (1 - \xi \gamma).\]  

**Proof:** We suppress \(s\) superscripts for brevity. Ignoring the irrelevant term \(pv_h I_0\) and redundant control variables \(L, I_1\), the planner’s objective is
\[
\sup_{\{B^b_1, D^b_1, S^b\}, q_1, \gamma, \rho} \quad pg_h(q_1 \sum_b D^b_1) + (1-p) \int (1-\rho(v))v I_0 dF - \sum_b B^b_1 \\
+ g_a \left( (1-p) I_0 \int \rho(v) dF \right) - \gamma I_0 (1-p) \int \rho(v) dF
\]
subject to

\begin{align}
B_1^b &\leq \phi^b \rho q_1 \left( \frac{D_0}{N^b} + D_1^b \right) \quad \text{(D.13)} \\
pq_1 D_1^b + I_0 (1 - p) \int (1 - \rho(v)) S^b(v) C_dF(v) &\leq B_1^b - B_0 + I_0 - (1 - p) \int S^b(v) \rho(v) \gamma dF(v) \\
1 &= q_1 g'_h \left( \sum_b q_1 D_1^b \right) \quad \text{(D.14)} \\
g'_a \left( (1 - p) I_0 \int \rho(v) dF \right) &= \gamma \quad \text{(D.15)}
\end{align}

An argument identical to the one used in the proof of Proposition 2 shows that the optimal rule is a threshold rule.

Introducing Lagrange multipliers \( \{ \theta^b, \delta^b \}, \kappa, \lambda \), the Lagrangian is

\begin{align}
g_h(q_1 \sum_b D_1^b) + (1 - p) \int_{v \leq v_L} v I_0 dF - \sum_b B_0^b + g'_a \left( (1 - p) I_0 F(v_L) \right) - \gamma I_0 (1 - p) F(v_L) \\
+ \sum_b \theta^b \left( \phi^b \rho q_1 \left( \frac{D_0}{N^b} + D_1^b \right) - B_1^b \right) \\
+ \sum_b \delta^b \left( B_1^b - B_0 + (1 - p) \int_{v \leq v_L} S^b(v) \gamma I_0 dF(v) - \left[ pq_1 D_1^b + (1 - p) \int_{v \geq v_L} S^b(v) C I_0 dF(v) \right] \right) \\
+ \kappa \left( 1 - q_1 g'_h (q_1 \sum_b D_1^b) \right) \\
+ \lambda \left( g'_a \left( (1 - p) F(v_L) I_0 \right) - \gamma \right)
\end{align}

Differentiating with respect to \( B_1^b \),

\begin{align}
-1 - \theta^b + \delta^b &= 0. \quad \text{(D.17)}
\end{align}

Note that the only piece depending on \( S^b \) is then:

\begin{align}
\sum_b (1 + \theta^b) \left( B_1^b - B_0 + (1 - p) \int_{v \leq v_L} S^b(v) \gamma I_0 dF(v) - \left[ pq_1 D_1^b + (1 - p) \int_{v \geq v_L} S^b(v) C I_0 dF(v) \right] \right).
\end{align}

The optimal seniority rule is thus

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\[ S^b(v) = 1(b = \arg\max_z \theta^z)1(v < v_L) + 1(b = \arg\min_z \theta^z)1(v > v_L) \quad \text{(D.19)} \]

Now, plugging in \( \theta^b + 1 = \delta^b \), differentiating with respect to \( v_L \), and dividing by \( I_0 \),

\[
-(1 - p)v_L f(v_L) + g_a' \left( (1 - p)F(v_L)I_0 \right) (1 - p)f(v_L) - \gamma(1 - p)f(v_L) \\
+ (1 - p)f(v_L) \left[ (1 + \max_b \theta^b)\gamma + (1 + \min_b \theta^b)C \right] \\
+ \lambda g_a'' \left( (1 - p)F(v_L)I_0 \right) (1 - p)f(v_L). 
\]

Dividing by \( (1 - p)f(v_L) \) and setting this equal to 0,

\[
0 = -v_L + \gamma + C + g_a' \left( (1 - p)F(v_L)I_0 \right) - \gamma + \left( \max_b \theta^b \gamma + \min_b \theta^b C \right) + \lambda g_a'' \left( (1 - p)F(v_L)I_0 \right). \quad \text{(D.20)}
\]

Next, differentiating the Lagrangian with respect to \( \gamma \), setting it equal to 0, and adding \( \lambda \) to both sides,

\[
\lambda = -I_0(1 - p)F(v_L) + \max_b \delta^b I_0(1 - p)F(v_L) = \max_b \theta^b I_0(1 - p)F(v_L), \quad \text{(D.21)}
\]

Noting that market clearing in the liquidation market implies \( \gamma = g_a' \left( (1 - p)F(v_L)I_0 \right) \),

\[
0 = -v_L + \gamma + C + \left( \max_b \theta^b \gamma + \min_b \theta^b C \right) + \lambda g_a'' \left( (1 - p)F(v_L)I_0 \right). \quad \text{(D.22)}
\]

Applying the above equation and adding \( v_L \) to both sides,

\[
v_L = \gamma + C + \min_b \theta^b C + \max_b \theta^b \left( \gamma + I_0(1 - p)F(v_L)g_a'' \left( L \right) \right). \quad \text{(D.23)}
\]

Following the same steps as in the proof of Proposition 2,

\[
\xi = \frac{-Lg_a''(L)}{\gamma}. \quad \text{(D.24)}
\]

Plugging this in,

\[
v_L = \gamma + C + \min_b \theta^b C + \max_b \theta^b \gamma(1 - \xi). \quad \text{(D.25)}
\]

Finally, we characterize \( \theta^b \) and derive \( M^b \). As in the proof of Proposition 2,
\[ \xi_q = \frac{-I_1 g''_h(I_1)}{g'_h(I_1)}. \]  

(D.26)

Now, differentiate the Lagrangian with respect to \( D^b_1 \):

\[ 0 = pq_1 g'_h(I_1) + \theta^b \phi^b_h p q_1 - \delta^b q_1 p - \kappa q_1^2 g''_h(I_1). \]  

(D.27)

Dividing by \( q_1 \) and plugging in \( \delta^b = 1 + \theta^b \),

\[ 0 = p (g'_h(I_1) - 1) + \theta^b p (\phi^b_h - 1) - \kappa q_1 g''_h(I_1). \]  

(D.28)

Multiplying by \( D^b_1 \),

\[ 0 = D^b_1 p (g'_h(I_1) - 1) + D^b_1 \theta^b p (\phi^b_h - 1) - D^b_1 \kappa q_1 g''_h(I_1). \]  

(D.29)

Since this holds for each \( b \), we can sum and rearrange:

\[ \sum_b D^b_1 \kappa q_1 g''_h(I_1) = \sum_b D^b_1 p (g'_h(I_1) - 1) + \sum_b D^b_1 \theta^b p (\phi^b_h - 1). \]  

(D.30)

Differentiating the Lagrangian with respect to \( q_1 \) and setting it equal to zero,

\[ 0 = pg'_h(I_1) \sum_b D^b_1 + \sum_b \theta^b (\phi^b_h p (\frac{D_0}{NB} + D^b_1)) - p \sum_b \delta^b D^b_1 - \kappa \left( g'_h (q_1 \sum_b D^b_1) + q_1 \sum_b D^b_1 g''_h (q_1 \sum_b D^b_1) \right). \]  

(D.31)

Simplifying with \( \delta^b = 1 + \theta^b \),

\[ 0 = p (g'_h(I_1) - 1) \sum_b D^b_1 + \sum_b \theta^b (-p D^b_1 + (\phi^b_h p (\frac{D_0}{NB} + D^b_1)) - \kappa \left( g'_h (q_1 \sum_b D^b_1) + q_1 \sum_b D^b_1 g''_h (q_1 \sum_b D^b_1) \right). \]  

(D.32)

Using equation (D.30),

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\[ 0 = \sum_b \theta^b \phi^b_h p \frac{D_0}{N^B} - \kappa g'_h(q_1 \sum_b D^b_p). \]  
(D.34)

Define

\[ \hat{C}^\text{old} \equiv \sum_b \theta^b \phi^b_h p q_1 D_0 \frac{N}{N^B}. \]  
(D.35)

Then

\[ \kappa = \frac{\hat{C}^\text{old}}{q_1 g'_h(I_1)}. \]  
(D.36)

Plugging this into equation (D.28),

\[ 0 = p(g'_h(I_1) - 1) + \theta^b p(\phi^b_h - 1) - \frac{\hat{C}^\text{old}}{q_1 g'_h(I_1)} q_1 g''_h(I_1) \]  
(D.37)

\[ \theta^b p(1 - \phi^b_h) = p\left(\frac{1}{q_1} - 1\right) - \frac{\hat{C}^\text{old}}{g'_h(I_1)} g''_h(I_1) \]  
(D.38)

\[ \theta^b = \theta^{p,b} + \frac{\phi^b_h}{1 - \phi^b_h p \phi^b_f} \hat{C}^\text{old}. \]  
(D.39)

Suppose that \( \phi^b_h = \phi_h \) for all banks. Clearly \( \theta^{p,b} = \theta^p \). We also have that \( \hat{C}^\text{old} = \theta C^\text{old} \) and from that we see that \( \theta^b = \theta^s \).

## E Policy Proposal Discussion

### E.1 Related policies

In response to previous crises, governments have implemented policies to deter liquidations. In the wake of the COVID-19 pandemic, numerous state and local governments instituted moratoriums on the eviction of commercial tenants. For example, the “COVID-19 Emergency Protect Our Small Businesses Act of 2021,” which was signed into law on March 6th 2021, banned evictions and foreclosure actions relating to certain small commercial properties in the state of New York.\(^\text{30}\)

These moratoriums are analogous to an extreme version of the policy of Proposition 3 in which an infinite tax is levied on liquidations.

Similarly, Section 4013 of the Coronavirus Aid, Relief, and Economic Security (CARES) Act of 2020 encouraged banks to engage in restructurings, rather than liquidations, with distressed borrowers. Prior to this intervention, if banks engaged in troubled debt restructurings (TDRs) with distressed borrowers to avoid liquidations, banks had to categorize the borrowers’ loans as impaired. The value of an impaired loan must be revised downward to the expected discounted value of the future cashflows generated by the loan. Such a downward revision of loan value could harm a bank by depleting its regulatory capital, potentially forcing the bank to sell other assets at fire-sale prices to reduce its leverage and satisfy capital requirements (Laux and Leuz, 2010). Thus, the CARES Act encouraged the continuation of insolvent firms by alleviating negative regulatory consequences for banks negotiating with firms that were adversely affected by COVID.

Historically, governments have been more hesitant to subsidize or encourage liquidations during a crisis. However, outside of crises, governments have provided incentives for liquidations. For example, asset sales that are part of a liquidating Chapter 11 bankruptcy plan are exempt from any transfer or “stamp” taxes (11 U.S.C. §1146(a)). This tax exemption is effectively a subsidy for liquidations. Other policies have simply made it easier for lenders to liquidate firms. For example, a 2001 reform of Article 9 of the Uniform Commercial Code made it easier for secured lenders to foreclose on assets (Benmelech, Kumar, and Rajan, 2020). Likewise, antirecharacterization passed in states like Texas and Louisiana made it easier for creditors to seize assets associated with bankrupt firms (Ersahin, 2020). Prior to these laws, bankruptcy judges would sometimes recharacterize a debtor’s bankruptcy remote assets, typically held in a nonbankrupt affiliated special purpose vehicle, as part of the bankrupt firm, subjecting the assets to the automatic stay. By banning this practice of recharacterization, these laws effectively encouraged liquidations.

Internationally, the Czech Republic, France, Germany, Italy, Luxembourg, Portugal, Spain, Switzerland, and Turkey responded to COVID-19 by implementing temporary bankruptcy moratoriums. These interventions prevented creditors from initiating involuntary bankruptcy proceedings against insolvent firms (Gómez et al., 2020). Some of these moratoriums also suspended existing laws imposing personal liability on managers who fail to file for bankruptcy when their firms become insolvent.

### E.2 Other Related Policies

The COVID-19 pandemic led to academic proposals for government interventions aimed at mitigating social losses caused by liquidations. For example, as discussed in the main text, Blanchard, Philippon, and Pisani-Ferry (2020) propose that governments could accept lower recovery in re-

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structurings that allow firms to continue operating. Greenwood and Thesmar (2020) propose a tax credit for landlords and lenders that agree to such restructurings. Our model (Proposition 2) implies that such policies are socially optimal if banks are not financially constrained and fire-sale externalities are nontrivial.\textsuperscript{33} Given the health of the banking sector during the COVID-19 pandemic (Greenwood, Iverson, and Thesmar, 2020), it is likely that the social planner in our model would enact a similar policy in a crisis resembling the COVID-19 pandemic. More broadly, firms in the US have relied less on bank lending (Crouzet, 2018) in recent years and banks have been well capitalized (Corbae, D’Erasmo et al., Forthcoming). According to our model, this bolsters the case for subsidizing continuation in crises. Nonetheless, future crises or crises in different countries could call for the optimal subsidization of liquidation.

DeMarzo, Krishnamurthy, and Rauh (2020) propose that the Federal Reserve and the Treasury create a special purpose vehicle (SPV) to provide DIP loans to bankrupt firms. The SPV would provide highly subsidized loans to bankrupt firms, which would be fully collateralized, priming existing liens if necessary (11 U.S.C. §364(d)). While we do not formally model such a policy, our analysis nonetheless illustrates the potential benefits of the policy proposed by DeMarzo, Krishnamurthy, and Rauh (2020). Recall that when banks are financially constrained, the social planner faces a tradeoff between reorganizing viable firms and preserving funding to solvent firms at reasonable credit spreads. If the government were to exogenously increase the supply of date-one loans to bankrupt firms at subsidized rates, then more reorganizations could be achieved without exacerbating the loan-price externality, in which privately supplied DIP loans crowd out funding to solvent firms. As such, our model suggests that this proposal may be especially effective when banks are also in financial distress and hence loan price externalities are strong. Outside of our model, it is possible that creditors may force marginally solvent firms into bankruptcy, increasing deadweight losses, in order to take advantage of subsidized government funding. However, this incentive for inefficient behavior would be mitigated by the fact that government DIP loans would be senior to all unsecured debt, and even potentially secured debt through priming liens.

Finally, other academics proposed policies aimed at paying the debt of all firms to prevent deadweight losses associated with liquidation (Saez and Zucman, 2020; Hanson et al., 2020b). Specifically, Saez and Zucman (2020) recommend expanding unemployment insurance and paying a fraction of the maintenance costs of businesses in sectors that are affected by pandemic-induced shutdowns. Hanson et al. (2020b) propose an intervention in which the government would “provide payment assistance to enable impacted businesses to meet their recurring fixed obligations—including interest, rent, lease, and utility payments.” We do not formally model interventions like these that entail paying the expenses of all operating firms in troubled sectors. Such an

\textsuperscript{33}Indeed, the government accepting a larger haircut on distressed debt is analogous to the implementation that we describe in Proposition 3.
intervention is analogous to the implementation we describe in Proposition 3, in which the government subsidizes the continuation of insolvent businesses. Unlike the policy described in Proposition 3, these proposed interventions also subsidize the continuation of solvent businesses, making them potentially expensive. However, a policy like ours that makes aid contingent on demonstrated insolvency could incentivize firms to file for bankruptcy, incurring deadweight losses, just to take advantage of the government subsidy. In deciding how to allocate assistance, the government thus faces a tradeoff between the expense of helping solvent firms and the greater incentive for firms to demonstrate their insolvency through value-destroying bankruptcy filings.