A Theory of Dynamic Inflation Targets

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Abstract

Should central banks’ inflation targets remain set in stone? We study a dynamic mechanism design problem between a government (principal) and central bank (agent). The central bank has persistent private information about structural shocks affecting the optimal rate of inflation. Firms learn about the state of the economy from the central bank, whose reports affect inflation expectations through firms’ beliefs about both the persistent state and the future conduct of monetary policy. A dynamic inflation target implements the full-information commitment allocation: the central bank is delegated the authority to adjust its own target as long as it does so one period in advance. The dynamic inflation target takes the form of a linear incentive scheme for deviations from the target, and both the target level and its flexibility adjust in response to persistent shocks. An informational divine coincidence arises because firms and the government both learn from the central bank, and more complex mechanisms are necessary if firms directly observe the state. We apply our theory to study lower bound spells, a declining natural interest rate, and a flattening Phillips curve. We leverage our framework to study longer horizon time consistency problems and optimal mechanisms with enforcement costs.

JEL codes: E52, D82

Keywords: inflation targeting, persistent private information, dynamic mechanism design, monetary policy, time consistency

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1 Introduction

Since their inception in the early 1990s, many central banks’ inflation targets have evolved substantially. For example, the Bank of New Zealand has announced at least four major updates to its target definition since 1990.\(^1\) The Bank of Canada undergoes regular reviews of its inflation target at 5-year intervals. In 2020 and 2021, the U.S. Federal Reserve and the European Central Bank both updated their inflation target frameworks. Overall, central banks have exercised substantial discretion over target adjustments during this period.

In academic discourse, an important motivation for inflation targets is the interaction between a time consistency problem and central bank private information (Barro and Gordon, 1983).\(^2\) Prior work has shown that a static inflation targeting mechanism can resolve the resulting inflationary bias in either fully static environments or when shocks are uncorrelated across time (Walsh, 1995; Athey et al., 2005). These frameworks motivate inflation targets as desirable mechanisms but do not speak to the empirical regularity that central banks regularly update their targets. When deliberating target adjustments, central banks in practice often invoke persistent economic change, which presupposes that shocks are correlated over time.\(^3\)

In this paper, we study a dynamic monetary policy game in the presence of persistent shocks and private information. As in previous work, the central bank faces a time consistency problem; unlike in previous work, persistent shocks make the central bank’s private information persistent. This gives rise to additional information frictions because firms learn about the persistent state from the central bank, which they use to form inflation expectations. Our main result is that a time-varying, dynamic inflation target mechanism implements the efficient, full-information commitment allocation. The dynamic inflation target is a two-parameter mechanism, featuring both a target level and a target flexibility. Our result generalizes the canonical work on inflation targets to environments with persistent private information. An informational divine coincidence emerges because strategic target adjustments offset strategic information dissemination to manipulate firm beliefs. We leverage our theory to study how the dynamic inflation target optimally responds to lower bound spells, a declining natural rate of interest, and a flattening Phillips curve. We extend our theory to longer-duration time consistency problems and develop a second-best theory for dynamic inflation targets in practice, arguing that our mechanism closely resembles the Bank of Canada’s current

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\(^1\) See for example McDermott and Williams (2018). The Bank of New Zealand’s initial target postulated an inflation band of 0-2%. The band was revised in 1996 to 0-3% and again in 2002 to 1-3%. Another revision in 2012 added an explicit focus on the 2% target midpoint.

\(^2\) There is much empirical support for the existence of central bank private information. For example, see Romer and Romer (2000), Kuttner (2001), Gürkaynak et al. (2005), Campbell et al. (2012), Krishnamurthy and Vissing-Jorgensen (2012), and Lucca and Moench (2015) among many others.

\(^3\) The long-term strategic review that preceded the Federal Reserve’s target adjustment in 2020 was partly motivated by the persistent decline in the natural rate of interest and the accompanying concern of an increase in the incidence of future ZLB spells. See for example Clarida (2019). In August 2020, the Fed concluded its review by adopting a target that aims to “achieve inflation that averages 2% over time” (Powell, 2020). At the same time, commentators have also suggested an explicit upward revision in the inflation target, for example Blanchard et al. (2010), Ball (2013) and Krugman (2014).
operating framework.

Our infinite horizon model features persistent but evolving economic states and general social preferences over inflation and output. Small firms determine the current inflation-output relationship based on their expectations about future allocations and policy, giving rise to a forward-looking “Phillips curve”. The standard time consistency problem emerges (Kydland and Prescott, 1977; Barro and Gordon, 1983). Neither firms nor the government observe the underlying economic state, which is persistent private information of a central bank that sets monetary policy under discretion. A Ramsey government (principal) designs a transfer/punishment mechanism to incentivize the central bank’s (agent) policy design, which is otherwise set under discretion. The central bank’s behavior under the mechanism reveals its persistent private information to both the government and to firms. Firms in turn use the information revealed by the central bank to update their beliefs about the distribution of future shocks and the conduct of future policy, and so to form inflation expectations. An incentive compatible mechanism must account for both the time consistency problem of the central bank that sets policy under discretion, as well as its strategic incentives to use information revelation under the mechanism to influence firm’s inflation expectations.

We develop our main result in Section 3: a dynamic inflation target mechanism implements the full-information Ramsey commitment allocation. This mechanism is incentive compatible—it overcomes both the central bank’s time consistency problem and the central bank’s informational reporting problem under persistent private information. Formally, the dynamic inflation target is a two-parameter slope-intercept transfer rule: the central bank faces a linear penalty for inflation in excess of a target level, with the slope of the penalty representing the target flexibility. The linear penalty for inflation is set so that the central bank internalizes the marginal cost of inflation in the prior period, which manages the time consistency problem. Crucially, our mechanism delegates to the central bank the authority to update its own target—both level and flexibility—as long as it does so one period in advance. That is, the target parameters for date \( t \) are set at date \( t - 1 \). Intuitively, advanced updating leads the central bank to internalize its own future time consistency problem when it updates its future target. This incentivizes the central bank to report truthfully in updating its target.

A surprising aspect of the dynamic inflation target is that it also overcomes the central bank’s strategic misreporting incentives under persistent private information. This informational divine coincidence lies at the heart of our paper. Intuitively, it arises because both the government and firms have the same information set, meaning that they also learn in the same manner from the central bank’s reports. The central bank would benefit from biasing the government’s beliefs about future inflation upward, which increases the future target level and so reduces future penalties for high inflation. At the same time, such misreporting is costly to the central bank because biasing government beliefs upwards also biases firm beliefs upwards, which worsens the contemporaneous inflation-output tradeoff. The marginal benefit of biasing government beliefs upwards is the target
flexibility, which under the Ramsey allocation is exactly equal to the marginal cost of biasing firm beliefs upwards. Thus the informational incentives of the central bank exactly offset one another under the dynamic inflation target.

The informational divine coincidence is essential for the simple dynamic inflation target to be able to implement the Ramsey allocation in this environment with persistent private information. It arises not only due to the fact that firms and government have the same information set, but also because (as stressed above) the allocation being implemented is the Ramsey allocation. We revisit throughout the paper the idea that when the informational divine coincidence breaks down, more complex mechanisms than the dynamic inflation target are needed. This emphasizes a part of our contribution, which is assumptions on information revelation needed for simple inflation targets to be robust to persistent private information.

We study how a change in the informational environment leads to a breakdown of the informational divine coincidence and necessitates a more complex mechanism. In particular, we allow a subset of firms are directly informed about the underlying state. The dynamic inflation target is no longer an incentive compatible mechanism for implementing the Ramsey allocation. The intuition is that with informed firms but an uninformed government, the central bank earns an additional information rent from revealing to the government what firms’ knowledge of the persistent state is. We characterize the wedge in incentive compatibility that arises, and in particular show that informed firms create an excess incentive for the central bank to report and bias expectations upward. This implies that a controlled adjustment process with penalties for target changes is necessary when the informational divine coincidence fails. It also suggests a novel complexity-based argument for central banks to be responsible for collecting and disseminating information about the economic state: when firms also learn from the central bank, the simpler dynamic inflation target is incentive compatible.

We develop four applications of our theory in Section 4. Each of our applications is motivated by recent empirical evidence and monetary policy debates on structural change in the U.S., and emphasizes in each case the relevance of persistent private information. First, we study lower bound spells, where the central bank finds itself stuck at a zero nominal interest rate. It is well known that optimal monetary policy at the lower bound features history dependence, promising to keep interest rates low even once the economy has exited the liquidity trap. We show that policy under a dynamic inflation target replicates the full-information optimal commitment solution even in the presence of persistent private information. Under a dynamic inflation target, lower bound spells motivate adjustments in both target level and flexibility; in particular, the infinitely forward-looking commitment solution is implemented via iterated one-period commitments in the form of one-period-ahead target adjustments.4

4 When confronting the effective lower bound, central banks have recently resorted to unconventional policy instruments, focusing largely on forward guidance, asset purchases, and to a lesser degree negative rates. Some commentators have explicitly raised the question whether changes in the targeting framework could and should be seen as a potential additional unconventional monetary policy instrument. Our theory provides a natural framework to ask
In our second application, we study how the dynamic inflation target responds to a declining natural rate of interest in the presence of an occasionally-binding effective lower bound on interest rates. With mounting empirical evidence for a historically low natural rate after the Great Recession (Laubach and Williams, 2016), this question has received ample attention. Many observers in the U.S. have explicitly advocated for an increase in the Federal Reserve’s inflation target level.\footnote{The declining natural rate of interest has in part motivated the Federal Reserve to undergo a long-term strategic review of its operating framework between 2018 and 2020, which concluded with an adoption of an average inflation target.} We show that a decline in the natural interest rate leads to an increase in the dynamic inflation target’s level as well as, more surprisingly, to an increase in its flexibility. A key insight of our analysis is that the presence of an occasionally-binding lower bound may even lead to a sign switch in the optimal target flexibility in the risky steady state: intuitively, a large enough probability of hitting the occasionally binding constraint means that the desire to promote inflation to lift nominal rates dominates the standard time consistency problem, leading to too little inflation under discretion.

Our third application studies the impact of a flattening Phillips curve. The changing slope of the U.S. Phillips curve has garnered much attention since the Great Recession (Blanchard, 2016; Galí and Gambetti, 2019; Rubbo, 2020; Del Negro et al., 2020), and the ensuing debate has engulfed monetary policy discourse in recent years (Brainard, 2015). We show that a persistent flattening of the Phillips curve leads to a decrease in the dynamic inflation target’s level and flexibility. Intuitively, a flattening Phillips curve increases the output cost of expected inflation, and so leads to a larger time consistency problem. Interestingly, the constriction of the inflation target on both dimensions in response to the flattening Phillips curve yields a sharply different policy prescription than the first two applications, suggesting competing forces when these trends happen simultaneously.

Finally, we revisit the classical inflation-conservative central banker paradigm of Rogoff (1985). In the presence of persistent shocks, we show that a dynamic inflation target could be implemented by appointing a more inflation-conservative central banker when the target flexibility is meant to fall, and a less inflation-conservative central banker when the target flexibility is meant to rise.

Having shown that a dynamic inflation target delegates to the central bank the authority to update its own target one period in advance, a natural question emerges when considering the implications of our theory for policy design in practice: How long is a period and what is the appropriate time horizon for target adjustments? The time consistency problem of the baseline model has a duration of one period – between times when inflation is set – because it only features one period ahead expectation dependence. This means a dynamic inflation target takes the form of iterated one period commitments, but offers little guidance on the interval of time that these commitments covers. We tackle this question in Section 5, where we generalize our framework to allow for longer horizon forward-looking models where output today depends on forecasts of inflation for multiple periods. We show that in this environment, a K-horizon dynamic inflation target.
target implements the Ramsey allocation. In particular, the K-horizon target forms an inflation target using a weighted average of the commitments and forecasts made for period $t$ over the past $K$ periods. The informational divine coincidence continues to hold.

In this environment, we introduce the commitment curve, which formalizes the horizon and persistence over which the central bank finds it valuable to make promises to improve the contemporaneous input-output tradeoff. The commitment curve formally represents the size of the commitment the central bank makes at date $t$ for all future periods $t + k$. Under the classical New Keynesian Phillips Curve, the commitment curve is sharply downward sloping: it is positive at $k = 1$ and zero at every $k > 1$, representing the intuition of the dynamic inflation target that the central bank need only make commitments for the next period. This is a special case that only obtains linearizing around the 0-inflation steady state, and more generally the nonlinear Calvo pricing equation has infinitely many forward-looking expectation terms. In the more general environment, a flatter commitment curve means the K-horizon dynamic inflation target calls for the central bank at date $t$ to make substantial commitments over a long horizon.

Our main application in this environment is to study dynamic inflation targets in the context of a generalized New Keynesian Phillips Curve that obtains when we linearize the standard Calvo pricing equation with positive trend inflation (Ascari, 2004; Ascari and Sbordone, 2014). This is of particular policy relevance as it allows us to shed light on the horizon for target adjustments, that is “how long is a period?” Formally, we show that the commitment curve’s shape is that of quasi-hyperbolic discounting (Laibson, 1997). The central bank’s largest magnitude commitment at date $t$ is for the next date $t + 1$, but it also makes a thereafter exponentially declining series of commitments for dates $t + 2$ and onward. If the quasi-hyperbolic rate and the exponential rate are not too large, then the mechanism involves making substantial commitments over a long horizon, suggesting a longer horizon adjustment period for targets is desirable. By contrast if the quasi-hyperbolic rate is large, then commitments made for the next period are large relative to commitments for the future, suggesting a shorter horizon for target adjustments is desirable. Interpreting the Bank of Canada’s regular target review process as an approximate dynamic inflation target of this form, we show that the commitment curve can guide the optimal horizon of this review process.

The majority of this paper focuses on implementable mechanisms that achieve the Ramsey solution. In Section 6, we study optimal mechanisms when the transfer scheme is costly to the government. The dynamic inflation target is incentive compatible but not optimal. The optimal mechanism trades off Ramsey efficiency against dynamic information rents from costly transfers. The central bank earns an information rent not only from contemporaneous inflation, but also from expected inflation for next period. This means that the optimal allocation is distorted by backward looking information rents, much as the Ramsey allocation is affected by the Phillips curve. In the canonical case of multiplicative taste shocks $\theta_t u_t$, we show that the optimal allocation is that of a Ramsey government subject to the Phillips curve, but whose type is the government’s virtual value. The optimal allocation favors higher inflation relative to Ramsey, when the virtual value today is
high relative to the virtual value yesterday. The dynamic inflation target is recovered only at the extremes of the shock distribution, that is there is no distortion (relative to Ramsey) at the top and bottom.

We use this analysis to shed light on the differences between the allocations under the optimal mechanism and the dynamic inflation target in our applications. We show that costly enforcement calls for less aggressive unconventional policies (e.g., forward guidance) when the economy experiences a lower bound spell. It calls for more aggressive policies (e.g., raising the inflation target) in response to a decline in \( r^* \). Finally, it calls a less aggressive response to the flattening Phillips curve.

While monetary policy is the primary focus of this paper, our results could be applied more broadly to principal-agent settings where “moving goal posts” are desirable due to a combination of persistent private information and time consistency problems arising through expectations.\(^6\)

**Related literature.** The paper most closely related to ours is Halac and Yared (2014). Their paper uses dynamic mechanism design techniques to study the effects of persistent private information in the fiscal policy context in a delegation framework, with time inconsistency driven by present bias due to quasi-hyperbolic discounting. By contrast, we study persistent private information in the monetary policy context with transfers, with a time consistency problem resulting from inflation expectations in a Phillips curve relationship. Novel to our environment is that firms form expectations based on the report of the persistent state, and we highlight the importance of information in this context.\(^7\) We build off of the dynamic mechanism design literature with persistent private information. In particular, Pavan et al. (2014) provide provide conditions for implementability in a general principal-agent framework with transfers and persistent shocks.\(^8\) We deploy these techniques to study central bank inflation targeting.

We also relate to the literature on time consistency and the commitment flexibility trade-off in monetary policy.\(^9\) This literature approaches this question both using transfer or penalty (i.e. separable money burning) based mechanisms, and delegation mechanisms. In the former approach, Walsh (1995) shows that an inflation target is an optimal mechanism in a static context with transferable utility. The linear form of their static target follows the same intuition as the within-period linear form of our dynamic target. In the dynamic context, as is well understood the full-information Ramsey allocation of our model can be implemented with a linear inflation penalty whose slope is the recursive multiplier on the Phillips curve implementability condition (Marcet

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\(^6\) For example, the sovereign debt literature commonly features a time consistency problem that arises because long-term debt prices depend on the government’s future fiscal policy decisions.

\(^7\) More broadly, we related to the literature on the commitment versus flexibility trade-off with quasi-hyperbolic agents studies both transfer mechanisms (DellaVigna and Malmendier, 2004; Galperti, 2015; Beshears et al., 2020) and delegation mechanisms (Amador et al., 2006; Halac and Yared, 2018; Sublet, 2022) to control these agents.

\(^8\) A similar condition is found, for example, in Farhi and Werning (2013).

\(^9\) For example, see Kydland and Prescott (1977), Barro and Gordon (1983), Canzoneri (1985), Rogoff (1985), Cukierman and Meltzer (1986), Persson and Tabellini (1993), Svensson (1995) and Walsh (1995) for the former among many others. More broadly, there has been a long tradition considering the implications of private information for the design of policy. For example, see Backus and Driffill (1985), Sleet (2001), and Angeletos et al. (2006) among many others.
and Marimon 2019, Svensson 1997, Svensson and Woodford 2004). We contribute by studying the impact of persistent private information in a principal-agent environment, and our framework emphasizes a role of the target level in overcoming our model’s core informational frictions.\textsuperscript{10} Halac and Yared (2019) use a framework with socially costly penalties that can be imposed on the central bank to study the trade-off between instrument-based rules and target-based rules. In the delegation context, Athey et al. (2005) considers a dynamic monetary policy framework with independent shocks and show the optimal mechanism is static bounds on inflation. Waki et al. (2018) considers a related framework with independent shocks in which the optimal mechanism consists of history dependent bounds on inflation. Relative to this literature, we study inflation setting with transfers (or penalties) and persistent private information, which gives rise to novel information frictions due to inflation expectations in a Phillips curve type relationship. Similar to Walsh (1995) and Halac and Yared (2019), we adopt an approach using transfers and penalties.

Our discussion also connects to a long literature that studies the optimal rate of inflation (Schmitt-Grohé and Uribe, 2010). More recently, many papers have investigated quantitatively whether a fall in the natural rate of interest in the presence of a zero lower bound constraint could quantitatively justify higher inflation targets (Coibion et al., 2012; Kiley and Roberts, 2017; Andrade et al., 2018; Eggertsson et al., 2019). In our paper, we take as given that persistent structural shocks can alter the welfare implications of inflation and, consequently, the socially desired rate of inflation. We ask if and how a central bank should respond to such shocks in the presence of persistent private information and time consistency problems.

## 2 Model

Our economy is populated by a government, a monetary authority or central bank, and a continuum of small firms. The central bank learns about persistent changes in the state of the economy. It uses this private information, which we also refer to as the central bank’s type, to set monetary policy under discretion. The central bank is subject to a time consistency problem in the tradition of Kydland and Prescott (1977) and Barro and Gordon (1983): Firms determine the relationship between inflation and output in a forward looking manner, which gives rise to a Phillips curve. The government (principal) designs a mechanism to control the inflation policies of the central bank (agent), taking as given the price-setting behavior of firms.

Time is infinite and discrete, indexed by $t = 0, 1, \ldots$ We summarize allocations by inflation $\pi_t \in [\underline{\pi}, \overline{\pi}]$ and output $y_t \in [\underline{y}, \overline{y}]$. There is a state of the economy, $\theta_t \in \Theta = [\underline{\theta}, \overline{\theta}]$, that follows a Markov process described by the conditional transition density $f(\theta_t | \theta_{t-1})$. The central bank observes the state $\theta_t$ at the beginning of $t$ and is tasked with setting inflation for that period. Firms

\textsuperscript{10} More broadly, several papers have extended the Marcet and Marimon (2019) approach for providing a recursive representation of a planner’s problem to environments with moral hazard and incomplete information (Messner et al. 2012, Mele 2014, Pavoni et al. 2018). We contribute by study the mechanism design problem of a principal designing a mechanism for an agent, rather than giving a recursive representation to the principal’s problem.
do not observe the state but form posterior beliefs $\mu_t$ on its distribution based on behavior of the central bank in that period.\footnote{There is a long tradition in macroeconomics to motivate and study monetary policy games when the central bank has private information. For example, see Sargent and Wallace (1975), Barro and Gordon (1983), Canzoneri (1985), Rogoff (1985), Walsh (1995), Athey et al. (2005) and many others. There exists much empirical support for the existence of central bank private information. Romer and Romer (2000) show that the difference between the Federal Reserve’s private inflation forecasts and commercial inflation forecasts is a significant predictor of commercial forecast errors. Lucca and Moench (2015) document sizable excess returns on U.S. equities leading up to scheduled Federal Open Market Committee (FOMC) meetings. These findings suggest the existence of substantial (private) information content in FOMC announcements. Krishnamurthy and Vissing-Jorgensen (2012) find strong empirical support for a signaling channel of unconventional monetary policy, whereby large-scale asset purchases between 2009 and 2012 worked to a large extent by conveying private information to financial market participants. Similarly, work by Kuttner (2001) and Gürkaynak et al. (2005) shows that Federal Reserve announcements are associated with significant price effects that are not due to changes in the policy rate itself. Building on this work, Campbell et al. (2012) show that asset prices and commercial macroeconomic forecasts respond strongly to the information content in FOMC announcements. L’Huillier and Schoenle (2019).\label{fn:1} Our Phillips curve relationship is robust to a Lucas critique provided that expected future (next period) inflation is sufficient for determining how changes in future policies affect firm behavior. For example, higher expected inflation may lead firms to increase the frequency with which they update prices, altering the slope of the Phillips curve.\label{fn:2} Persson and Tabellini (1993) also consider a general social welfare function where shocks enter that change the welfare consequences inflation.} We denote by $\mathbb{E}_t[\pi_{t+1} | \mu_t]$ firms’ expectation of next-period inflation, given their posterior beliefs $\mu_t$ about the current state $\theta_t$. Firms’ price setting determines output based on future inflation expectations, giving rise to a “Phillips curve”\footnote{Although we use linear expectations $\mathbb{E}_t[\pi_{t+1}]$, it is easy to see our framework can be adapted for nonlinear expectations. For example, suppose that we had $y_t = F_t(\pi_t, \mathbb{E}_t[\pi_{t+1}])$ for a nonlinear function $g_t$. Then define $\pi^*_t = g_t(\pi_{t+1})$, define the Phillips curve as $y_t = F_t(\pi^*_t, \mathbb{E}_t[\pi^*_{t+1}]) = F_t(g_t^{-1}(\pi^*_t), \mathbb{E}_t[\pi^*_{t+1}])$, and similarly for the preference function. More generally if we have $\mathbb{E}_t[\pi_{t+1}, y_{t+1}]$, then we can define a new variable $\pi^*_t = g_t(\pi_t, y_t)$, and define the problem over $(\pi^*_t, y_t)$ where $y_t = Y_t(\pi^*_t, \mathbb{E}_t[\pi^*_{t+1}])$, where $Y_t$ solves $Y_t(\pi^*_t, \mathbb{E}_t[\pi^*_{t+1}]) = F_t(g_t^{-1}(\pi^*_t | \pi^*_t, \mathbb{E}_t[\pi^*_{t+1}]), \mathbb{E}_t[\pi^*_{t+1}]).$\label{fn:3} A key concern of this Phillips curve relationship is a Lucas critique—firms’ price-setting behavior may change in response to changes in the monetary policy regime, such as target changes (L’Huillier and Schoenle, 2019). Our Phillips curve relationship is robust to a Lucas critique provided that expected future (next period) inflation is sufficient for determining how changes in future policies affect firm behavior. For example, higher expected inflation may lead firms to increase the frequency with which they update prices, altering the slope of the Phillips curve.\label{fn:4} Persson and Tabellini (1993) also consider a general social welfare function where shocks enter that change the welfare consequences inflation.} firms’ expectation of next-period inflation, given their posterior beliefs $\mu_t$ about the current state $\theta_t$. Firms’ price setting determines output based on future inflation expectations, giving rise to a “Phillips curve”\footnote{Although we use linear expectations $\mathbb{E}_t[\pi_{t+1}]$, it is easy to see our framework can be adapted for nonlinear expectations. For example, suppose that we had $y_t = F_t(\pi_t, \mathbb{E}_t[\pi_{t+1}])$ for a nonlinear function $g_t$. 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$$y_t = F_t(\pi_t, \mathbb{E}_t[\pi_{t+1} | \mu_t]).$$

(1)

Because shocks are persistent, inflation expectations $\mathbb{E}_t[\pi_{t+1} | \mu_t]$ depend not only on firm beliefs about the future conduct of monetary policy but also about the distribution of future shocks $\theta_{t+1}$.

The per-period social welfare function for the central bank and government over inflation and output is $U_t(\pi_t, y_t, \theta_t)$. To simplify exposition, we internalize the Phillips curve relationship (1) and write reduced-form preferences as $U_t(\pi_t, \mathbb{E}_t[\pi_{t+1}], \theta_t) = U_t(\pi_t, F_t(\pi_t, \mathbb{E}_t[\pi_{t+1} | \mu_t]), \theta_t)$. The lifetime social welfare function of the central bank and government over inflation can then be written as

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t U_t(\pi_t, \mathbb{E}_t[\pi_{t+1} | \mu_t], \theta_t),$$

(2)

where $\beta$ is the discount factor.\footnote{Although we use linear expectations $\mathbb{E}_t[\pi_{t+1}]$, it is easy to see our framework can be adapted for nonlinear expectations. For example, suppose that we had $y_t = F_t(\pi_t, \mathbb{E}_t[\pi_{t+1}])$ for a nonlinear function $g_t$. Then define $\pi^*_t = g_t(\pi_{t+1})$, define the Phillips curve as $y_t = F_t(\pi^*_t, \mathbb{E}_t[\pi^*_{t+1}]) = F_t(g_t^{-1}(\pi^*_t), \mathbb{E}_t[\pi^*_{t+1}])$, and similarly for the preference function. More generally if we have $\mathbb{E}_t[\pi_{t+1}, y_{t+1}]$, then we can define a new variable $\pi^*_t = g_t(\pi_t, y_t)$, and define the problem over $(\pi^*_t, y_t)$ where $y_t = Y_t(\pi^*_t, \mathbb{E}_t[\pi^*_{t+1}])$, where $Y_t$ solves $Y_t(\pi^*_t, \mathbb{E}_t[\pi^*_{t+1}]) = F_t(g_t^{-1}(\pi^*_t | \pi^*_t, \mathbb{E}_t[\pi^*_{t+1}]), \mathbb{E}_t[\pi^*_{t+1}]).$\label{fn:3} A key concern of this Phillips curve relationship is a Lucas critique—firms’ price-setting behavior may change in response to changes in the monetary policy regime, such as target changes (L’Huillier and Schoenle, 2019). Our Phillips curve relationship is robust to a Lucas critique provided that expected future (next period) inflation is sufficient for determining how changes in future policies affect firm behavior. For example, higher expected inflation may lead firms to increase the frequency with which they update prices, altering the slope of the Phillips curve.\label{fn:4} Persson and Tabellini (1993) also consider a general social welfare function where shocks enter that change the welfare consequences inflation.}
changes.

2.1 Benchmark: Full-Information Ramsey Allocation

We begin by providing a benchmark allocation for efficiency. In particular, we characterize the efficient allocation that arises when: (i) the central bank has full commitment (Ramsey problem); and (ii) firms have full information, i.e., they observe the shock at date $t$. Given full information, firms’ posterior beliefs are the degenerate distribution which places all mass on $\theta_t$, which we denote by $\mu_t = \theta_t$, abusing notation slightly. We refer to this allocation as the full-information Ramsey allocation. It provides an efficiency benchmark that respects the Phillips curve relationship between inflation and output determined by firms.$^{15}$

Proposition 1. (Full-Information Ramsey Allocation) The full-information Ramsey allocation is characterized by

$$
\frac{\partial U_t}{\partial \pi_t} = v_{t-1},
$$

(3)

where $v_{t-1} = -\frac{1}{\beta} \frac{\partial U_{t-1}}{\partial E_{t-1}(\pi | \theta_{t-1})}$ for $t \geq 1$ and $v_{-1} = 0$.

The optimality condition for inflation at date $t$ equates the marginal utility from inflation $\frac{\partial U_t}{\partial \pi_t}$, with the marginal (dis)utility from the effect of inflation on the previous period’s output, summarized by $v_{t-1}$. The left-hand side (LHS) of equation (3) is date $t$ adapted, whereas the right-hand side (RHS) is date $t-1$ adapted. Therefore, the RHS is constant from the perspective of time $t$, implying that the marginal (flow) utility from inflation is constant at date $t$ in histories $\theta_t$ proceeding from the same history $\theta_{t-1}$. This means that $v_{t-1}$ is a date $t-1$ adapted constant.

The wedge $v_{t-1}$ is a sufficient statistic for the shock history $\theta^{t-1}$ in determining the Ramsey allocation rule $\pi_t, \pi_{t+1}, ...$ for inflation. In other words, the Ramsey allocation from dates $t$ and onward can be calculated with the knowledge of the wedge $v_{t-1}$, without knowing the exact shock history $\theta^{t-1}$ that gave rise to it. Note that since the economy starts at $t = 0$, then $v_{-1} = 0$.

It is helpful to contrast the full-information commitment (Ramsey) allocation of Proposition 1 with the full-information discretion (Markov) policy. Under discretion, the central bank finds it optimal to set $\frac{\partial U_t}{\partial \pi_t} = 0$ state by state. In particular at date $t$, the central bank neglects the impact of inflation on the previous period’s Phillips curve, which no longer serves as a constraint of the problem. This results in inflationary bias and reflects a standard Barro and Gordon (1983) time consistency problem. $v_{t-1}$ is precisely the wedge between the full-information Ramsey and Markov allocations. It reflects the severity of the central bank’s time consistency problem. We therefore refer to $v_{t-1}$ as the inflationary bias of the central bank at time $t$. In the presence of persistent shocks, this inflationary bias is potentially time-varying.

$^{15}$ This allocation is constrained efficient in the sense that it must still respect the Phillips curve relationship between inflation and output.
The inflationary bias of the central bank (discrepancy between Ramsey and Markov solutions) motivates studying how the government could design a mechanism to control the inflation setting decisions of the central bank. Such a mechanism can potentially motivate the central bank to adopt the Ramsey outcome, but has to account for the asymmetric information problem stemming from the central bank’s persistent private information. We now turn to studying this mechanism design problem.

2.2 Mechanism Structure

Our framework is a principal-agent problem in which the central bank privately observes the state of the economy \( \theta_t \) and then sets inflation under discretion. Because \( \theta_t \) is private information and the central bank has a time consistency problem, the government (principal) designs a mechanism to control the decision making process of the central bank (agent). The mechanism the government establishes can specify transfers (or punishments) \( T_t \) based on inflation policy.\(^{16}\) Although explicit monetary transfers are one interpretation, the practical analogs of the control mechanism \( T_t \) may be closer to policies such as Congressional scrutiny, reputational risk, or firing (not reappointing) the central banker.\(^ {17}\) For example, a central bank that is awarded high \( T_t \) may face a low degree of Congressional scrutiny in its policy determination.

The lifetime preferences of the central bank over social welfare and transfers are given by

\[
E \sum_{t=0}^{\infty} \beta^t \left[ U_t(\pi_t, E[\pi_{t+1}|\mu_t, \theta_t]) + T_t \right].
\]

Our main focus will be on characterizing a mechanism that implements the full-information Ramsey allocation. This mechanism will also be optimal in the limiting case where the costs to the government of providing incentives are negligible relative to the underlying social welfare problem. In Section 6, we study the case where transfers are not neutral from the perspective of the government.

The mechanism requires the central bank to make a report of the observed shock at date \( t \). We denote the reported type \( \tilde{\theta}_t \) and say that reporting is truthful when \( \tilde{\theta}_t = \theta_t \). We study direct and full-transparency mechanisms, under which the central bank truthfully reports its type each period.\(^ {18}\) Full transparency implies that there is no pooling of central bank types in reporting in a manner that shrouds the private information. Along the equilibrium path, agents’ posterior will therefore be the degenerate distribution at the reported type, or \( \mu_t = \tilde{\theta}_t \). Note that we abuse

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\(^{16}\) This approach is in the tradition of a vast literature studying inflation targets in monetary policy games. Walsh (1995) studies a static setting and shows that the optimal transfer mechanism takes the shape of an inflation target.

\(^{17}\) In the U.S., for example, this process is multifaceted. The central bank Chair is directly held accountable by Congress in the form of bi-annual, as well as extraordinary, Congressional testimonies. The institutions adopted by modern central banks to allow for active monitoring by stakeholders and maintain accountability vary across countries but most are highly complex.

\(^{18}\) Once we restrict to full transparency, the Revelation Principle as usual allows us to focus on mechanisms where the central bank truthfully reports its type.
notation here because $\mu_t$ is a full distribution in general.\textsuperscript{19}

We denote by $\Theta^t$ the space of shock histories up to date $t$. A mechanism in our model is a mapping from the history of reported types into a transfer and allocation, given by $(\pi_t, T_t) : \Theta^t \rightarrow \mathbb{R}^2$. Although the date $t$ allocations depend on the entire history of reported types, we will show state space reduction results that allow us to characterize sufficient statistics for information histories.

### 2.3 Incentives, Time Consistency, and Information

At every date $t$, the central bank makes a report $\hat{\theta}_t$ of its true type $\theta_t$. We define the value function of the central bank under the mechanism $(\pi, T)$ by

$$W_t(\theta_t) = \max_{\hat{\theta}_t} T_t + U_t(\pi_t, \mathbb{E}_t[\pi_{t+1}|\hat{\theta}_t], \theta_t) + \beta \mathbb{E}_t \left[ W_{t+1}(\theta_t^t-1, \hat{\theta}_t, \theta_{t+1}) | \theta_t \right],$$

where $\theta_t^t-1$ is the history of reported types whereas $\theta_t$ is always the current true type. Note that $\pi_t$ and $T_t$ are functions of the current reported type and the history of reported types. The incentive constraint of a central bank at date $t$ with history $\theta_t^t-1$ is

$$U_t(\pi_t, \mathbb{E}_t[\pi_{t+1}|\theta_t], \theta_t) + T_t + \beta \mathbb{E}_t \left[ W_{t+1}(\theta_t^t+1) | \theta_t \right] \geq U_t(\pi_t^*, \mathbb{E}_t[\pi_{t+1}^*|\theta_t^*], \theta_t) + T_t^* + \beta \mathbb{E}_t \left[ W_{t+1}(\theta_t^t-1, \theta_t^*, \theta_{t+1}) | \theta_t \right]$$

(5)

for all $t$, $\theta_t$, and $\theta_t^*$, and where we denote $\pi_t^* = \pi_t(\theta_t^t-1, \theta_t^*)$ and so on. Equation (5) is the truthful reporting incentive constraint: at date $t$, a central bank should find it preferable to truthfully report its type $\theta_t$ as opposed to reporting any alternate type $\theta_t^* \in \Theta$. As usual, incentive compatibility is characterized using a one-shot deviation along a path of truthful reporting, which is why the continuation value includes the true continuation type. The global incentive constraint (5) is high-dimensional, as there is an incentive constraint for each $\theta_t^* \in \Theta$ and every history $\theta_t^t \in \Theta^t$. As usual, we will employ a first order approach to incentive compatibility in deriving results.\textsuperscript{20} The required envelope condition associated with global incentive compatibility—derived in the proof of our main result in Appendix A—is given by

$$\frac{\partial W_t(\theta_t)}{\partial \theta_t} = \frac{\partial U_t(\pi_t, \mathbb{E}_t[\pi_{t+1}|\theta_t], \theta_t)}{\partial \theta_t} + \beta \mathbb{E}_t \left[ W_{t+1}(\theta_t^t+1) \frac{\partial f(\theta_t^t+1|\theta_t)}{\partial \theta_t} | \theta_t \right]$$

(6)

where, for clarity, $\frac{\partial U_t(\pi_t, \mathbb{E}_t[\pi_{t+1}|\theta_t], \theta_t)}{\partial \theta_t}$ is the derivative of $U_t$ in the direct type $\theta_t$, but \textit{not} including the Phillips curve expectation (which is based on the reported type). The familiar integral incentive

\textsuperscript{19} Restricting attention to full transparency mechanisms is not without loss of generality. In principle, the government could want to pool central bank types to manipulate firms’ posterior beliefs. By considering mechanisms under which the central bank truthfully reveals its type, we assume away such motivations. Given that central bank transparency has become an increasingly prominent focal point over the last two decades, we view the full transparency benchmark as important and realistic (Powell, 2019).

\textsuperscript{20} We make use of techniques in Pavan et al. (2014) for a first order approach with persistent shocks.
constraint is obtained by integrating and iterating forward (see the proof of Proposition 14 for this formulation).

The global incentive constraint (5) and its envelope formulation (6) reveal three principal driving forces of the model. The first two are conventional forces. First, there is a standard Barro and Gordon (1983) time consistency problem, marked by the absence of any terms that capture the impact of inflation at date $t$ on the Phillips curve at date $t−1$.21

Second, there is an information rent from persistent private information (Pavan et al., 2014). The standard information rent, which arises even absent persistent shocks, is captured in the firm term of equation (6) seen from the second term on the RHS of equation (6). It captures the gain in welfare that central bank achieves from an increase in its type $θ_t$, while holding fixed its report. An incentive compatible allocation must maintain this information rent to ensure that a central bank with a higher type does not report a lower type. The second term captures the information rent associated with persistence: revealing $θ_t$ gives up private information about the distribution of future shocks, captured by the term $\frac{∂f(θ_t+1|θ_t)}{∂θ_t}$. If high $θ_t$ produces high $θ_t+1$ and high continuation values $W_{t+1}$ due to persistence, then the information rent earned by $θ_t$ is higher because the central bank knows it will receive high continuation values even without changing its report. This means that incentive compatibility requires awarding the central bank more for reporting higher values $\tilde{θ}_t$ today. If shocks are not persistent, then the entire second term on the RHS of equation (6) drops out and the information rent is fully captured in the current period.

The third and novel force in our model is that the central bank has an incentive to manipulate firm beliefs. Firms form inflation expectations, which appear in the Phillips curve, based on their beliefs about next period’s shock distribution. The central bank’s report today affects these beliefs, i.e., $E_t[\pi_{t+1} | \tilde{θ}_t]$. Global incentive compatibility (5) reflects that a change in reported type alters the central bank’s current flow utility indirectly by changing firms’ inflation expectations. In the conventional New Keynesian framework, an increase in expected inflation generally lowers current flow utility. The central bank therefore has an incentive to bias firm expectations downward in order to improve the contemporaneous inflation-output trade-off. The fact that expectations are formed based on the reported type also means that the central bank does not earn an information rent from this channel. Formally this is seen in the fact that the first information rent term on the RHS of equation (6) is only the direct derivative in the type, and does not include a term for inflation expectations.

### 3 Dynamic Inflation Target

In this section, we show the main result of our paper: A “dynamic inflation target” mechanism can implement the full-information Ramsey allocation when the target is set by the central bank one
period in advance. This mechanism overcomes the time consistency and informational problems we identified in Section 2.3.

Equation (3), along with its sufficient statistic implications, suggests a mechanism that uses the transfer rule $T_t$ to penalize inflation deviations from a target. An inflation target of this form seeks to correct the time consistency problem in the central bank’s inflation policies by incentivizing it to set inflation close to the target. In the presence of persistent structural shocks, however, the target itself might need to be adjusted over time to accommodate a changing efficient level of inflation. That is, the optimal inflation rate may drift far from the central bank’s target in a persistent manner, implying large potential gains from letting the target adjust. The commitment-flexibility trade-off that motivated the inflation target in the first place may itself be subject to structural change. Indeed, this is precisely reflected in the time variation of the full-information Ramsey allocation in the presence of persistent $\theta_t$ shocks.

3.1 Inflation Targets as Dynamic Mechanisms

We look over a class of mechanisms defined by the affine transfer rule

$$T_t = -b_{t-1}(\pi_t - \tau_{t-1}).$$

We say that $\tau_{t-1}$ is the level of the transfer, and $b_{t-1}$ is the slope of the punishment for increasing inflation.\footnote{The structure of our target mechanism and, in particular, its linearity are similar to Walsh (1995) in a static setting.} This class of mechanisms specifies affine transfers at date $t$ based on inflation at date $t$, where the affine function parameters $(b_{t-1}, \tau_{t-1})$ are determined at date $t - 1$.

We define in particular a class of mechanisms with affine transfer rules under which the level is expected next-period inflation.

**Definition 2** (Dynamic Inflation Target). A dynamic inflation target is an affine transfer rule mechanism whose target level equals expected inflation, $\tau_{t-1} = \mathbb{E}_{t-1}[\pi_t | \theta_{t-1}]$, and whose target flexibility is the slope $b_{t-1}$.

Under our proposed dynamic inflation target, two things happen when the central bank reports its type $\theta_t$ at date $t$. First, its type report maps into a contemporaneous inflation policy $\pi_t$, which in turn generates a transfer $T_t$ based on the target parameters $(b_{t-1}, \tau_{t-1})$ specified in the previous period. The mechanism establishes a target in the sense that $\tau_{t-1} = \mathbb{E}_{t-1}[\pi_t | \theta_{t-1}]$; that is, the level of the mechanism is always equal to expected inflation. Second, the report also maps into target parameters $(b_t, \tau_t)$ for the transfer rule in the next period, i.e., the new target. In sum, the mechanism is a mapping $(\pi_t, b_t, \tau_t) : \Theta^t \rightarrow \mathbb{R}^3$ from the history of reported types into inflation for the current period and the target for the next period. In this sense, we can also think of the central
bank as directly choosing inflation and its own future target, represented by \((\pi_t, b_t, \tau_t)\), from among the set of triples that follow from the same history \(\theta^{t-1}\) of reported types prior to date \(t\).

The target level \(\tau_{t-1}\) and target flexibility \(b_{t-1}\) capture two distinct facets of the inflation target mechanism. The target level is the level of inflation the central bank is expected to hit on average. The target flexibility characterizes how severe the punishment is when the central bank exceeds its inflation target. An increase in target level means that the central bank incurs lower penalties for higher average inflation. An increase in target flexibility means the central bank incurs lower penalties for higher marginal inflation.

Our main result is that this dynamic inflation target implements the full-information Ramsey allocation in a locally incentive compatible mechanism. Moreover, it admits a key state space reduction property.

**Proposition 3. (Dynamic Inflation Target Implements Efficient Allocation)** A dynamic inflation target implements the full-information Ramsey allocation in a locally incentive compatible mechanism, with target flexibility \(b_{t-1} = \nu_{t-1}\). The target \((\tau_{t-1}, b_{t-1})\) is a sufficient statistic at date \(t\) for the history \(\theta^{t-1}\) of past types.

Proposition 3 shows that the full-information Ramsey allocation can be implemented by a simple dynamic inflation target. Inflation always meets the target level in expectation, that is \(\tau_{t-1} = \mathbb{E}_{t-1} \pi_t\), while the target flexibility is set to the inflationary bias, \(b_{t-1} = \nu_{t-1}\). The inflation target prescribed by our mechanism is dynamic in the sense that both its level and flexibility are time-varying.

Intuitively, the mechanism serves two roles: It uses the inherited target from the prior period to correct the time consistency problem in the central bank’s contemporaneous inflation choice, and it provides incentives for correctly updating the target for the next period. The form of the inflation target follows the well-known logic from the static setting (Walsh, 1995). Since \(\nu_{t-1}\) is the central bank’s inflationary bias, the mechanism provides the correct incentives for the inflation choice by assigning a penalty \(b_{t-1} = \nu_{t-1}\) for raising inflation. This means the target’s flexibility is used as the means of correcting inflationary bias that arises from the fact that firm inflation expectations affect contemporaneous output.

In the presence of persistent shocks, the inflation target must be updated to accommodate persistent changes in the full-information Ramsey allocation. Proposition 3 yields two key insights. First, the central bank optimally resets its target one period in advance. That is, when the central bank observes a persistent shift in the efficient inflation level, it adjusts its inflation target for the next period in response to this shift. The current target, on the other hand, remains in effect for the current period and governs contemporaneous inflation policy. Second, both the target level \(\tau_{t-1}\) and the target flexibility \(b_{t-1}\) are subject to change when the target is updated.

Dynamic target adjustments under our mechanism are best understood in relation to the underlying frictions discussed in Section 2.3. Consider first a change in the target flexibility. When
the central bank updates $b_t$ in period $t$—to go into effect and govern inflation policy in period $t+1$—it internalizes that expectations about future inflation affect output today via the Phillips curve. In other words, even though the central bank takes the behavior of its future self as given under discretion, it understands that the target it sets in period $t$ will constrain the inflation policy of its future self in period $t+1$. The central bank consequently internalizes its future time consistency problem and corrects it by setting the appropriate penalty, $b_t = \nu_t$, for its future self—one period in advance.23

Our mechanism uses changes in the target level $\tau_t$ to overcome the core informational frictions of our model, in particular the central bank’s incentive to manipulate firm beliefs in the presence of persistent private information. While it is surprising that a simple dynamic inflation target is able to account for these complex effects, the affine transfer rule of our mechanism is designed so that the two information forces exactly offset each other. To illustrate our result, consider the effect of a perturbation in inflation expectations on the central bank’s lifetime value. The two relevant terms in the central bank’s Bellman equation are

$$U_t(\pi_t, E_t[\pi_{t+1} | \tilde{\theta}_t], \theta_t) + \beta v_t E_t[\pi_{t+1} | \tilde{\theta}_t]$$

where the third line follows from Proposition 1. In economic terms, the central bank wishes to bias downward the inflation expectations of firms in order to economize on the Phillips curve relationship and improve the contemporaneous inflation-output trade-off. By setting next period’s target level to also equal inflation expectations, i.e., $\tau_t = E_t \pi_{t+1}$, the government provides the central bank with a distinct incentive to bias upward inflation expectations: increasing expected inflation raises the target level and so reduces average future penalties for high inflation. The marginal benefit of this upward bias is equal to the target flexibility, $v_t$. But under the full-information Ramsey allocation, the target flexibility is precisely equal to the inflationary bias. Thus these two forces exactly offset each other.

This property is an informational divine coincidence that arises in implementing the Ramsey outcome of this model. The informational divine coincidence arises because firms and the government

23 This is also similar to the static setting, where the central bank is willing “ex ante” to set up a targeting mechanism for itself. It is also closely related to the literature on optimal mechanisms to control present bias (e.g. Amador et al. 2006), where agents are willing to set up mechanisms to control their own time consistency problems.
have the same information set in every period, that is firms learn from the mechanism in exactly the same way as governments do. This information structure is crucial to the ability for a dynamic inflation target to be able to implement the Ramsey allocation. We revisit this in Section 3.3, where we study the impact that different information sets for firms and government has on the structure of the mechanism.

Finally, a key point of tractability for our mechanism is that the current target \((\nu_{t-1}, \tau_{t-1})\) is a sufficient statistic for the entire history \(\theta^{t-1}\) of shock realizations. This means that our mechanism admits a recursive formulation where the date \(t\) state variables are the inherited target, \((\nu_{t-1}, \tau_{t-1})\), and the current state, \(\theta_t\). This sufficient statistic property follows precisely because the target flexibility \(\nu_{t-1}\) summarizes the inflationary bias from the previous period, while the target level \(\tau_{t-1}\) summarizes a form of promised utility to the central bank for truthfully revealing its persistent type. This property greatly reduces the knowledge required for the central bank to adjust its target: the central bank only needs to know its current target, and not the history under which that target arose.

3.2 Evolution of the Target

The characterization of the target in Proposition 3 can be used to understand the evolution of the target. Combining the first order condition of the constrained efficient allocation (3) with the definition of the updated target \(\nu_t\), and making the natural assumption that \(\frac{\partial F_t}{\partial \pi_t} + 1 > 0\ local to the constrained efficient allocation, we obtain a law of motion,

\[
\nu_t = \delta_t \left( \nu_{t-1} - \frac{\partial U_t}{\partial \pi_t} \right)
\]

where the derivative \(\frac{\partial U_t}{\partial \pi_t}\) holds output fixed, and where \(\delta_t = -\frac{\partial y_t}{\partial y_t} / \partial \pi_t + \beta \frac{\partial y_t}{\partial \pi_t}\) is the relative impacts of inflation expectations and current inflation on output. \(\delta_t = 1\) is the standard NKPC, while \(\delta_t < 1\) implies contemporaneous inflation has a larger effect on output than inflation expectations. \(\delta_t < 1\) implies contemporaneous inflation has a larger effect on output than inflation expectations.

Consider first the case where \(\delta_t = 1\), as in the conventional NKPC. In this case, the target flexibility evolution is \(\nu_t = \nu_{t-1} - \frac{\partial U_t}{\partial \pi_t}\). Intuitively, the mechanism starts from the flexibility afforded in the current period, and then adjusts \(\nu_t\) upward as the central bank incurs greater disutility from inflation today, \(\frac{\partial U_t}{\partial \pi_t}\). Intuitively, the central bank achieving greater disutility from inflation today means the central bank stimulated output, exacerbating the time consistency problem tomorrow and requiring a less flexible target.

\(\delta_t\) scales the target adjustment by the relative impacts of current and future inflation on output. If \(\delta_t < 1\), current inflation has a larger impact on output than future inflation, meaning that the time consistency problem is less severe. This leads the target to become more flexible over time, in the sense that \(\nu_t\) is a decaying process. By contrast if \(\delta_t > 1\), future inflation has a larger impact on output.
output, the time consistency problem is more severe, and the target tends to become less flexible over time.

Finally, we can consider how the target intercept changes in response to a structural shock. Considering a marginal increase in the structural shock, we have

\[
\frac{d\tau_{t-1}}{d\theta_{t-1}} = \mathbb{E}_{t-1}\left[\frac{\partial f(\theta_t|\theta_{t-1})}{\partial \theta_{t-1}}\right] + \frac{\partial \nu_{t-1}}{\partial \theta_{t-1}} \mathbb{E}_{t-1}\left[\frac{\partial \tau_t}{\partial \nu_{t-1}} \theta_{t-1}\right].
\]

(9)

There are two effects of the structural shock on the target. The first effect, “expectations,” reflects that the probability measure over future states changes in response to the shock. If a higher \(\theta_t\) raises the probability of high-inflation states, then the target intercept \(\tau_{t-1}\) increases as well. The expectations effect therefore implies that, in response to persistent shocks, the target intercept can change even when the target slope remains constant. If shocks were fully transitory, on the other hand, the probability measure would not be affected and no adjustment in the target intercept would be required. The second effect, “target slope adjustment,” reflects the extent to which a change in the slope of the target impacts optimal future inflation. In the natural case where \(\frac{\partial \tau_t}{\partial \nu_{t-1}} < 0\), the upward adjustment of the target intercept is amplified (dampened) when the target slope is reduced (increased). In economic terms, if a structural shock leads to a decrease in the slope \(\nu_{t-1}\), then the central bank will find it optimal to generate higher average levels of inflation in the next period, since the penalty for exceeding the target has been reduced. Firms anticipate this, so that inflation expectations increase for a given state \(\theta_{t-1}\) and associated conditional density \(f\). As a result, the target level \(\tau_{t-1}\), which is set equal to firm inflation expectations, also increases.

In sum, when the economy experiences a structural shock \(\theta_t\), both components of the target may be affected. The intercept of the target is directly affected by changes in expectations, but is also affected indirectly if the structural shock leads to a change in target slope. The target slope updates as a result of the structural shock if it leads to a fundamental change in either the motivation of the central bank to generate excess inflation, or if it alters the nature of the time consistency problem.

### 3.3 The Importance of Information

An important observation is the informational divine coincidence that gives rise to the simple dynamic inflation target rests upon the government and firms having the same information set: neither observes \(\theta\) directly, and relies on the central bank to report it.

Consider the following illustrative example. Suppose instead that a fraction \(\gamma \in [0, 1]\) of firms are informed, and directly observe the state \(\theta\). This means that the reduced-form preferences of the central bank are now given by

\[
U_t\left(\tau_t, \gamma \mathbb{E}_t[\tau_{t+1}|\theta_t] + (1 - \gamma)\mathbb{E}_t[\tau_{t+1}|\tilde{\theta}_t], \theta_t\right).
\]
The benchmark Ramsey allocation of Proposition 1 is the same as before and defines the same values $v_{t-1}$. However, the dynamic inflation target defined above no longer implements the Ramsey allocation. To understand why, the envelope condition of the central bank can now be written as

$$\frac{\partial W_t(t')}{\partial \theta_t} = \frac{\partial U_t(\pi_t, E_t[\pi_{t+1}|\theta_t], \theta_t)}{\partial \theta_t} + \gamma \frac{\partial U_t(\pi_t, E_t[\pi_{t+1}|\theta_t], \theta_t)}{\partial \theta_t} E_t[\pi_{t+1} \frac{\partial f(\theta_{t+1}|\theta_t)}{\partial \theta_t} | \theta_t]$$

(10)

The first and third terms are the same information rents as before. However, the second term reflects an information rent that the central bank earns from informed firms. This information rent reflects how information about the type affects firms’ expectations of future inflation, in much the same manner as the continuation value.

Equation (10) is instructive in why a dynamic inflation target no longer implements the Ramsey allocation. In particular, suppose that we construct the value function $W_t(t')$ associated with the allocation and transfer rule under the dynamic inflation target. Since under this rule we have $\gamma E_t[\pi_{t+1}|\theta_t] + (1 - \gamma) E_t[\pi_{t+1}|\theta_t] = E_t[\pi_{t+1}|\theta_t]$, then as before our conjectured value function has

$$\frac{\partial W_t^{DIT}}{\partial \theta_t} = \frac{\partial U_t(\pi_t, E_t[\pi_{t+1}|\theta_t], \theta_t)}{\partial \theta_t} + \beta E_t\left[W_{t+1}(\theta^{t+1}) \frac{\partial f(\theta_{t+1}|\theta_t)}{\partial \theta_t} | \theta_t\right].$$

Thus, our DIT no longer satisfies the required envelope condition, and is not an incentive compatible mechanism. In particular, we can define the incentive compatibility wedge $\omega_t$ at date $t$ such that $\frac{\partial W_t^{DIT}}{\partial \theta_t}$ satisfies the required envelope condition at date $t$. We see that, substituting in the definition of $\omega_t$, this wedge is simply the extra information rent,

$$\omega_t = \gamma \frac{\partial U_t(\pi_t, E_t[\pi_{t+1}|\theta_t], \theta_t)}{\partial \theta_t} E_t[\pi_{t+1} \frac{\partial f(\theta_{t+1}|\theta_t)}{\partial \theta_t} | \theta_t]
- \frac{\gamma}{\beta} v_{t} \omega_t \Lambda(\theta_{t+1}|\theta_t)$$

where $\Lambda(\theta_{t+1}|\theta_t) = \frac{\partial f(\theta_{t+1}|\theta_t)}{\partial \theta_t} | \theta_t$ is the derivative of the likelihood ratio.

$\omega_t$ informs us about the behavior of the missing information rent that arises when trying to use the dynamic inflation target. In particular, it is the information rent that arises because a central bank with a higher type earns an information rent from informed firms’ knowledge of that type, which feeds into how they form expectations for future inflation. The information rent depends on how firms’ expectations of inflation covaries with the shock structure. If high types $\theta_t$ signal high future types $\theta_{t+1}$ (monotone likelihood) and high future types signal high inflation $(\pi_{t+1})$, this $\omega_t$ corresponds to a negative portion of the information rent that is unaccounted for by the dynamic inflation target. This means that the dynamic inflation target gives too much surplus to high $\theta$ types, motivating lower types to deviate upwards. In effect, target increases are too attractive in
this case under the dynamic inflation target, leading to excess upward revisions. Intuitively, the presence of informed firms, combined with the positive correlation pattern, implies that an increase in type $\theta$ means that informed firms revise their inflation expectations upwards. This upward revision reduces the information rent earned by the high $\theta$ type central bank. By contrast, the dynamic inflation target treats the upward revision as coming only upon the report of the central bank. Thus it provides the offsetting effect: a high $\theta$ type knows that by reporting upwards, it pushes up expectations of all firms.

$\omega_t$ informs the form an augmented dynamic inflation target would need to take to remain incentive compatible. In the case of positive correlation, it implies that a contemporaneous tax is needed in the transfer rule for reporting a higher type. This acts as a disciplining device on upward target adjustments that increase inflation expectations. The tax prevents lower types from misreporting and engaging in upwards adjustments by providing the required information rent. In effect, it reduces the value of upward adjustment of the target in recognition that the presence of informed firms reduces the central bank’s information rent.

These observations yield important insights on the design of central bank inflation targets. The immediate consequence is the presence of information heterogeneities among firms (informed and uniformed) gives a need for a penalized target adjustment process, where upwards target adjustments must be accompanied by additional penalties. These penalties play an intuitive role of ensuring that central banks that should be conducting low inflation, are not incentivized towards excessive upward adjustments. A more nuanced perspective is that this provides a complexity based argument for central banks to be responsible for collecting and disseminating information about the structural state of the economy to firms. When all firms are uninformed and learn from the central bank, the dynamic inflation target provides a simple implementation of the Ramsey allocation. By contrast when some or all firms are informed, a dynamic inflation target is only able to achieve the Ramsey allocation when a penalty process is enacted to control target adjustments.

### 3.4 Welfare Gains from a Dynamic Inflation Target

We now look to understand what the possible welfare gains might be from instituting a dynamic inflation target, relative to a permanent and static target. Suppose that instead of a dynamic target, the central bank instead adopted a static target $(\nu^*, \tau^*)$. For simplicity, assume full information. The following proposition describes the first order welfare gains (in terms of allocative efficiency) from moving from the static target to a dynamic inflation target.

**Proposition 4.** To first order, the welfare gains in allocative efficiency from moving from a static target $(\nu^*, \tau^*)$ to the dynamic inflation target $(\nu_{t-1}, \tau_{t-1})$ of Proposition 3 are

$$
\mathbb{E} \sum_{t=1}^{\infty} \beta^t \begin{bmatrix}
\nu^*_t - \nu^* & \mathbb{E}_{t-1} \tau^*_t - \tau_{t-1}
\end{bmatrix}
\begin{bmatrix}
\text{Cost of Excess Inflation} \\
\text{Amount of Excess Inflation}
\end{bmatrix}.
$$
The first order welfare gains available from moving to a dynamic inflation target depend on two forces. The first, $\nu^*_t - \nu^*$, is the intertemporal variation in the time consistency problem under the static target (where $\nu^*_t - 1$ is the time consistency wedge evaluated at the allocation obtained under the static target). When $\nu^*_t > \nu^*$, the time consistency problem is more severe than the slope imposed $\nu^*$, and hence inflation is too high relative to the efficient tradeoff. In other words, the first term reflects the cost of excess inflation. The second term, $E_{t-1} \pi^*_t - \tau_{t-1}$, is the difference between inflation expectations under the static target and inflation expectations under the dynamic target. High welfare gains are therefore available when a large excess time consistency problem, $\nu^*_t - \nu^*$, coincides with substantial excess inflation, $E_{t-1} \pi^*_t - \tau_{t-1}$, relative to the constrained efficient inflation level. The dynamic inflation target thus allows welfare gains not only by allowing for greater inflation when the static target would be too severe, but also by allowing for lower inflation when the static target would be too flexible.

4 Applications

This section presents five applications of the theory we have developed in Sections 2 and 3. Each application stresses in a different way the relevance of persistent private information in first-order questions about the design and conduct of monetary policy.

In our applications, we study special cases of a linearized New Keynesian model. The model consists of a standard New Keynesian Phillips curve, given by

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_t y_t, \quad (11)$$

and a dynamic IS equation, given by

$$y_t = E_t y_{t+1} - \frac{1}{\sigma} \left( i_t - E_t \pi_{t+1} - r^*_t + \rho_t \right), \quad (12)$$

where $\pi_t$ and $y_t$ denote inflation and the output gap, respectively. This formulation of the model potentially allows for time variation in several parameters. We think of $\{\kappa_t, r^*_t, \rho_t\}$ as exogenously specified stochastic processes, given some initial conditions $\{\kappa_0, r^*_0, \rho_0\}$. In particular, $\kappa_t$ is the slope of the Phillips curve, $r^*_t$ denotes the natural rate of interest, and $\rho_t$ captures a demand shock. Each of our applications will associate the shock $\theta_t$ we have studied in Sections 2 through 3 with one of these stochastic processes and turn off time variation in the others.

Equations (11) and (12) also take as given a process for the nominal interest rate $\{i_t\}$. In our applications, the nominal interest rate, which we consider the conventional instrument of monetary policy, will be determined as part of the dynamic inflation target mechanism. Finally, our applications will consider alternative loss functions $U_t(\pi_t, y_t, \theta_t)$, with differing interpretations of $\theta_t$. 
4.1 Lower Bound Spells

When the economy is at the effective (zero) lower bound—which we refer to as a “lower bound spell”—the central bank loses its conventional policy instrument (short-term interest rates). Historically, CBs have then resorted to unconventional policy instruments, focusing largely on forward guidance and asset purchases. Some commentators have explicitly raised the question whether changes in the targeting framework could and should be seen as a potential additional unconventional monetary policy instrument. Our theory provides a natural framework to ask this question. Crucially, we implicitly abstract from asset purchases: That is, we do not allow the CB to use any other unconventional tool that essentially allows it to make the ZLB slack again. We assume that instruments are incomplete to such an extent that the economy experiences a ZLB episode.

Zero lower bound spells are commonly represented by a constraint \( i_t \geq 0 \) in the literature (Eggertsson and Woodford, 2003; Werning, 2011). Consider a canonical loss function at a distorted steady state,

\[
U(\pi_t, y_t) = -\frac{1}{2} \pi_t^2 - \frac{1}{2} \alpha y_t^2 + \lambda y_t.
\]

When explicitly accounting for the zero lower bound constraint, \( i_t \geq 0 \), social welfare can be associated with the Lagrangian \( E \sum_{t=0}^{\infty} \beta^t [U(\pi_t, y_t) + \theta_t i_t] \).

The Lagrange multiplier \( \theta_t \) can be interpreted as the shadow value of being able to set negative nominal rates. In other words, when the economy falls into a liquidity trap, the shadow value on policies that push the economy away from the constraint rises—for example by raising inflation expectations, lowering current output, or raising future expected output.

In this application, we represent the mechanism design problem directly over the reduced-form loss function \( U_t(\pi_t, y_t) + \theta_t i_t \), which encodes the shadow value of being able to set negative rates directly into utility. A positive innovation to \( \theta_t \) qualitatively captures the same economics as an explicit lower bound spell: the shadow value of higher nominal interest rates or, as we show below, higher inflation expectations rises. We associate a lower bound spell that is expected to be persistent, for example due to a persistent negative demand shock, with a persistently high shadow value \( \theta_t \). This representation captures the economics of a lower bound spell precisely because the shadow value of being able to set negative interest rates rises when the economy enters a liquidity trap.

We assume that \( \theta_t \in [\theta, \bar{\theta}] \) follows a Markov process with \( E_t \theta_{t+1} = \rho \theta_t \) for \( 0 \leq \rho \leq 1 \). We associate \( \rho = 0 \) with a transitory liquidity trap, where the lower bound constraint is expected not to bind in the following period. In this application, we abstract from shocks to the slope of the Phillips curve, \( \kappa_t = \kappa \), innovations in the natural rate, \( r^*_t = r^* \), and demand shocks, \( \rho_t = 0 \). Substituting the NKPC (11) into the dynamic IS equation (12) then implies

\[
i_t = E_t \pi_{t+1} + r^* + \frac{\sigma}{\kappa} \left[ -\pi_t + (1 + \beta) E_t \pi_{t+1} - \beta E_t \pi_{t+2} \right].
\]

This means that, after substituting out for \( i_t \) and \( y_t \) in preferences \( U_t(\pi_t, y_t) + \theta_t i_t \), we can represent reduced-form preferences by \( U_t(\pi_t, E_t \pi_{t+1}, E_t \pi_{t+2}, \theta_t) \). Since \( E_t \pi_{t+2} \) appears in this implementability condition, the resulting time consistency problem has a horizon of more than one period. We
study longer-horizon time consistency problems in Section 5, where we revisit this application for general \( \sigma \neq 0 \). In this application, we set \( \sigma = 0 \) so that the time consistency problem reverts to a single period. We can then rewrite the reduced-form utility function as

\[
U_t(\pi_t, E_t \pi_{t+1}, \theta_t) = -\frac{1}{2} \pi_t^2 - \frac{1}{2} \lambda \left( \pi_t - \beta E_t \pi_{t+1} \right)^2 + \lambda \left( \pi_t - \beta E_t \pi_{t+1} \right) + \theta_t \left( E_t \pi_{t+1} + r^* \right)
\]

where \( \lambda = \frac{\sigma}{\kappa} \) and \( \hat{\lambda} = \frac{1}{\kappa} \).

**Proposition 5.** The dynamic inflation target that implements the full-information Ramsey allocation is

\[
v_t = \gamma_0 + \gamma_1 \theta_t + \gamma_2 v_{t-1}
\]

\[
E_t \pi_{t+1} = \gamma_0 + (\gamma_2 - 1) v_t + \left( \gamma_1 + \frac{1}{\beta} \right) \rho \theta_t
\]

where \( \gamma_0 = \frac{\lambda_1 \gamma_2}{1 - \beta \gamma_2} > 0 \), where \( \gamma_1 = \frac{\lambda_2}{1 - \gamma_2 \beta \rho} \left[ \rho - \frac{1 + \lambda}{\lambda} \right] < 0 \), and where \( \gamma_2 = \frac{1 + \lambda (1 + \lambda) - \sqrt{(1 + \lambda (1 + \lambda))^2 - 4 \lambda^2 \beta}}{2 \lambda \beta} \)

with \( 0 < \gamma_2 < 1 \). Optimal inflation sets \( \pi_t = v_t - v_{t-1} + \frac{1}{\beta} \theta_t \).

Proposition 5 characterizes the dynamic inflation target of Proposition 3 when the economy experiences a lower bound spell.

To illustrate the economic forces that govern the dynamic inflation target mechanism, consider the following scenario: We initialize the economy at its risky steady state. Formally, we consider a particular realization of the stochastic process where \( \theta_t = 0 \) for sufficiently many periods such that the economy and the mechanism asymptotically converge. It is straightforward to see that the target flexibility converges to \( \theta_t \to \nu = \frac{\gamma_0}{\gamma_2} = \frac{1}{1 - \gamma_2 \beta} \kappa \lambda > 0 \) in this limit. In the language of Svensson (1997), the distorted steady state \( \lambda > 0 \) implies that there is an *average inflationary bias*, which \( \nu > 0 \) corrects. Similarly, the target level converges to \( \tau_t = E_t \pi_{t+1} \to \tau = \gamma_0 + (\gamma_2 - 1) \nu = 0 \) in the risky steady state limit. This reflects a common Ramsey intuition: with a distorted steady state, the central bank achieves a better inflation-output trade-off today by promising lower inflation tomorrow, and subsequently achieves a better inflation-output-trade-off tomorrow by promising future lower inflation, and so on. This pushes optimal inflation under commitment towards zero in the long run, absent shock innovations. Formally, the allocation rule implies \( \pi_t = v_t - v_{t-1} + \frac{1}{\beta} \theta_t \to \nu - \nu = 0 \).

25 In both this application and the ones that follow, the proof shows that there are two linear solutions that satisfy the first order conditions of the optimum, and we take the non-explosive solution to remain consistent with the transversality condition.

26 We define the risky steady state of a dynamic stochastic general equilibrium model as comprising the allocation, prices, and multipliers that the model converges to if a shock sequence of \( \theta_t = 0 \) for all \( t \) is realized. This is distinct from the standard deterministic steady state definition because agents understand that the environment is stochastic. It is also distinct from the notion of the stochastic steady state, which describes the distribution that allocation, prices, and multipliers converge to (in probability / distribution) as the model is simulated for a sufficiently long period of time under the ergodic stochastic process \( \{ \theta_t \} \).
Figure 1. Impulse Responses: Lower Bound Spell

Note. Figure 1 plots the impulse responses of inflation and the dynamic inflation target after a lower bound shock $\theta_0 > 0$. Panels (A) through (D) show target flexibility, target level, inflation, and the shock, respectively. We target a quarterly calibration, staying as close as possible to Galí (2015), setting $\beta = 0.99$, $\alpha = 0.75$, and $\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}$. The blue solid line corresponds to a persistent shock ($\rho = 0.6$) and the red dashed line to a transitory shock ($\rho = 0$). In each case, we initialize the economy at the risky steady state and consider a shock at time 0.

Our dynamic inflation target implements the long-run Ramsey allocation in the risky steady state of this economy with a target level of $\tau = 0$ and a positive target flexibility $\nu > 0$ that exactly offsets the central bank’s time inconsistent incentive to respond to the steady state distortion.\(^{27}\)

We now initialize the economy at its risky steady state and consider a positive realization of the shock, $\theta_0 > 0$.\(^{28}\) Intuitively, we consider the economy as having entered a lower bound spell of uncertain duration at date 0. We plot the resulting impulse response functions (IRFs) under the dynamic inflation target mechanism in Figure 1.

Suppose first that the ZLB spell is purely transitory, and hence $E_0\theta_1 = 0$. We consider a realization of the shock path such that $\theta_t = 0$ for all $t \geq 1$. Panel (a) of Figure 1 plots the dynamics of the target under this path. Target flexibility $\nu_0$ features a negative innovation in response to $\theta_0 > 0$, since $\gamma_1 < 0$, that is the target becomes more flexible. Intuitively, the transitory lower bound spell increases the shadow value of future inflation, which depresses the real interest rate today and pushes the economy away from the liquidity trap. This policy motive pushes against the usual

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\(^{27}\) Similarly, we have $i_t \to r^*$ and $y_t \to 0$. The allocation in the risky steady state is therefore the same as in the deterministic steady state of this model. This follows from certainty equivalence under a first-order linearization.

\(^{28}\) Formally, given some initial conditions, consider some date $T$ large enough such that the economy has asymptotically converged to the risky steady state under some given convergence criterion. We renormalize this date $T$ to be our new period 0 for ease of exposition.
problem of inflationary bias and calls for a lower penalty on future inflation.

And even though the lower bound spell ends at date 1, the added target flexibility is persistent and decays only at the rate $\gamma_2$. This endogenous persistence captures the common intuition that optimal monetary policy in a liquidity trap makes long-lived promises to keep interest rates even after the economy moves away from the lower bound (Werning, 2011). Intuitively, promising high inflation at date 1 means that unless the central bank promises high inflation at date 2, the central bank will have to experience a significant output contraction at date 1. Hence the central bank smooths the output contraction by promising to maintain higher inflation. The increase in inflation expectations can be seen from the second equation: since $\gamma_1 < 0$ and $\gamma_2 < 1$, with a purely transitory shock an increase in $\theta_t$ raises inflation expectations by $\gamma_1 (1 - \gamma_2)$.

Suppose instead that there is a persistent shock $\theta_0$ that is expected to decay out at rate $\rho$. The target is more flexible over a longer period of time as the shock becomes more persistent. Intuitively, the lasting ZLB spell means the central bank expects a binding ZLB tomorrow, meaning there will be further value to increasing inflation expectations tomorrow, which filters back to today through the NKPC to avoid even deeper losses to output. Moreover, higher flexibility persists for longer as the shock decays along its path $\theta_t = \rho^t \theta_0$: not only does the greater flexibility persist directly, but it is enhanced because positive future shocks $\theta_t$ reinforce continued persistence. In general, the impact of persistent shocks on inflation expectations directly is more nuanced and combines two offsetting effects: higher inflation expectations because of greater flexibility, and lower inflation expectations because a persistent ZLB path creates value to a longer smooth path of high inflation, rather than a single period of high inflation.

Proposition 5 therefore demonstrates how a dynamic inflation target mechanism implements the well understood full-information commitment solution for optimal monetary policy in a liquidity trap but in a setting with persistent private information. In the language of Eggertsson and Woodford (2003), optimal monetary policy features history dependence and promises to overheat the economy beyond the duration of the lower bound spell. In summary, our result shows how to implement such a policy not only in the presence of persistent private information, but also relying only on one-period iterated commitments to a dynamic inflation target, rather than on a long-horizon commitment to the future path of interest rates.

A lower bound spell $\theta_0 > 0$ increases inflation expectations unambiguously if $\rho$ is sufficiently small. Intuitively, a transitory liquidity trap implies the shadow value of the constraint is high today but not tomorrow, promoting a more aggressive increase in expectations. By contrast if the shock is very persistent, a protracted lower bound spell makes for a long path of binding constraints.

4.2 $R^*$

A vibrant debate has recently emerged on the question of how monetary policy should respond to the perceived decline in the natural rate of interest (Laubach and Williams, 2016). In the presence of an Effective Lower Bound (ELB) on interest rates, a decline in the natural rate implies that
nominal interest rates are closer to the ELB on average and there is less room for central banks to use conventional monetary policy during recessions. Many observers in the U.S. have hence advocated for an increase in the Federal Reserve’s inflation target.\footnote{Concluding its longer-term strategic review in August 2020, the Fed adopted an average inflation target.}

In our second application, we study a persistent fall in the natural rate of interest in the stochastic steady state of an economy that might fall below the ELB with some probability in the future. We capture movements in the natural interest rate as movements in \(\theta_t = r_t^* - r^*\), that is deviations of the natural interest rate from a “long run” value. We assume that \(\theta_t \in [\underline{\theta}, \overline{\theta}]\) follows a Markov process with \(\mathbb{E}_t \theta_{t+1} = \rho \theta_t\) for \(0 \leq \rho \leq 1\), where high \(\rho\) indicates that movements in the natural interest rate are persistent. Finally, we set \(\kappa_t = \kappa\) and shut off shocks to the slope of the Phillips curve.

We model an effective lower bound as follows. At the beginning of each date \(t\), the central bank observes \(\theta_t\) and adopts a rule for monetary policy. After its monetary policy rule has been set for the period, an observable transitory demand shock \(\varrho_t \in [\underline{\varrho} + r^*, \overline{\varrho} + r^*]\) is realized, with \(\mathbb{E}_t \varrho_t = 0\). Defining \(\rho_t = \varrho_t - r^*\), so that \(\rho_t \in [\underline{\rho}, \overline{\rho}]\) and \(\mathbb{E}_t \rho_t = -r^*\), then we have

\[ i_t = \mathbb{E}_t \pi_{t+1} + \theta_t - \rho_t, \]

where as in Section 4.1 we have set \(\sigma = 0\) in the dynamic IS equation. We represent the effective lower bound as a penalty \(\lambda_0 - \lambda_1 i_t\) for negative realized nominal interest rates \(i_t \leq 0\), with \(\lambda_0, \lambda_1 \geq 0\). This means that the expected (dis)utility from the ELB is given by

\[ v(i_t^*) = -\int_{i_t^*}^{\overline{\varrho}} \left( \lambda_0 - \lambda_1 (i_t^* - \rho) \right) f(\rho) d\rho, \]

where \(i_t^* \equiv \mathbb{E}_t \pi_{t+1} + \theta_t\). We assume that \(\rho\) is uniformly distributed, \(f(\rho) = \frac{1}{\overline{\rho} - \underline{\rho}}\), so that we have

\[ v(i_t^*) = -\underline{\varrho} + \beta v_0 i_t^* - \frac{1}{2} \beta v_1 i_t^*^2, \]

for \(v_0 = \frac{1}{2\beta} \frac{\lambda_0 + 1}{\overline{\rho} - r^*}\) and \(v_1 = \frac{1}{2\beta} \frac{\lambda_1}{\overline{\rho} - r^*}\).\footnote{We further have \(\underline{\varrho} = \lambda_0 \frac{\overline{\rho}}{\overline{\rho} - r^*} + \frac{1}{2} \lambda_1 \frac{\overline{\rho}^2}{\overline{\rho} - r^*}\).} This means we have \(v'(i_t^*) = \beta(v_0 - v_1 i_t)\). Intuitively, the effective lower bound generates welfare gains from setting \(i_t^* > 0\): this creates distance from the ELB and makes it less likely that a demand shock will push nominal rates below zero. This has implications for the design of the dynamic inflation target, as we show below.

The social preference function is \(U_t(\pi_t, y_t, i_t^*) = -\frac{1}{2} \pi_t^2 - \frac{1}{2} \alpha y_t^2 + v(i_t^*)\), reflecting both the loss from inflation and output gaps, and the benefit to the central bank from creating distance from the ELB and reducing the probability that a demand shock places the central bank at the ELB. Given
this setup, we can represent the reduced form utility function as
\[
U_t(\pi_t, \mathbb{E}_t \pi_{t+1}, \theta_t) = \frac{1}{2} \pi_t^2 - \frac{1}{2} \bar{\rho} \left( \pi_t - \beta \mathbb{E}_t \pi_{t+1} \right)^2 + v(\mathbb{E}_t \pi_{t+1} + \theta_t)
\]
for $\bar{\rho} = \frac{\alpha}{\sigma^2}$. We assume changes in the natural interest rate are persistent, with $\mathbb{E}_t \theta_{t+1} = \rho \theta_t$, and where $\rho = 0$ corresponds to transitory shocks. We obtain the following result.

**Proposition 6.** The dynamic inflation target that implements the full-information Ramsey allocation is

\[
v_t = \delta_0 + \delta_1 v_{t-1} + \delta_2 \theta_t
\]

where $\delta_0 = -\delta_1 \frac{1+\hat{\delta}(1-\bar{\beta})}{\alpha(1-\beta\rho)} v_0 < 0$, where $\delta_1 = \frac{1+\hat{\delta}(1-\bar{\beta})+\nu_1-\sqrt{(1+\hat{\delta}(1-\bar{\beta})+\nu_1)^2-4\hat{\delta}\bar{\beta}}}{2\hat{\delta}\bar{\beta}}$ with $0 \leq \delta_1 \leq 1$, where $\delta_2 = \frac{1+\hat{\delta}(1-\bar{\beta})}{\alpha(1-\beta\rho)} \nu_1 v_1 < v_1$, and where $\zeta = \frac{\hat{\delta}\bar{\beta}}{(\alpha\beta\nu_1)(1-\hat{\beta})-\hat{\delta}\bar{\beta}} > 0$. Under the dynamic inflation target, the central bank raises the target level $\tau_t$ and lowers target flexibility $v_t$ in response to a fall in the natural interest rate $\theta_t$. Inflation is given by

\[
\frac{1}{\zeta} \pi_t = v_t - \frac{\hat{\delta}\beta + \nu_1}{\alpha\beta} v_{t-1} + v_0 - v_1 \theta_t.
\]

**Proposition 6** characterizes the dynamic inflation target in the presence of a time-varying natural interest rate $\theta_t$.

To illustrate the economic forces at play, we again start by analyzing the risky steady state of the economy under the dynamic inflation target mechanism, which corresponds formally to the limit under a sufficiently long shock realization with $\theta_t = \theta_t = 0$. In this limit, the target level converges to

\[
\tau_t \to \tau = \zeta \left( v_0 + \delta_0 \right) + \zeta \left( \delta_1 - \frac{\hat{\delta}\beta + \nu_1}{\alpha\beta} \right) v > 0,
\]

which is positive. Unlike in Section 4.1, the risk of a lower bound spell in this economy leads to a positive steady state inflation target. It is well understood that proximity to an occasionally-binding effective lower bound provides an economic motive for a higher long-run inflation target level. **Proposition 6** formalizes this argument by showing that, in the presence of persistent private information, the dynamic inflation target indeed features a positive target level in the risky steady state. Formally, if we shut off the risk of hitting the occasionally-binding effective lower bound, i.e., take the limit as $\lambda_0, \lambda_1 \to 0$, we have $\tau \to 0$ as in Section 4.1.31

While the implications of the effective lower bound for the optimal inflation target level may be well appreciated, our qualitatively novel result is that optimal long-run target flexibility is also

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31 Even in this limit, we study an economy at a non-distorted steady state in this subsection, whereas we considered a distorted steady state in Section 4.1. While the time consistency problem that stems from this distorted affects target flexibility in the risky steady state, this discussion highlights that the target level is unaffected. The commitment Ramsey allocation features 0 long-run inflation for the same economic rationale as we discussed in Section 4.1.
affected by the risk of lower bound spells. In particular, in the risky steady state we have

\[ \nu_t \rightarrow \nu = \frac{1}{1 - \delta_t} \delta_0 = -\frac{\delta_1}{1 - \delta_t} \frac{1 + \alpha(1 - \beta)}{\alpha(1 - \beta \delta_t)} v_0 < 0. \]

A particularly interesting feature of this model is that the target flexibility is negative in the stochastic steady state. Although this may seem surprising, its intuition follows from the effective lower bound. In particular, if there is no effective lower bound then the divine coincidence holds in this model, which has a nondistorted steady state, and so \( \pi_t = y_t = 0 \) for all \( t \). But given then divine coincidence, than \( \nu_t = 0 \) for all \( t \) as well, that is commitment is not needed. Now, consider what happens from the presence of the effective lower bound. In this model with an effective lower bound, the central bank can still implement \( \pi_t = y_t = 0 \) by allowing the nominal interest rate to fall below zero is response to demand shocks. However, this generates a first order ELB cost and a second order inflation/output benefit. Hence, optimal policy pushes away from the divine coincidence by raising inflation expectations. The desire to raise inflation expectations implies there is a positive value at date \( t \) to future inflation, much like in a lower bound spell, and so the central bank increases target flexibility. Since in this case target flexibility was already at zero, as a result the central bank now adopts a negative slope that promotes inflation, rather than discouraging it. More generally if the central bank were at a distorted steady state as in the previous application, the distorted steady state would call for a positive slope while the ELB would call for a negative slope. In such a case, a high natural rate would lead the distorted steady state to dominate and give rise to a positive slope, while a low natural rate would cause the ELB to dominate and give rise to a negative slope.

Having characterized this economy’s risky steady state, we now study the dynamics of the inflation target mechanism. We initialize the economy at the risky steady state and consider a negative innovation, i.e., a fall, in the natural rate of interest \( \theta_0 < 0 \). In Figure 2, we plot the impulse responses of mechanism and allocations in response to this shock, considering both the case with a persistent (panel (a), \( \rho \rightarrow 1 \)) and transitory (panel (b), \( \rho \rightarrow 0 \)) shock. A decline in the natural interest rate leads to an increase in target flexibility, that is lower \( \nu_0 \). Intuitively, a lower interest rate pushes the central bank towards the ELB and promotes increasing inflation expectations. At the same time, a decline in the natural rate of interest also raises the target level. This captures that both of the parameters of the target adjust to accommodate higher inflation as the central bank approaches the ELB.

### 4.3 Flattening Phillips Curve

The changing slope of the U.S. Phillips curve has garnered much attention since the Great Recession. See for example Blanchard (2016), Gali and Gambetti (2019), Rubbo (2020), and Del Negro et al. (2020) among others. This debate on the recent flattening of the Phillips curve has also engulfed monetary policy discourse in recent years. See for example Brainard (2015). In this application,
we analyze the implications of a persistent flattening of the Phillips curve for the central bank’s dynamic inflation target. We associate $\theta_t$ with a persistent shock to the social benefit of stimulating output, which corresponds to time variation in the slope of the NKPC.

Social welfare is characterized by a New Keynesian loss function around a distorted steady state, $U_t(\pi_t, y_t, \theta_t) = -{1 \over 2} \pi_t^2 - {1 \over 2} \alpha y_t^2 + \theta_t y_t$. For tractability, we set $\alpha = 0$. In this case, internalizing the NKPC yields reduced-form utility

$$U_t(\pi_t, E_t^{\pi_{t+1}}, \theta_t) = -{1 \over 2} \pi_t^2 + {1 \over \theta_t} \left( \pi_t - \beta E_t^{\pi_{t+1}} \right)$$

Note that an increase in $\theta_t$ corresponds to an effective reduction in the slope $\kappa/\theta_t$ of the Phillips curve. We assume that $E_t^{\theta_{t+1}} = 1 - \rho + \rho \theta_t$ with $0 \leq \rho \leq 1$, reflecting reversion of the slope of the Phillips curve towards $\kappa$ over time.
Figure 3. Impulse Responses: Flattening Phillips Curve

Note. Figure 3 plots the impulse responses of inflation and the dynamic inflation target after a flattening of the Phillips curve. Panels (A) through (D) show target flexibility, target level, inflation, and the shock, respectively. We target a quarterly calibration, staying as close as possible to Galí (2015), setting $\beta = 0.99, \alpha = 0.75, \text{ and } \kappa = (1 - \alpha)(1 - \alpha \beta) / \alpha$. The blue solid line corresponds to a persistent shock ($\rho = 0.6$) and the red dashed line to a transitory shock ($\rho = 0$). In each case, we initialize the economy at the risky steady state and consider a shock at time 0.

Proposition 7. The dynamic inflation target that implements the full-information Ramsey allocation is

$$v_t = \frac{1}{\kappa / \theta_t}$$

$$\tau_t = (1 - \rho) \left( \frac{1}{\kappa} - \frac{1}{\kappa / \theta_t} \right)$$

Optimal inflation is $\pi_t = \frac{1}{\kappa / \theta_t} - \frac{1}{\kappa / \theta_{t-1}}$.

Proposition 7 characterizes the response of the dynamic inflation target to a persistent flattening of the Phillips curve. We again start our discussion of the economic forces by studying the risky steady state that obtains in this application after a sufficiently long realization of $\theta_t = 1$ (that is, a slope $\kappa / \theta = 1$). In this limit the slope of the Phillips curve is constant and given by $\kappa$. It is therefore easy to verify that $v_t \to v = \frac{1}{\kappa}$ and $\tau_t \to \tau = 0$. Intuitively, in the stochastic steady state there is a constant inflationary bias, $v = \frac{1}{\kappa}$, that arises from the distorted steady state. The distorted steady state with a constant Phillips curve slope implies that the marginal value of stimulating output today is equal to the marginal cost of reducing output yesterday, resulting in zero expected inflation.
being optimal, that is $\tau_t = 0$. It is only when the economy experiences a shock in the stochastic stead state that inflation and the target deviate.

Suppose that a positive shock $\theta_t$ hits, flattening the Phillips curve. Proposition 7 highlights that the temporary reduction in Phillips curve slope enhances the value of stimulating output, and leads to positive inflation. In conjunction, this means that the marginal cost of expected inflation has also risen, leading the target to become less flexible, that is $\nu_t$ increases. The higher cost of expected inflation causes the target level $\tau_t$ to decrease in proportion to the persistence.

Figure 3 plots an impulse response to a reduction in the slope of the NKPC (higher $\theta_t$, that is a fall in $\kappa/\theta_t$) which documents the pattern described. More persistent shocks means that the reduction in slope persists, and leads to a longer path of less flexible targets with lower levels. At the same time, it also means that the inflation response is less deflationary in the near term.

### 4.4 Revisiting Rogoff’s Inflation-Conservative Central Banker

In our final application for this section, we ask whether dynamic inflation targets can be implemented by inflation-conservative central bankers in the spirit of Rogoff (1985). In particular, our inflation-conservative central banker places a greater penalty on inflation than the government. After appropriate intertemporal rearrangement of terms, we represent this by assuming central bank preferences equal to

$$V_t = U_t - c(\pi_t - E_{t-1}\pi_t),$$

where as before $U_t$ denotes the preferences of society and the government, and where $c$ is the constant linear cost to the conservative central banker of inflation exceeding firm inflation expectations. We characterize the optimal mechanism with preference disagreement between government and central bank more generally in Section 6.2, of which the following proposition is a special case.

**Proposition 8.** Suppose an inflation-conservative central banker has flow utility $V_t = U_t - c(\pi_t - E_{t-1}\pi_t)$. The full-information Ramsey allocation can then be implemented by a dynamic inflation target with $b_{t-1} = \nu_{t-1} - c$ and $\tau_{t-1} = E_{t-1}\pi_t$.

Proposition 8 demonstrates that the appointment of an inflation-conservative central banker does not obviate the fundamental need for a dynamic inflation target. Intuitively, the inflation-conservative central banker applies a constant penalty to inflation, given by $c$. In the presence of persistent shocks, the target flexibility $\nu_t$ of the dynamic inflation target changes over time. While an inflation-conservative central bank raises target flexibility on average, in the sense that $b_{t-1} = \nu_{t-1} - c < \nu_{t-1}$, the total implied inflation penalty $b_{t-1} + c$ is $\nu_{t-1}$ just as before. The inflation target mechanism that implements the full-information Ramsey allocation is still time-varying and responds to persistent shocks.
In the language of Svensson (1997), however, appointing an inflation-conservative central banker can resolve average inflationary bias when \( c \) is set equal to the average value of \( \nu_t \) in the stochastic steady state. When this average penalty is large (e.g., in the presence of a distorted steady state) but time variation in \( \nu_t \) is small, approximating the dynamic inflation target with an inflation-conservative central bank may result in relatively small welfare losses.

Proposition 8 suggests that an alternative implementation of the dynamic inflation target might be to appoint new central bank chairs with appropriate inflation preferences in response to changes in \( \nu_t \). The inflation conservativeness of the central bank would then be time-varying and correspond to \( c_t = \nu_{t-1} \). If in response to a shock at date \( t - 1 \) the dynamic inflation target requires \( \nu_{t-1} > \nu_{t-2} \), then a more dovish central banker at date \( t - 1 \) should be replaced by a more hawkish central banker at \( t \). Just as the dynamic inflation target must be updated one period in advance, the appointment of a new central banker would also be announced one period in advance.\(^32\)

5 Duration and Persistence of Time Inconsistency

We have shown that a dynamic inflation target mechanism implements the constrained efficient allocation in an economy with persistent shocks and persistent private information. This result generalizes the benchmark result on static inflation target mechanisms that was the subject of a vast literature following Barro and Gordon (1983) and has shaped monetary policy discussion since then. To consider the implications of our result for monetary policy design in practice, a natural question emerges: How long is a period? In this section, we generalize our theory in the necessary dimensions to tackle this question. Our discussion will yield new insights on the duration and persistence of time inconsistency, which ought to shape the design of dynamic inflation target mechanisms in practice.

The time consistency problem that emerges in the baseline model of Section 2 has a duration of one period. That is, the forward-looking implementability condition that the planner faces—the Phillips curve in equation (1)—features one-period-ahead inflation expectations. Under discretion, the central bank consequently fails to internalize that policy decisions at time \( t \) affect inflation expectations formed at time \( t - 1 \)—this was the source of the time consistency problem in the baseline model. A dynamic inflation target that takes the form of repeated one-period commitments can therefore resolve this time consistency problem.

We extend the duration of the time consistency problem by assuming that output now depends on \( K \geq 1 \) periods of inflation expectations

\[
y_t = F_t(\pi_t, E_t[\pi_{t+1} | \tilde{\theta}_t], ..., E_{t+K}[\pi_{t+K} | \tilde{\theta}_t])
\]

Importantly, just as a fixed central bank under the optimal mechanism was tasked with updating its own target, in an implementation with time varying conservativeness a central banker would be tasked with appointing her own replacement one period in advance (or at the least, would be responsible for naming her successor). However, this institutional arrangement is not typical (if used at all) in practice. For example, in the U.S. the president is tasked with appointing members of the Board of Governors, who must then be confirmed by the Senate.
where $\mathbb{E}_{t}[\pi_{t+k} | \tilde{\theta}_t]$ is firms’ $k$ period ahead inflation expectation. The model is otherwise unchanged. This means that substituting in to social preferences $U_t(\pi_t, y_t, \theta_t)$, we have the lifetime social welfare of the government,

$$
\mathbb{E} \sum_{t=0}^{\infty} \beta^t U_t(\pi_t, \mathbb{E}_t[\pi_{t+1} | \tilde{\theta}_t], \ldots, \mathbb{E}_t[\pi_{t+K} | \tilde{\theta}_t], \theta_t),
$$

(16)

The case of $K = 1$ is that of the baseline model. Other than the change in reduced form preferences, all other aspects of the model are the same.

We again start by describing the full-information Ramsey allocation. As in the baseline model, this serves as an efficiency benchmark which we then seek to implement through a mechanism.

**Proposition 9.** The full-information Ramsey allocation of the government is characterized by

$$
\frac{\partial U_t}{\partial \pi_t} = \sum_{k=0}^{K-1} v_{t-1,k}
$$

where $v_{t-1,k} = -\frac{1}{\beta^{t-1-k}} \frac{\partial U_{t+k}}{\partial \mathbb{E}_{t+k}[\pi_{t+k} | \tilde{\theta}_{t+k}]}$ for $t-1-k \geq 0$, and where $v_{t-1,k} = 0$ for $t-1-k < 0$.

Proposition 9 provides a simple generalization of Proposition 1. Inflation at date $t$ affects not only flow utility at date $t-1$, but also flow utility at all dates backward to $t-K$, which is the earliest date at which expected date $t$ inflation appears in output determination. Thus as the duration of the time consistency problem increases in the sense that $K$ increases, the first order condition becomes increasingly backward looking. $v_{t-1,k}$ is defined analogously to $v_{t-1}$ in the baseline model for $k$ periods back. In particular, $v_{t-1,k}$ is a date $t-1-k$ adapted constant. In this environment, we can think of

$$
v_{t-1} \equiv \sum_{k=0}^{K-1} v_{t-1,k}
$$

as the total time consistency problem that needs to be corrected at date $t$ in order to implement the constrained efficient allocation.

### 5.1 Implementation with Dynamic Inflation Target

We now develop the main result of this section, showing that a variant of the dynamic inflation target implements the full-information Ramsey allocation. In particular, we consider affine transfer mechanisms as before. Recall that an affine transfer rule mechanism has form $T_t = -b_{t-1}(\pi_t - \tau_{t-1})$.

We define a $K$-horizon dynamic inflation target as follows.

**Definition 10** (K-horizon Dynamic Inflation Target). A $K$-horizon dynamic inflation target is an affine transfer rule mechanism whose target level equals a weighted average of the past $K$ inflation
forecasts,
\[ \tau_{t-1} = \sum_{k=0}^{K-1} \omega_{t-1,k} \mathbb{E}_{t-1-k}[\pi_t | \theta_{t-1-k}] , \]
for some weights \( \omega_{t-1,k} \), and whose target flexibility is the slope \( b_{t-1} \).

The K-horizon dynamic inflation target is the usual dynamic inflation target when \( K = 1 \). By contrast when \( K > 1 \), the target level \( \tau_{t-1} \) is based on the last \( K \) expectations for date \( t \) inflation. We are now ready to prove the following generalization of our main result.

**Proposition 11.** A K-horizon dynamic inflation target implements the full-information Ramsey allocation in a locally incentive compatible mechanism. The weights for the target level are \( \omega_{t-1,k} = \frac{v_{t-1,k}}{v_{t-1}} \), and the target flexibility is \( b_{t-1} = v_{t-1} \).

Proposition 11 gives us a key generalization of our main result. Intuitively, the target flexibility corrects the total time consistency problem \( v_{t-1} = \sum_{k=0}^{K-1} v_{t-1,k} \) that arises because expected date \( t \) inflation affects output determination at dates \( t - K \) through \( t - 1 \). In addition to the longer-horizon determination of the target flexibility, the target level is determined by \( K \) periods of inflation expectations for date \( t \), with weights \( \omega_{t-1,k} = \frac{v_{t-1,k}}{v_{t-1}} \). Intuitively, at date \( t - 1 - k \) the value to the central bank of manipulating firm beliefs about inflation expectations is \( v_{t-1,k} \), analogously to the intuition in the baseline model. This means the relative contribution to determination of the target level \( \tau_{t-1} \) is highest for inflation expectations in past dates \( t - 1 - k \) when the time consistency problem was relatively large, that is large \( v_{t-1,k} \). In sum, the average expectation is determined by the magnitude of the time consistency problem when that expectation is formed.

The argument for why the K-horizon dynamic inflation target implements the Ramsey allocation follows analogous steps. Critical to it is that the information effects still exactly cancel each other out. This is seen from the simple generalized version of the effect of a perturbation of the reported type, \( d\tilde{\theta} \), on expectations, which parallels the argument from before:

\[
\frac{\partial}{\partial \mathbb{E}_t[\pi_{t+k}|\tilde{\theta}_t]} \left[ U_t + \beta^k v_{t+k,k} \mathbb{E}_t[\pi_{t+k}|\tilde{\theta}_t] \right] \frac{\partial \mathbb{E}_t[\pi_{t+k}|\tilde{\theta}_t]}{\partial \tilde{\theta}_t} = \left( \frac{\partial U_t}{\partial \mathbb{E}_t[\pi_{t+k}|\tilde{\theta}_t]} + \beta^k v_{t+k,k} \right) \frac{\partial \mathbb{E}_t[\pi_{t+k}|\tilde{\theta}_t]}{\partial \tilde{\theta}_t}
\]

\[
= \left( -\beta^k v_{t+k,k} + \beta^k v_{t+k,k} \right) \frac{\partial \mathbb{E}_t[\pi_{t+k}|\tilde{\theta}_t]}{\partial \tilde{\theta}_t} = 0
\]

where the third line follows from Proposition 9. This reflects the same notion of informational divine coincidence as in the baseline model: the incentive to bias firm inflation expectations downward is
exactly offset by the incentive to bias government transfers upward at the equivalent horizon. It holds precisely because the slope \( v_{t+k,k} \) for future transfers is equal to the marginal cost through the Phillips curve under the Ramsey allocation.

**Sufficient Statics.** An important aspect of the \( K \)-horizon dynamic inflation target is that iterated one period commitments no longer suffice, that is the current target is no longer a sufficient statistic for the entire history of shocks. This is because the target parameters today depend on the past \( K \) periods, both in terms of the magnitude of past time consistency problems (appearing in the slope) and past inflation forecasts (appearing in the intercept). Nevertheless, the sufficient statistics notion has a natural generalization to this setting. In particular, the target evolution in this framework can be represented as two \( K \times 1 \) vectors, \( V \) and \( T \).

We use the vector \( V \) to keep track of evolution of the target slope over time. In particular, \( V_t(k) \) tracks what the target slope will be in \( k - 1 \) periods, so that \( V_t(1) = v_{t-1} \) is always the current period slope. At date \( t \), \( V_t(2) \) tracks the cumulative value of past time consistency problems \( v_{t,k} \) for the next period, that is \( V_t(2) = \sum_{k=1}^{K-1} v_{t,k} \). To this we need to add the current wedge \( v_{t,0} \) determined this period in order to get \( V_{t+1}(1) = V_t(2) + v_{t,0} = v_t \), which provides the correct next period slope. More generally, this implies that the vector \( V_t \) is updated according to,

\[
V_{t+1}(k) = V_t(k+1) + v_{t+k-1,k},
\]

where by convention \( V_t(K+1) = 0 \). We can similarly update the target level as a weighted combination of past expectations and the current expectation, in particular

\[
T_{t+1}(k) = \frac{V_t(k+1)}{V_t(k+1)+v_{t+k-1,k}} T_t(k+1) + \frac{v_{t+k-1,k}}{V_t(k+1)+v_{t+k-1,k}} E_t \pi_{t+k}.
\]

This captures the weighted average updating process.

To implement the \( K \)-period dynamic inflation target, the central bank must keep track of only the pair \((V, T)\) of \( K \times 1 \) vectors. Intuitively, these two vectors encode a notion of forward guidance in the form of partial commitments for what the central bank will do for the next \( K \) periods. At date \( t \), the central bank takes as given its target for the current date, \( \tau_{t-1} = T_t(1) \) and \( b_{t-1} = V_t(1) \), and lacks any ability to update this target. However, the central bank has partial ability to update its target for date \( t+1 \), taking as given its prior commitments that are encoded in \( V_t(2) \) and \( T_t(2) \). However, its updates are muted relative to the history: a slope \( v_{t,0} \) today only generates a percent revision \( \frac{v_{t,0}}{\sum_{k=1}^{K-1} v_{t,k}} \) to its target flexibility, and only assigns a weight \( \frac{v_{t,0}}{v_t} \) to the new inflation forecast in the target level. Intuitively, there are \( K - 1 \) past commitments that the current period must be weighed against. Thus a substantial update to the existing commitment is the consequence of either a large current slope \( v_{t,0} \) or a larger deviation in inflation expectations relative to the historical average.

In contrast, the central bank at date \( t \) has no prior commitment over inflation at date \( t + K \),
which is now appearing for the first time in output determination. Thus, its new commitment for date \( t + K \) is \((\nu_{t+K}, E_t[\pi_{t+K}])\). Nevertheless, the central bank expects this commitment to be updated potentially substantially over the next \( K \) periods, as new shocks affect the severity of the time consistency problem and the inflation expectation.

At any date \( t \), the central bank makes and updates commitments for periods \( t + 1, ..., t + K \), that is for the duration of the time consistency problem. However, the central bank makes no commitments for periods \( t + k \) for \( k > K \), whose inflation levels do not yet appear in current period output. This provides a generalized notion of iterated one period commitments of the baseline model: the central bank here makes iterated \( K \)-period partial commitments, with full commitment only provided for the next period.

### 5.2 Sluggish Target Adjustment: Duration and Persistence of Time Inconsistency

Facing an implementability condition of the form \( y_t = F_t(\pi_t, E_t[\pi_{t+1}], ..., E_{t+K}[\pi_{t+K}]) \), a (Ramsey) planner under commitment finds it valuable to make promises about inflation \( K \) periods into the future. Such promises affect output \( y_t \) contemporaneously through firms’ forward-looking pricing decisions. And this is valuable to the planner because it improves the contemporaneous inflation-output tradeoff: When there is a tradeoff between output and inflation stabilization in period \( t \) (away from the Divine Coincidence benchmark), then being able to backload inflation adjustments (for a given desired output level) into periods \( t + 1 \) through \( t + K \) smooths the cost of inflation across these periods.\(^{33}\)

These promises that the Ramsey planner finds it valuable to make are time inconsistent, in the sense that a planner waking up and reoptimizing in period \( t + s \) would have an incentive to reneg on them. In the context of our mechanism, the promises that implement the full-information Ramsey commitment allocation are enforced in a time consistent manner by confronting the discretionary central bank’s future selves with penalties. In the context of the generalized \( K \)-horizon Phillips curve, these penalties must align the central bank’s incentives with promises made up to \( K \) periods in the past.

The duration of time inconsistency is then naturally defined as the largest integer \( S \) such that expectation terms about future period \( t + S \) appear in forward-looking implementability conditions. In the case of the generalized Phillips curve we study in this section, the duration of time inconsistency is therefore \( K \). In terms of the economics, the duration tells us for how many

\(^{33}\) This intuition is true even in the standard linearized New Keynesian model, where the NKPC features only a single forward-looking expectation term. And we know that optimal policy under commitment in this benchmark is fully history-dependent: The planner ends up making promises for all dates into the future. But this is not because promises arbitrarily far into the future improve the contemporaneous inflation-output tradeoff the planner faces in period \( t \). Instead, the planner smooths the cost of inflation adjustments between periods \( t \) and \( t + 1 \) initially, which is possible due to firm expectations, but then finds it valuable to again smooth the promised inflation adjustment for period \( t + 1 \) across period \( t + 2 \), and so forth. This is why the Ramsey planner ends up making promises infinitely far into the future even though the NKPC features only a single expectation term. What’s different in the context of the generalized \( K \)-horizon Phillips curve is that promises \( K \) periods into the future directly improve the contemporaneous inflation-output tradeoff.
periods into the future the planner finds it valuable to make promises in order to improve the *contemporaneous* period-$t$ inflation-output tradeoff.

Now, not all of these promises are created equal. The planner will find it valuable to make stronger promises about future inflation in some periods and weaker promises for other periods. We introduce the notion of *persistence* of time inconsistency to capture this: Formally, the persistence of time inconsistency in period $t$ is a $K \times 1$ vector that captures the strength of the promises the planner finds it valuable to make for each of the following $K$ periods. Conversely, this corresponds precisely to the severity of the time consistency problem present in period $t + K$ for promises that were made $k$ periods in the past. In other words, the *persistence* captures the amount of promise that is made in period $t$ for period $t + k$.

It is easy to see now how persistence also maps directly into the size of the penalty that our mechanism must impose for period $t + k$ in order to align the incentives of the discretionary central bank for the promise that was made in period $t$. This is precisely $\nu_{t+k,k}$. We thus formally define the curve.

**Definition 12 (Commitment Curve).** The commitment curve at date $t$ is the curve $(k, \nu_{t+k,k})$ of commitments made at date $t$ for all $k \geq 1$.

The commitment curve defined in Definition 12 provides a natural method of thinking about the persistence of time inconsistency and commitment under the K-horizon dynamic inflation target. Intuitively, its shape conveys how long the horizon of commitments being made truly is.

Suppose first that we return to the simple case of Section 3. In this case, the commitment curve has a starkly downward shape: we have $(1, \nu_{t-1})$ and $(k, 0)$ for all $k > 1$. The commitment curve therefore completely phases out after the first point, meaning that no promises are being made at a horizon beyond one period ahead.

In the framework of the current section, to build intuition suppose instead that the commitment curve is flat for $K$ periods: $\nu_{t+k,k} = \nu_t$ for $k = 1, \ldots, K$ and $\nu_{t+k,k} = 0$ for $k > K$. In contrast to the case of low persistence ($\nu_{t,k} = 0$ for $k > 1$), a flat time consistency curve means the central bank makes equal sized commitments for each of $t+1, \ldots, t+K$ at date $t$. This affects the innovations in the target. When the time consistency curve is steeply downward sloping, past commitments for date $t+1$ are small relative to the commitment that will be made today, meaning that the weight assigned to inflation expectations today has a relatively larger impact on the target level, while the flexibility is principally driven by the short end of the time consistency curve. By contrast when the commitment curve is flat, cumulative past commitments are large relative to the current commitment. This means that long-horizon inflation expectations formed in the past have a greater influence on the target level for $t+1$ relative to current short-horizon inflation expectations for the next period.

These two new notions will determine the dynamic properties of our mechanism in structural models. And therefore, these two concepts are critical to answer the policy-relevant question of “How long is a period?”
In this section we develop our main policy application, leveraging the framework we introduced in Sections 5.1 and 5.2. Under the dynamic inflation target mechanism of Section 3, the planner delegates to the central bank the authority to change its own target—as long as it does so one period in advance. In order to make our framework suitable for policy analysis, the natural question then becomes *how long is a period*: In practice, at what horizon should central banks pre-commit to changes in their dynamic inflation targets?

In the standard model, the New Keynesian Phillips curve (11) with one expectation term, $E_t \pi_{t+1}$, emerges when linearizing around a 0-inflation steady state. The non-linear pricing equation of the Calvo model, on the other hand, features an infinite series of forward-looking expectation terms (Galí, 2015). In fact, even linearizing the standard model around a steady state with non-zero or trend inflation yields a generalized Phillips curve of the form (15). Planning problems with implementability conditions of this form place us squarely in the theoretical framework we have thus far developed in Section 5.1, allowing us to investigate the duration and persistence of time inconsistency and, consequently, of dynamic inflation target mechanisms.  

We develop our main policy application in the context of a generalized New Keynesian Phillips curve (GNKPC) that emerges when linearizing the standard Calvo model around positive steady state or trend inflation, denoted $\gamma = 1 + \pi$ (Ascari, 2004; Ascari and Sbordone, 2014).

Following closely Ascari and Ropele (2007), we study a linearized New Keynesian model that comprises a standard dynamic IS equation of the form (12), with $EIS_{\sigma} = 1$, and a GNKPC, together given by

$$y_t = E_t y_{t+1} - (i_t - E_t \pi_{t+1})$$

$$\pi_t = \kappa y_t + (\beta \gamma + \tilde{\beta})E_t \pi_{t+1} + \tilde{\beta} E_t \left[ \sum_{s=1}^{\infty} \delta^s \pi_{t+1+s} \right].$$

where $\tilde{\beta} = (\gamma - 1) \beta (1 - \alpha \gamma^{-1}) (1 - e^{\gamma})^{-1}$, $\tilde{\delta} = \alpha \beta \gamma^{-1}$, and $\kappa = \frac{1 - \alpha \gamma^{-1}}{\alpha \gamma^{-1}}$ denotes the slope of the GNKPC and captures the contemporaneous inflation-output sensitivity. We denote by $\alpha$ the Calvo parameter, so that $(1 - \alpha)$ is the probability that a firm can reset its price each period, and by $\frac{\epsilon - 1}{\epsilon}$ the elasticity of substitution between intermediate inputs. Note that in the case with no trend

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34 Many other prominent models of nominal rigidities yield pricing equations of the form (15). Starting with Fischer (1977) and Taylor (1980), multi-period staggered wage and price contracts have become a popular model of nominal rigidities in forward-looking rational expectations models. An influential paper in this tradition is Chari et al. (2000). More generally, the non-linear pricing equation that emerges from time-dependent rational expectations models of nominal rigidities (Calvo, 1983) features an infinite series of forward-looking expectation terms. It is only when linearizing around a 0-inflation steady state that these terms collapse and give rise to the standard NKPC. More recently, Werning (2022) studies the pass-through of inflation expectations and considers Phillips curve with generalized beliefs that also take a form similar to (15). Generalized K-horizon Phillips curves are therefore an important benchmark in the literature.

35 In fact, Ascari and Sbordone (2014) argue that “the conduct of monetary policy should be analyzed by appropriately accounting for the positive trend inflation targeted by policymakers.”
inflation, \( \gamma = 1 \), we recover the standard New Keynesian Phillips curve (11).\(^{36}\) Unlike in previous sections, we now denote by \( \pi_t \) and \( y_t \) the percent deviations from a deterministic steady state with trend inflation \( \gamma \).

Given a preference function \( U(\pi_t, y_t, \theta_t) \), we can characterize the following shape of the commitment curve associated with the GNKPC above.

**Proposition 13.** The commitment curve has a quasi-hyperbolic shape, that is

\[
v_{t+k,k} = \beta^* \delta^*(k-1)v_{t+1,1}
\]

where \( \beta^* = \frac{\hat{\beta}}{\beta + \hat{\gamma}} \) and \( \delta^* = \frac{\delta}{\beta} \).

Proposition 13 reveals that the commitment curve in the GNKPC model has a shape associated with quasi-hyperbolic discounting (Laibson 1997). The quasi-hyperbolic shape means that there is a particularly large drop in the commitment curve between \( k = 1 \) and \( k = 2 \), where the drop is the quasi-hyperbolic parameter \( \beta^* \). After the first point, the shape of the commitment curve is simple exponential discounting, with \( v_{t+k+1,k+1} = \delta^* v_{t+k,k} \). Thus after the large initial drop, commitments phase out more gradually.

The quasi-hyperbolic shape of the commitment curve reveals an interesting connection of the evolving process of commitments in the case of a canonical form of time-inconsistent preferences. In particular, it implies that the promise made at date \( t \) for date \( t + 1 \) receives a relatively higher weight, by a factor \( \beta^* \), than promises made at date \( t - k \) for date \( t \). This reflects that making commitments exhibits an efficient “dynamic inconsistency”: much as in quasi-hyperbolic models excessive weight is inefficiently placed on current flow utility, in the GNKPC a quasi-hyperbolic weight is efficiently placed on current commitments for the next period. Proposition 13 further illuminates the shape of the commitment curve under the standard NKPC. Under the standard NKPC, we have \( \hat{\beta} = 1 \) and hence \( \beta^* = 0 \). This corresponds to an extreme case of quasi-hyperbolic discounting, where only the first point on the commitment curve is nonzero.

Proposition 13 illustrates a simple manner of thinking about the important policy question of “how long is a period.” In particular, it embeds an important trade-off. The first is the quasi-hyperbolic discount factor \( \beta^* \) leads to a more sharply declining commitment curve. This means that the current period always plays a particularly important role in determining the optimal

---

\(^{36}\) Ascari and Ropele (2007) represent the GNKPC in terms of an auxiliary variable

\[
\pi_t = \kappa y_t + \beta \gamma \mathbb{E}_t \pi_{t+1} + (\gamma - 1)\beta (1 - \alpha \gamma^{e-1}) \mathbb{E}_t \left[ (\epsilon - 1) \pi_{t+1} + \phi_{t+1} \right]
\]

\[
\phi_t = \alpha \beta \gamma^{e-1} \mathbb{E}_t \left[ (\epsilon - 1) \pi_{t+1} + \phi_{t+1} \right]
\]

where we have already set the EIS to \( \sigma = 1 \). Defining \( \hat{\beta} \) and \( \hat{\delta} \) as above, then dividing through the second equation by \( e - 1 \), defining \( \phi_t = \frac{1}{e-1} \), and solving forward we have \( \phi_t = \sum_{s=1}^{\infty} \delta^s \pi_{t+s} \). Substituting into the first equation and reallocating terms, we get \( \pi_t = \kappa y_t + (\beta \gamma + \hat{\beta}) \mathbb{E}_t \pi_{t+1} + \hat{\beta} \mathbb{E}_t \sum_{s=1}^{\infty} \delta^s \pi_{t+s} \).
commitment for the next period, giving a role for shorter-horizon commitments that happen on a period-by-period basis. On the other hand, the exponential part of the curve $\delta^*$ calls means that longer-horizon commitments can also be important when $\beta^*$ is not too low and $\delta^*$ is high enough. This means that the history of past commitments is important to determining the current commitment, and gives a role for longer-horizon adjustment processes.

One manner of visualizing this trade-off is to ask what fraction of current commitment comes from periods $k \geq k^*$ in the past, for $k^* > 1$. This precisely measures the importance of long-horizon commitments in determining the total commitment $v_{t-1}$ today. Formally, let us assume that we have entered a stochastic steady state where $v_{t+1,1} = v$. In this stochastic steady state, the total fraction of the commitment $v_{t-1}$ that has come from historical periods $k \geq k^* > 1$ periods, $V_{t-1,k^*}$, is given by

$$V_{t-1,k^*} = \frac{\beta^* \sum_{k=k^*}^{\infty} (\delta^*)^{k-1}}{1 + \beta^* \sum_{k=2}^{\infty} (\delta^*)^{k-1}} = \frac{\beta^* \frac{\delta^*}{1-\delta^*}}{1 + \beta^* \frac{\delta^*}{1-\delta^*}}.$$

### 6 Optimal Mechanisms

In Section 3, we showed that a dynamic inflation target can implement the full-information Ramsey allocation. In this section, we study optimal mechanisms when enforcing the mechanism is costly to the government, and when the government and central bank have different preferences. In our baseline model, the optimal allocation under our dynamic inflation target mechanism coincides (by construction) with the full-information Ramsey allocation. Although a dynamic inflation target is an implementable mechanism, it is not generally optimal. However, many of the important qualitative...
insights of the dynamic inflation target carry over to understanding the optimal mechanism. In Section 6.3, we show that costly enforcement has surprising implications for the applications we have studied so far.

6.1 Optimal Mechanisms with Costly Enforcement

We begin by maintaining preference alignment $U_t$ between the government and central bank, but introduce costly enforcement. We capture the social cost of implementing and enforcing a monetary policy mechanism by assuming that transfers are now costly to the government. Social preferences of the government are now

$$\max E \left[ \sum_{t=0}^{\infty} \beta^t \left( U_t(\pi_t, E_t[\pi_{t+1}|\tilde{\theta}_t], \theta_t) - \kappa T_t \right) \right],$$

(19)

where $\kappa \geq 0$ captures the importance of this social cost.\(^{37}\) In conjunction, we introduce the central bank participation constraint, given by

$$W_0 \geq 0.$$ Without loss of generality, we have normalized the outside option in the participation constraint to 0.\(^{38}\)

Recall that a mechanism is a mapping $(\pi_t, T_t) : \Theta_t \to \mathbb{R}^2$. A mechanism must be incentive compatible, as defined in Section 2.3. As in the baseline model, the solve for the optimal relaxed mechanism that enforces the envelope characterization of local incentive compatibility (6). The following result characterizes the allocation rule under the optimal mechanism in this environment.

**Proposition 14.** The solution to an optimal allocation rule of the relaxed problem is given by the first-order conditions

$$\frac{\partial U_t}{\partial \pi_t} - K \Gamma_t \frac{\partial U_t}{\partial \theta_t} \frac{\partial \pi_t}{\partial \theta_t} = \lambda_{t-1}$$

where $\lambda_{t-1} = -\frac{1}{\beta} \frac{\partial U_{t-1}}{\partial \pi_{t-1}} + K \Gamma_{t-1} \frac{1}{\beta} \frac{\partial^2 U_{t-1}}{\partial \theta_{t-1} \partial \pi_{t-1}}$ with $\lambda_{-1} = 0$, $K = \frac{\kappa}{1+\kappa}$, and where $\Gamma_t(\theta^t)$ is given by the recursive sequence

$$\Gamma_t(\theta^t) = \Gamma_{t-1}(\theta^{t-1})h^{-1}(\theta_t|\theta_{t-1})E_{t-1} \left[ \Lambda(s_t|\theta_{t-1}) \right]$$

where $h^{-1}(\theta_t|\theta_{t-1}) = \frac{1-F(\theta_t|\theta_{t-1})}{f(\theta_t|\theta_{t-1})}$ is the inverse hazard rate, $\Lambda(s_t|\theta_{t-1}) = \frac{\partial f(s_t|\theta_{t-1})/\partial \theta_{t-1}}{f(s_t|\theta_{t-1})}$ is the derivative of the likelihood ratio, and where $\Gamma_0(\theta^0) = h^{-1}(\theta_0)$.

If transfers were not costly ($\kappa = K = 0$), the optimal allocation would be the Ramsey commitment allocation, and the dynamic inflation target would be an optimal mechanism. However, there are

\(^{37}\)This corresponds to a standard (quasilinear) transferable utility model. As usual, $T_t$ may also correspond to non-quasilinear utilities, provided they are transferable in this form.

\(^{38}\)At the end of the proof of Proposition 14, we show that a dynamic inflation target is an optimal mechanism when there is instead an average participation constraint, $E W_0 \geq 0$.  

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two effects that arise from the fact that transfers are now costly.

The first can be seen from the LHS of the equation. If there were no time consistency problem, this equation would collapse to \( \frac{\partial U_t}{\partial \pi_t} - K_{\pi} \frac{\partial U_t}{\partial \theta_t \partial \pi_t} = 0 \). Intuitively, this reflects a standard trade-off between allocative efficiency and information rents. A social planner that sets \( \frac{\partial U_t}{\partial \pi_t} > 0 \) achieves an inefficient allocation from the perspective of the inflation-output tradeoff. However, choosing the efficient allocation \( \frac{\partial U_t}{\partial \pi_t} = 0 \) has the consequence of increasing the information rent earned by high \( \theta \) types, \( \frac{\partial U_t}{\partial \theta_t \partial \pi_t} \). If shocks are persistent, this information rent carries forward to future types in the normal manner, according to \( \Gamma_t \), which reflects persistence of a higher type. In a standard case of a multiplicative taste shock, \( U_t = \theta_t u_t \), the LHS collapses to a usual virtual value representation, \( (\theta_t - K_{\pi}) \frac{\partial u_t}{\partial \pi_t} \), while at date 0 the virtual value is the usual \( \theta_0 - h^{-1}(\theta_0) \).

The RHS side of the equation reflects the impact of time consistency in terms of both allocative efficiency and its interaction with information rents. The first term in \( \lambda_{t-1} \) is the usual time consistency wedge, \( v_{t-1,t} \), required for allocative efficiency. This motivates a dynamic inflation target when \( K = 0 \). The second term represents the interaction of the time consistency problem and past information rents. Intuitively, offering a higher \( \pi_t \) to the central bank at date \( t \) not only affects the information rent \( \frac{\partial U_t}{\partial \theta_t \partial \pi_t} \) today, but also affects the information rent in the prior period through inflation expectations. If in general higher expected inflation lowers the information rent by worsening the prior period inflation-output trade-off, then this effect in fact calls for a higher inflation rate at date \( t \) than under allocative efficiency. Intuitively, the higher inflation rate improves planner welfare by lowering the central bank’s information rents in the prior period, even though it worsens social surplus.

Proposition 14 highlights the importance of shock persistence to the optimal mechanism with costly enforcement. If shocks were not persistent, then \( \Gamma_0 = \frac{1 - F(\theta_0)}{f(\theta_0)} \) but \( \Gamma_t = 0 \) for all \( t \geq 1 \), since \( \Lambda = 0 \) (past shocks convey no information about the current shock). This implies that the optimal allocation satisfies \( \frac{\partial u_t}{\partial \pi_t} = -\frac{1}{\beta} \frac{\partial u_{t-1}}{\partial \pi_{t-1}} \) for all \( t \geq 2 \). Thus the optimal mechanism reverts to a dynamic inflation target along any history for all date \( t \geq 2 \). In other words absent persistent shocks, all rents are extracted by distorting allocations at dates 0 and 1. Note that if additionally there is no time consistency problem, all rents are extracted at date 0 and the date 1 allocation is not distorted. Time consistency means the allocation at date 1 is distorted precisely because \( E_{01} \pi_1 \) affects the information rent earned by the central bank at date 0.

Revisiting the standard case of multiplicative taste shocks, \( \theta_t u_t \), then defining virtual value as \( \vartheta_t = \theta_t - K_{\pi} \), the optimal allocation rule collapses to

\[
\vartheta_t \frac{\partial u_t}{\partial \pi_t} = \vartheta_{t-1} - \frac{1}{\beta} \frac{\partial u_{t-1}}{\partial E_{t-1} \pi_t}.
\]

The allocation rule in this case mirrors that of the Ramsey allocation, but uses the virtual value \( \vartheta \) in place of the true type \( \theta_t \). This tells us that the direction of distortion relative to the Ramsey allocation depends on the relative distortion of the virtual value relative to the true type. In particular, the
central bank promotes more inflation on the margin when

\[ \frac{\theta_t}{\theta_{t-1}} > \frac{\theta_{t-1}}{\theta_{t-1}} \]

that is the virtual value of the central bank at date \( t \) is higher relative to the true type, than at date \( t - 1 \).

The above logic on the relationship between virtual values in general, but the virtual value is endogenous to the allocation rule more generally outside the multiplicative taste shock. We study this further in our applications below.

It should be noted that the Ramsey allocation is still implementable, but is no longer optimal. In particular, at the full-information Ramsey allocation, the informational effects still exactly offset one another, and a dynamic inflation target still implements that allocation. However, at the full-information Ramsey allocation the marginal benefit of higher inflation is zero, while the marginal cost of the enforcement mechanism is not. As a result, the government no longer finds it optimal to implement the full-information Ramsey allocation. This helps to understand why the optimal mechanism here deviates from a precise dynamic inflation target: once moved away from the full-information Ramsey allocation, the informational effect on firms is no longer exactly offset by the informational effect on government transfers.

Nevertheless, the properties of the optimum still bear similarities to the dynamic inflation target. In particular, the marginal impact of inflation on flow utility net of information rents today, \( \frac{\partial U_t}{\partial \pi_t} - K \Gamma_t \frac{\partial U_t}{\partial \theta_t} \partial \pi_t \), is equated with the marginal impact of inflation today on flow utility net of information rents the prior period, \( \lambda_{t-1} \). This historical impact is represented by the single statistic \( \lambda_{t-1} \). Thus, the history dependence of the mechanism can be encoded in the triple \( (\lambda_{t-1}, \Gamma_{t-1}, \theta_{t-1}) \). \( \lambda_{t-1} \) encodes the total time consistency problem, while \( (\Gamma_{t-1}, \theta_{t-1}) \) encodes the persistence of information rents (used to determine \( \Gamma_t \)). This triple is a sufficient statistic at date \( t \) for characterizing the allocation and transfer rule to implement the optimum of Proposition 14. In this respect, a key qualitative insight of the dynamic inflation target that carries over is that there is a simple sufficiently statistic, \( \lambda_{t-1} \), that carries forward in determining the influence of the time consistency problem on the evolution of the optimal mechanism. Unlike with the baseline model, however, this variable encapsulates not only the impact on allocative efficiency, but also the impact on past information rents.

Finally, we can use Proposition 14 to show that the optimal mechanism in fact reverts to a dynamic inflation target at both extremes of the shock distribution. In other words, there is both a no top distortion and a no bottom distortion result: if at period \( t \) we have \( \theta_t = \bar{\theta} \) or \( \theta_t = \underline{\theta} \), then \( \Gamma_t = 0 \), and hence \( \Gamma_{t+\delta} = 0 \ \forall \ t \geq s \).39 Along such histories, the optimal mechanism reverts to the dynamic inflation target.

39 See Pavan et al. (2014) for a related result on no distortion at the top or bottom.
Corollary 15. If $\theta_t \in \{\theta, \bar{\theta}\}$, then the optimal allocation at dates $t + 1 + s$ $(s \geq 0)$ can be implemented by a dynamic inflation target.

The no top distortion result closely resembles normal top distortion results in the absence of time consistency. It follows the standard logic: because there are no central bank types above $\bar{\theta}$, no types above $\bar{\theta}$ earn information rents from the allocation of type $\bar{\theta}$, meaning there is no reason to distort that allocation. In our model, the time consistency problem implies that the optimal allocation rule that is the full-information Ramsey allocation, rather than the rule under discretion $\partial U_t / \partial \pi_t = 0$. In addition, persistent private information implies there is also a no distortion at the bottom result. As a result, not only does the optimal mechanism have a component that resembles the dynamic inflation target throughout the shock distribution, but it reverts fully to the dynamic inflation target at the limits of the distribution.

6.2 Optimal Mechanisms with Preference Differences

We now allow for preference disagreement, that is the government has flow utility $V_t(\pi_t, E_t[\pi_{t+1} | \tilde{\theta}_t], \theta_t)$ whereas the central bank still has flow utility $U_t$. Maintaining costly enforcement, social preferences of the government are now

$$\max E \left[ \sum_{t=0}^{\infty} \beta^t \left( V_t(\pi_t, E_t[\pi_{t+1} | \tilde{\theta}_t], \theta_t) - \kappa T_t \right) \right],$$

where $\kappa \geq 0$ captures the importance of this social cost. As before there is a central bank participation constraint.

We define $K = \frac{k}{1 + k}$ as before, and define weighted reduced form preferences to be

$$Z_t = (1 - K)V_t + KU_t.$$

Weighted reduced form preferences average the preferences of the government and central bank. A higher weight is assigned to central bank preferences the more costly enforcement is, that is $K$ rises in $\kappa$. The optimal mechanism can be described as follows.

Proposition 16. The solution to an optimal allocation rule of the relaxed problem is given by the first-order conditions

$$\frac{\partial Z_t}{\partial \pi_t} - K \Gamma_t \frac{\partial U_t}{\partial \theta_t \partial \pi_t} = \lambda_{t-1}^*$$

where $\lambda_{t-1}^* = -\frac{1}{\beta} \frac{\partial Z_{t-1}}{\partial E_{t+1} \pi_t} + K \Gamma_{t-1} \frac{1}{\beta} \frac{\partial^2 U_{t-1}}{\partial \theta_t \partial E_{t+1} \pi_t}$ and $\Gamma_t$ is defined as in Proposition 14.

The optimal allocation rule of Proposition 16 is similar to that of Proposition 14, but with one important difference: the weighted preference $Z_t$ replaces the planner’s utility. Note that the
information rent terms still involve central bank preferences \( U_t \), as information rents accrue based on central bank preferences.

Suppose that \( K = 0 \), and hence enforcement is costless. In this case, we have \( Z_t = V_t \) and hence we obtain an optimal allocation \( \frac{\partial V_t}{\partial \pi_t} = -\frac{1}{\beta} \frac{\partial Z_t}{\partial E_{t-1}\pi_t} \). In this case, the government implements the Ramsey allocation from the government’s perspective, rather than the central bank’s perspective. This follows intuitively: the principal has different preferences than the agent but has a costless method of enforcing her preferred allocation. Hence, the principal does so. In this case, \( \Lambda_{t-1} \) encodes the combination of the impact of inflation today on the welfare of the principal in the prior period, and the impact of inflation today on the information rent earned by the central bank in the prior period.

Suppose in the other extreme than \( K = 1 \), that is transfers are tremendously costly to the principal relative to direct utility. This is equivalent to assuming that \( V_t = 0 \), that is the planner only values transfers. In this case, we have \( Z_t = U_t \) and the optimal allocation of Proposition 16 collapses to that of Proposition 14. Intuitively when the principal only cares about transfers, the principal on the one hand wants to make utility as high as possible to the agent in order to relax the central bank’s participation constraint and extract greater transfers. This means that the principal values the central bank’s time consistency problem in the sense that providing an allocation that accounts for the time consistency problem enhances central bank surplus and raises transfers. On the other hand, the principal also internalizes that higher promised utility increases agent information rents. This leads to the same allocation rule as in the case where principal and agent preferences are aligned except for transfers.

At intermediate values of \( K \), the optimal allocation rule trades off the two extremes. On the one hand, the planner wishes to push the allocation closer to her Ramsey allocation, which increases her direct utility from allocations. At the same time, the planner wishes to push the allocation closer to the central bank’s Ramsey allocation in order to relax the participation constraint and extract greater transfers. This leads to a balancing act determined by \( K \), which encodes a relative weight the principal assigns to the different motivations.

It is helpful to return to the multiplicative taste shock case. Suppose that \( U_t = \theta_t u_t \) and \( V_t = \theta_t v_t \). Then, we can write \( Z_t = \theta_t z_t \) for \( z_t = (1 - K)v_t + K u_t \). Hence, we recover an optimal rule,

\[
(1 - K) \theta_t \frac{\partial v_t}{\partial \pi_t} + K \theta_t \frac{\partial u_t}{\partial \pi_t} = (1 - K) \theta_{t-1} \frac{-1}{\beta} \frac{\partial v_{t-1}}{\partial E_{t-1}\pi_t} + K \theta_{t-1} \frac{-1}{\beta} \frac{\partial u_{t-1}}{\partial E_{t-1}\pi_t}.
\]

Thus in this case, the central bank implements the allocation rule that maximizes the weighted average of the efficient rule for government with true type \( \theta_t \), and the central bank with virtual value \( \theta_t \). In effect, this is a case where \( K \) serves as a Pareto weight on the central bank in determining the Ramsey allocation. A high virtual value \( \theta_t \) has two effects. The first is that it increases the relative weight placed on central bank preferences relative to government preferences in weighing the marginal benefit to inflation at date \( t \). The second is that it increases the importance of date \( t \).
flow utility relative to date $t-1$ flow utility, promoting higher inflation.

As in Corollary 15, following $\theta_t \in \{\theta, \bar{\theta}\}$ the optimal allocation reverts to the Ramsey allocation associated with weighted reduced-form preferences $Z_t$. If $K = 1$, then this allocation coincides with that of the dynamic inflation target.

6.3 Applications Revisited

It is instructive to revisit how the allocation rule changes in our applications to lower bound spells (Section 4.1), changing natural interest rates $r^*_t$ (Section 4.2), and changing slope of the Phillips curve (Section 4.3). We maintain the assumption of preference agreement but allow for costly enforcement of the inflation target mechanism (Section 6.1). In particular, we show that costly enforcement calls for less aggressive unconventional policies (e.g., forward guidance) when the economy experiences a lower bound spell, while it calls for more aggressive policies (e.g., raising the inflation target) in response to a decline in $r^*$.

**Lower bound spells.** In the case of lower bound spells (Section 4.1), reduced-form preferences satisfy

$$\frac{\partial U_t}{\partial \pi_t \theta_t} = 0 \quad \text{and} \quad \frac{\partial U_t}{\partial E_t \pi_t + 1 \partial \theta_t} = c_0 \text{ for a constant } c_0 > 0.$$ 

This reflects that high $\theta_t > 0$ corresponds to a binding lower bound and thus makes it valuable to promise more future inflation. However, because $\theta_t$ reflects a benefit of increasing the nominal rate and increasing inflation $\pi_t$ does not directly increase the nominal rate, changes in the allocation rule $\pi_t$ does not generate an information rent for the central bank at date $t$. In other words, this means that the virtual value at date $t$ is not distorted relative to the underlying type, whereas the virtual value at date $t-1$ is raised from the perspective of date $t$ by the backward looking information rent. Interestingly relative to the multiplicative taste shock case, this leads to a virtual value for a given date that changes from the perspective of different dates’ allocations.

With costly enforcement, the allocation rule under the optimal mechanism is given by

$$\frac{\partial U_t}{\partial \pi_t} = v_{t-1} + K \Gamma_{t-1} c_0,$$

where the RHS is $\lambda_{t-1}$. Consider the case where lower bound spells are persistent and higher current types signal higher future types. This implies $\Gamma_{t-1} > 0$, so that the marginal value of contemporaneous inflation is higher with costly enforcement, all else equal. Intuitively, in the lower bound spell application higher inflation expectations increase past information rents through $\theta_{t-1} E_{t-1} \pi_t$. This leads the planner to prefer a less aggressive policy for promoting future inflation.

To build intuition for this result, suppose the economy is at the lower bound with $\theta_0 > 0$. Without any mechanism, i.e., discretion, the central bank wants to revert to divine coincidence the moment we exit the lower bound. This leads to the desirability of using a dynamic inflation target to mitigate the time consistency problem. However, $\frac{\partial^2 U_t}{\partial E_t \pi_t + 1 \partial \theta_t} > 0$ tells us that an increase in inflation expectations is more valuable for central banks that are in a more binding lower bound
spell, and hence increases the information rent earned by these central banks. Thus, moving towards the Ramsey allocation has a social cost in the form of added costly transfers. As a result, the government weighs the benefits of allocative efficiency against the costs of enforcement, and chooses marginal utility from inflation that is higher than under Ramsey. This means that the central bank does not go all the way to the Ramsey inflation level, but rather wishes to lie at a distorted level where the marginal benefit of current inflation exceeds the marginal cost of current inflation on past output. In sum, costly enforcement implies the planner wants less aggressive unconventional policies to stimulate inflation expectations.

Declining $r^*$. In the case of changes in the natural rate $\theta_t = r_t^*$ (Section 4.2), reduced-form preferences satisfy $\frac{\partial U_t}{\partial \pi_t \partial \theta_t} = 0$ and $\frac{\partial U_t}{\partial E_t \pi_t \partial \theta_t} = -c_1$ for a constant $c_1 > 0$. High $\theta_t$ correspond to being further from the effective lower bound and imply a lower value to promising future inflation.

With costly enforcement, the allocation rule under the optimal mechanism is given by

$$\frac{\partial U_t}{\partial \pi_t} = \nu_{t-1} - K \Gamma_{t-1} c_1,$$

where again the RHS is $\lambda_{t-1}$. Consider again the case of persistent shocks, and assume high current types signal high future types and so $\Gamma_{t-1} > 0$. The marginal value of contemporaneous inflation is then lower with costly enforcement, all else equal. Intuitively, higher inflation expectations reduce past information rents by pushing higher inflation on central banks that were previously further from the effective lower bound. This leads the planner to prefer a more aggressive policy for promoting future inflation. Intuitively, in this case higher inflation expectations have less benefit for high $\theta$ types, i.e., central banks that are far away from the effective lower bound. Thus the planner reduces information rents by promoting inflation.

These results surprisingly highlight a contrast in the implications of costly enforcement for the optimal mechanism in these two applications. When the economy experiences a lower bound spell, costly enforcement calls for less aggressive unconventional policies to raise inflation expectations. Intuitively, less aggressive policies reduce information rents for central banks that have experienced zero lower bound spells in the past. When the economy is some distance away from the lower bound but experiences a persistent decline in $r_t^*$, costly enforcement calls for more aggressive policies to generate future inflation. Intuitively, more aggressive policies reduce information rents for central banks that are farther from the effective lower bound.

Flattening Phillips curve. In the case of a flattening Phillips curve (Section 4.3), reduced-form preferences satisfy $\frac{\partial U_t}{\partial \pi_t \partial \theta_t} = \frac{1}{k}$ and $\frac{\partial U_t}{\partial E_t \pi_{t+1} \partial \theta_t} = -\frac{\beta}{k}$. This reflects that high $\theta_t$ corresponds to a flattening Phillips curve, which increases the value of current inflation but reduces the value of generating future inflation and higher inflation expectations.
With costly enforcement, the allocation rule under the optimal mechanism is given by

$$\frac{\partial U_t}{\partial \pi_t} = \nu_{t-1} + \frac{K}{\kappa} \Delta \Gamma_t,$$

where again the RHS is $\lambda_{t-1}$ and $\Delta \Gamma_t = \Gamma_t - \Gamma_{t-1}$. In this case, the distortion to the optimal allocation rule, relative to the Ramsey condition, trades off two competing effects. On the one hand, high $\theta_t$ (flattening PC) means that the central bank’s value of stimulating output rises, promoting higher current inflation. This increases information rents to the central bank and calls for lower inflation. On the other hand, high inflation also increases past inflation expectations, which reduces information rents to past central banks and calls for higher inflation (similarly to the $r^*$ application). The relative magnitude of the two effects is determined by $\Delta \Gamma_t$, that is the change in the persistent portion of the information rent earned by the central bank between the two dates. From Proposition 14, we can write

$$\Delta \Gamma_t = \Gamma_{t-1} \left( h(\theta_t|\theta_{t-1}) \mathbb{E}_t \left[ \Lambda(s_t|\theta_{t-1}) \left| s_t \geq \theta_t \right. \right] - 1 \right).$$

where recall that $h^{-1}(\theta_t|\theta_{t-1}) = \frac{1-F(\theta_t|\theta_{t-1})}{f(\theta_t|\theta_{t-1})}$ is the inverse hazard rate and $\Lambda(s_t|\theta_{t-1}) = \frac{\partial f(s_t|\theta_{t-1})/\partial \theta_{t-1}}{f(\theta_t|\theta_{t-1})}$ is the derivative of the likelihood ratio. We know that the expected likelihood ratio derivative is zero at $\theta_t = \theta$ while we know that the inverse hazard rate is zero at $\theta_t = \bar{\theta}$. Thus local to the two extremes of the shock distribution, we have $\Delta \Gamma_t < 0$ and hence the optimal mechanism promotes higher inflation. Interestingly, this suggests a tendency in this environment for the backward looking information rent to dominate the contemporaneous information rent, and hence generate a tendency to promote higher inflation to generate lower past information rents (at the expense of promoting higher current information rents).

In the interior, two common assumptions are a monotone decreasing inverse hazard rate and a monotone increasing likelihood ratio (higher past types signal high future types). These have competing effects on the response to a flattening Phillips curve (high $\theta_t$). Intuitively, a lower inverse hazard rate reduces current virtual surplus whereas a higher likelihood ratio increases virtual surplus.

### 7 Conclusion

We provide a theory of how a central bank should update its inflation target in the presence of persistent economic shocks that are private information of the central bank. We show that a dynamic inflation targeting mechanism can implement the Ramsey allocation. The dynamic inflation target corrects for not only the central bank’s time consistency problem, but also its strategic incentives to reveal information to firms about the persistent underlying state. The target level and flexibility both are adjusted over time, with the key property that they are updated in advance. Our framework introduces the commitment curve, which summarizes the size of commitments the central bank
makes for the future and helps inform the persistence of commitment to the current target. Our results suggest that a mechanism of adjustment at restricted points in time, for example every five years as with the Bank of Canada, could be a desirable adjustment method.
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A Proofs

A.1 Proof of Proposition 1

Under full information, the objective function of the government is

$$\sup_{\pi_t} E_0 \sum_{t=0}^{\infty} \beta^t U_t (\pi_t, E_t [\pi_{t+1} | \theta_t], \theta_t).$$

Taking the FOC in $\pi_t$, we have

$$0 = \beta^{t-1} \frac{\partial U_{t-1}}{\partial E_{t-1} \pi_t} \frac{\partial E_{t-1} \pi_t}{\partial \pi_t (\theta^t)} f(\theta^{t-1}) + \beta^t \frac{\partial U_t}{\partial \pi_t} f(\theta^t)$$

From here, we have $\frac{\partial E_{t-1} \pi_t}{\partial \pi_t (\theta^t)} = f(\theta_t | \theta_{t-1})$, so that we have

$$0 = \frac{\partial U_{t-1}}{\partial E_{t-1} \pi_t} + \beta \frac{\partial U_t}{\partial \pi_t}$$

from which the result follows.

A.2 Proof of Proposition 3

The proof strategy is as follows. First, we derive the relevant envelope condition associated with local incentive compatibility, which defines necessary conditions on the value function associated with an incentive compatible mechanism.\footnote{\footnotemark} We then show that the value function generated by our proposed mechanism satisfies this envelope condition.

**Envelope condition.** Suppose that the central bank has a history $\tilde{\theta}^{t-1}$ of reports and a history $\theta^t$ of true types at date $t$. Given a mechanism with transfer rule $T_t(\tilde{\theta}_t)$ and allocation rule $\pi_t(\tilde{\theta}_t)$, the value function of a central bank that has truthfully reported in the past, assuming truthful reporting in the future, is given by

$$\mathcal{W}_t(\theta^t) = \max_{\tilde{\theta}_t} T_t + U_t (\pi_t, E_t [\pi_{t+1} | \tilde{\theta}_t], \theta_t) + \beta E_t \left[ \mathcal{W}_{t+1}(\theta^t, \tilde{\theta}_t, \theta_{t+1}) \right] \theta_t$$

Notice that the Phillips curve expectation $E_t [\pi_{t+1} | \tilde{\theta}_t]$ is based on the date $t$ reported type, not the date $t$ true type. Furthermore, notice that $\mathcal{W}_{t+1}$ depends on the reported type $\tilde{\theta}_t$, but not on the true type $\theta_t$. This is because flow utility at dates $t + s$ ($s \geq 0$) do not depend on past true types and because the shock structure is Markov. This implies that we can in fact write $\mathcal{W}_{t+1}(\theta^{t-1}, \tilde{\theta}_t, \theta_{t+1})$.

\footnotetext{This portion of the argument follows from the arguments in Farhi and Werning (2013), or more generally from Pavan et al. (2014), but we state it out for completeness and for clarity.}
As a result, the Envelope Condition in the true type $\theta_t$, evaluated at truthful reporting $\tilde{\theta}_t = \theta_t$, is

$$\frac{\partial \mathcal{W}_t(\theta^t)}{\partial \theta_t} = \frac{\partial U_t (\pi_t, \mathbb{E}_t [\pi_{t+1} | \theta_t], \theta_t)}{\partial \theta_t} + \beta \mathbb{E}_t \left[ \mathcal{W}_{t+1}(\theta^{t-1}, \tilde{\theta}_t, \theta_{t+1}) \right]$$

where we have

$$\frac{\partial \mathbb{E}_t \left[ \mathcal{W}_{t+1}(\theta^{t-1}, \tilde{\theta}_t, \theta_{t+1}) \right]}{\partial \theta_t} = \frac{\partial}{\partial \theta_t} \int_\mathcal{F} \mathcal{W}_{t+1}(\theta^{t-1}, \tilde{\theta}_t, \theta_{t+1}) f(\theta_{t+1} | \theta_t) d\theta_{t+1}
= \mathbb{E}_t \left[ \mathcal{W}_{t+1}(\theta^{t-1}, \tilde{\theta}_t, \theta_{t+1}) \frac{\partial f(\theta_{t+1} | \theta_t)}{\partial \theta_t} \bigg| \theta_t \right]$$

Substituting in and evaluating at truthful reporting, we obtain

$$\frac{\partial \mathcal{W}_t(\theta^t)}{\partial \theta_t} = \frac{\partial U_t (\pi_t, \mathbb{E}_t [\pi_{t+1} | \theta_t], \theta_t)}{\partial \theta_t} + \beta \mathbb{E}_t \left[ \mathcal{W}_{t+1}(\theta^{t-1}) \frac{\partial f(\theta_{t+1} | \theta_t)}{\partial \theta_t} \bigg| \theta_t \right]$$

which provides a conventional envelope condition for incentive compatibility. For clarity, note that $\frac{\partial U_t (\pi_t, \mathbb{E}_t [\pi_{t+1} | \theta_t], \theta_t)}{\partial \theta_t}$ is the derivative of $U_t$ in the direct type $\theta_t$, but not including the Phillips curve expectation, which is the derivative in the reported type.

**Verifying the envelope condition.** We now verify the value function under our mechanism satisfies the envelope condition. Our mechanism has a transfer rule $T_t(\theta^t) = -\nu_{t-1}(\theta^{t-1}) \left( \pi_t(\theta^t) - \mathbb{E}_{t-1}[\pi_t|\theta_{t-1}] \right)$ and an allocation rule given by the constrained efficient allocation of Proposition 1.

The value function associated with this mechanism is

$$\mathcal{W}_t(\theta^t) = -\nu_{t-1} \left( \pi_t - \mathbb{E}_{t-1}[\pi_t|\theta_{t-1}] \right) + U_t (\pi_t, \mathbb{E}_t [\pi_{t+1} | \theta_t], \theta_t) + \beta \mathbb{E}_t \left[ \mathcal{W}_{t+1}(\theta^{t+1}) \bigg| \theta_t \right]$$

where $\nu_{t-1}, \pi_t, \mathbb{E}_{t-1}[\pi_t|\theta_{t-1}]$ are the constrained efficient values associated with Proposition 1, given the realized shock history. From here, recall that $\nu_{t-1}$ and $\mathbb{E}_{t-1}[\pi_t|\theta_{t-1}]$ are only functions of $\theta^{t-1}$. Therefore, $\frac{\partial \nu_{t-1}}{\partial \theta_t} = \frac{\partial \mathbb{E}_{t-1}[\pi_t|\theta_{t-1}]}{\partial \theta_t} = 0$. Thus differentiating the value function in $\theta_t$, we have

$$\frac{\partial \mathcal{W}_t(\theta^t)}{\partial \theta_t} = \frac{\partial U_t (\pi_t, \mathbb{E}_t [\pi_{t+1} | \theta_t], \theta_t)}{\partial \theta_t} + \beta \mathbb{E}_t \left[ \mathcal{W}_{t+1}(\theta^{t+1}) \frac{\partial f(\theta_{t+1} | \theta_t)}{\partial \theta_t} \bigg| \theta_t \right]
- \nu_{t-1} \frac{\partial \pi_t}{\partial \theta_t} + \frac{\partial U_t (\pi_t, \mathbb{E}_t [\pi_{t+1} | \theta_t], \theta_t)}{\partial \theta_t} + \frac{\partial \mathbb{E}_t [\pi_{t+1} | \theta_t]}{\partial \theta_t} \bigg] + \beta \mathbb{E}_t \left[ \frac{\partial \mathcal{W}_{t+1}(\theta^{t+1})}{\partial \theta_t} \bigg| \theta_t \right]$$

The first line on the RHS are the terms associated with the envelope condition. The second line are derivatives that arise because in equilibrium, the reported type equals the true type, and we have
evaluated the value function given truthful reporting. It therefore remains to show that the second line sums to zero and hence our mechanism satisfies the required envelope condition.

It is helpful to write out the continuation value function $W_{t+1}$ in sequence notation. Iterating forward, we obtain

$$W_{t+1}(\theta^{t+1}) = -v_t \left( \pi_{t+1} - E_t[\pi_{t+1}|\theta_t] \right)$$

$$- E_{t+1} \left[ \sum_{s=1}^{\infty} \beta^s v_{t+s} \left( \pi_{t+1+s} - E_{t+s}[\pi_{t+1+s}|\theta_{t+s}] \right) | \theta_{t+1} \right]$$

$$+ E_{t+1} \left[ \sum_{s=0}^{\infty} \beta^s U_{t+1+s} \left( \pi_{t+1+s}, E_{t+1+s} [\pi_{t+2+s}|\theta_{t+1+s}], \theta_{t+1+s} \right) | \theta_{t+1} \right]$$

The first two lines on the RHS are total expected discounted value arising from transfers. The third line on the RHS is total expected discounted value arising from flow utility.

Notice from here that the second line is equal to zero. To see this, applying Law of Iterated Expectations, when $s \geq 1$ we have

$$E_{t+1} \left[ v_{t+s} \pi_{t+1+s} | \theta_{t+1} \right] = E_{t+1} \left[ E_{t+s} \left[ v_{t+s} \pi_{t+1+s} | \theta_{t+s} \right] | \theta_{t+1} \right] = E_{t+1} \left[ v_{t+s} E_{t+s} \left[ \pi_{t+1+s} | \theta_{t+s} \right] | \theta_{t+1} \right]$$

since $v_{t+s}$ is a function only of $\theta^{t+s}$, and so is known at date $t+s$. As a result, the second line is zero, and we can write

$$W_{t+1}(\theta^{t+1}) = -v_t \left( \pi_{t+1} - E_t[\pi_{t+1}|\theta_t] \right)$$

$$+ E_{t+1} \left[ \sum_{s=0}^{\infty} \beta^s U_{t+1+s} \left( \pi_{t+1+s}, E_{t+1+s} [\pi_{t+2+s}|\theta_{t+1+s}], \theta_{t+1+s} \right) | \theta_{t+1} \right]$$

From here, we differentiate the continuation value $W_{t+1}(\theta^{t+1})$ in the date $t$ type $\theta_t$, yielding

$$\frac{\partial W_{t+1}(\theta^{t+1})}{\partial \theta_t} = - \frac{\partial v_t}{\partial \theta_t} \left( \pi_{t+1} - E_t[\pi_{t+1}|\theta_t] \right) - v_t \left( \frac{\partial \pi_{t+1}}{\partial \theta_t} - \frac{dE_t[\pi_{t+1}|\theta_t]}{d\theta_t} \right)$$

$$+ E_{t+1} \left[ \sum_{s=0}^{\infty} \beta^s \left( \frac{\partial U_{t+1+s}}{\partial \pi_{t+1+s}} \frac{\partial \pi_{t+1+s}}{\partial \theta_t} + \frac{\partial U_{t+1+s}}{\partial E_{t+1+s} [\pi_{t+2+s}|\theta_{t+1+s}]} \frac{\partial E_{t+1+s} [\pi_{t+2+s}|\theta_{t+1+s}]}{\partial \theta_t} \right) | \theta_{t+1} \right]$$

Notice in the above derivation that only the first line includes a total derivative of firm expectations, $dE_t[\pi_{t+1}|\theta_t]/d\theta_t$, which accounts for the changes in probability density. All later lines only include the direct change in inflation policy. This is because conditional expectations at date $t+1$ are taken with respect to $\theta_{t+1}$, not $\theta_t$. 

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We now rearrange the first term on the second line as follows. In particular, we write
\[
\sum_{s=0}^{\infty} \beta^s \frac{\partial U_{t+1+s}}{\partial \nu_{t+1+s}} \frac{\partial \nu_{t+1+s}}{\partial \theta_{t}} = \frac{\partial U_{t+1}}{\partial \nu_{t+1}} \frac{\partial \nu_{t+1}}{\partial \theta_{t}} \quad + \sum_{s=0}^{\infty} \beta^s \frac{\partial U_{t+1+s}}{\partial \nu_{t+1+s}} \frac{\partial \nu_{t+1+s}}{\partial \theta_{t}}
\]
which extracts the first element of the sum, and relabels the remainder of the sum to continue to start from \(s = 0\). Substituting back in, we obtain
\[
\frac{\partial \mathcal{V}_{t+1}(\theta^{t+1})}{\partial \theta_{t}} = -\frac{\partial v_t}{\partial \theta_{t}} \left( \pi_{t+1} - E_t[\pi_{t+1}\theta_{t}] \right) - v_t \left( \frac{\partial \pi_{t+1}}{\partial \theta_{t}} - \frac{dE_t[\pi_{t+1}\theta_{t}]}{d\theta_{t}} \right) + \frac{\partial U_{t+1}}{\partial \nu_{t+1}} \frac{\partial \nu_{t+1}}{\partial \theta_{t}}
\]

By definition, we have \(v_{t+s+1} = -\frac{1}{\beta} \frac{\partial U_{t+s+1}}{\partial \pi_{t+s+1}}\) given the allocation rule we are using in constructing the value function is the constrained efficient allocation rule. By Proposition 1, we also have \(\frac{\partial U_{t+s+1}}{\partial \pi_{t+s+1}} = v_{t+s+1}\) for the same reason. Therefore, we can write
\[
\mathbb{E}_{t+1} \left[ \sum_{s=0}^{\infty} \beta^s \frac{\partial U_{t+1+s}}{\partial \nu_{t+1+s}} \frac{\partial \nu_{t+1+s}}{\partial \theta_{t}} \right] = \mathbb{E}_{t+1} \left[ \sum_{s=0}^{\infty} \beta^s \left( v_{t+s+1} \frac{\partial \pi_{t+s+2}}{\partial \theta_{t}} - v_{t+s+1} \frac{\partial \pi_{t+s+2}}{\partial \theta_{t}} \right) \right]
\]
\[
= 0
\]
where the last line follows by Law of Iterated expectations,
\[
\mathbb{E}_{t+1} \left[ v_{t+s+1} \mathbb{E}_{t+1+s} \left[ \frac{\partial \pi_{t+s+2}}{\partial \theta_{t}} \theta_{t+1+s} \right] \right] = \mathbb{E}_{t+1} \left[ v_{t+s+1} \mathbb{E}_{t+1+s} \left[ \frac{\partial \pi_{t+s+2}}{\partial \theta_{t}} \theta_{t+1+s} \right] \right]
\]
\[
= \mathbb{E}_{t+1} \left[ v_{t+s+1} \frac{\partial \pi_{t+s+2}}{\partial \theta_{t}} \theta_{t+1+s} \right]
\]
Therefore, we obtain
\[
\frac{\partial \mathcal{V}_{t+1}(\theta^{t+1})}{\partial \theta_{t}} = -\frac{\partial v_t}{\partial \theta_{t}} \left( \pi_{t+1} - E_t[\pi_{t+1}\theta_{t}] \right) - v_t \left( \frac{\partial \pi_{t+1}}{\partial \theta_{t}} - \frac{dE_t[\pi_{t+1}\theta_{t}]}{d\theta_{t}} \right) + \frac{\partial U_{t+1}}{\partial \nu_{t+1}} \frac{\partial \nu_{t+1}}{\partial \theta_{t}}
\]
Finally, notice that as before, by Proposition 1 we have \(v_t = \frac{\partial U_{t+1}}{\partial \pi_{t+1}}\), and therefore we can write
\[
\frac{\partial \mathcal{V}_{t+1}(\theta^{t+1})}{\partial \theta_{t}} = -\frac{\partial v_t}{\partial \theta_{t}} \left( \pi_{t+1} - E_t[\pi_{t+1}\theta_{t}] \right) + v_t \frac{dE_t[\pi_{t+1}\theta_{t}]}{d\theta_{t}}.
\]
We are now ready to substitute back in to the expression for \( \frac{\partial W_i}{\partial \theta_t} \). Substituting back in, we have

\[
\frac{\partial W_i(\theta^t)}{\partial \theta_t} = \frac{\partial U_t}{\partial \theta_t} + \beta E_t \left[ \mathcal{W}_{t+1}(\theta^{t+1}) \frac{\partial f(\theta_{t+1}\vert \theta_t)}{f(\theta_{t+1}\vert \theta_t)} \bigg| \theta_t \right] \\
- \nu_{t-1} \frac{\partial \pi_t}{\partial \theta_t} + \frac{\partial U_t}{\partial \pi_t} \frac{\partial \pi_t}{\partial \theta_t} + \frac{\partial U_t}{\partial E_t} \frac{\partial E_t}{\partial \pi_t} \frac{\partial \pi_t}{\partial \theta_t} + \beta E_t \left[ -\frac{\partial dE_t}{\partial \pi_t} \frac{\partial \pi_t}{\partial \theta_t} + \nu_t \frac{\partial E_t}{\partial \theta_t} \bigg| \theta_t \right]
\]

The arguments now are familiar. The first term on the third line is zero, since

\[
E_t \left[ -\frac{\partial dE_t}{\partial \pi_t} \frac{\partial \pi_t}{\partial \theta_t} \bigg| \theta_t \right] = -\frac{\partial dE_t}{\partial \pi_t} \frac{\partial \pi_t}{\partial \theta_t} = 0.
\]

From here, we can rearrange terms to get

\[
\frac{\partial W_i(\theta^t)}{\partial \theta_t} = \frac{\partial U_t}{\partial \theta_t} + \beta E_t \left[ \mathcal{W}_{t+1}(\theta^{t+1}) \frac{\partial f(\theta_{t+1}\vert \theta_t)}{f(\theta_{t+1}\vert \theta_t)} \bigg| \theta_t \right] \\
+ \left[ -\nu_{t-1} + \frac{\partial U_t}{\partial \pi_t} \right] \frac{\partial \pi_t}{\partial \theta_t} + \frac{\partial U_t}{\partial E_t} \frac{\partial E_t}{\partial \pi_t} \frac{\partial \pi_t}{\partial \theta_t} + \beta E_t \left[ \nu_t \frac{\partial E_t}{\partial \pi_t} \frac{\partial \pi_t}{\partial \theta_t} \bigg| \theta_t \right]
\]

By Proposition 1, we have \(-\nu_{t-1} + \frac{\partial U_t}{\partial \pi_t} = 0\). Likewise from the definition of \( \nu_t \), we have

\[
\frac{\partial U_t}{\partial E_t} = -\beta \nu_t.
\]

Therefore, we also have

\[
\frac{\partial U_t}{\partial E_t} \frac{\partial E_t}{\partial \pi_t} \frac{\partial \pi_t}{\partial \theta_t} = -\beta \nu_t \frac{\partial E_t}{\partial \pi_t} \frac{\partial \pi_t}{\partial \theta_t} + \beta \nu_t \frac{\partial E_t}{\partial \pi_t} \frac{\partial \pi_t}{\partial \theta_t} = 0.
\]

Thus, the entire second line is zero, and we are left with

\[
\frac{\partial W_i(\theta^t)}{\partial \theta_t} = \frac{\partial U_t}{\partial \theta_t} + \beta E_t \left[ \mathcal{W}_{t+1}(\theta^{t+1}) \frac{\partial f(\theta_{t+1}\vert \theta_t)}{f(\theta_{t+1}\vert \theta_t)} \bigg| \theta_t \right]
\]

which is the required envelope condition. This concludes the proof.

### A.3 Proof of Proposition 4

To first order, the welfare gains of an inflation perturbation from the static target is

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{\partial U_t}{\partial \pi_t} d\pi_t + \frac{\partial U_t}{\partial E_t} \frac{d\pi_t}{\pi_t+1} \right].
\]

\[\text{[41] For completeness, note that when considering the date 0 value function, we have } \nu_{-1} = 0 \text{ and have } \frac{\partial U_t}{\partial \pi_t} = 0 \text{ by Proposition 1.} \]
From here, the first order condition of the central bank is 
\[ \nu^* = \frac{\partial U_t}{\partial \pi_t} \], while by definition 
\[ \frac{\partial U_t}{\partial \pi_t} = -\beta \nu_t^* \]. We have \( \frac{\partial U_0}{\partial \pi_0} = 0 \), so that we have

\[ \mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t \left[ \nu^* - \nu^*_t \right] d\pi_t. \]

Finally, we have \( \mathbb{E}_{t-1} d\pi_t = \tau_{t-1} - \mathbb{E}_{t-1} \pi^*_t \), giving the result.

**A.4 Proof of Proposition 5**

Using reduced form preferences, our two key equations are

\[ \nu_{t-1} = -\pi_t - \hat{\alpha} \left( \pi_t - \beta \mathbb{E}_t \pi_{t+1} \right) + \hat{\lambda} \]
\[ \nu_t = -\hat{\alpha} \left( \pi_t - \beta \mathbb{E}_t \pi_{t+1} \right) + \hat{\lambda} - \frac{1}{\beta} \theta_t \]

Summing the two equations, we get

\[ \nu_t = \nu_{t-1} + \pi_t - \frac{1}{\beta} \theta_t \]

Now, we guess and verify a linear solution of the form,

\[ \nu_t = \gamma_0 + \gamma_1 \theta_t + \gamma_2 \nu_{t-1} \]

Using our key equation, we get

\[ \pi_t = \nu_t - \nu_{t-1} + \frac{1}{\beta} \theta_t \]

Leading one period and taking expectations,

\[ \mathbb{E}_t \pi_{t+1} = \gamma_0 + (\gamma_2 - 1) \nu_t + \left( \gamma_1 + \frac{1}{\beta} \right) \rho \theta_t \]

Now, substituting back in to the equation for \( \nu_t \) and rearranging, 

\[ \left( 1 + \hat{\alpha} - \hat{\alpha} \beta (\gamma_2 - 1) \right) \nu_t = \hat{\alpha} \beta \gamma_0 + \hat{\lambda} + \left[ \hat{\alpha} \beta \left( \gamma_1 + \frac{1}{\beta} \right) \rho - \frac{1 + \hat{\alpha}}{\beta} \right] \theta_t + \hat{\alpha} \nu_{t-1} \]

Now, we solve by coefficient matching. Coefficient matching on \( \gamma_2 \), we have

\[ \left( 1 + \hat{\alpha} - \hat{\alpha} \beta (\gamma_2 - 1) \right) \gamma_2 = \hat{\alpha} \]
\[
0 = \hat{\alpha} \beta \gamma^2 - \left(1 + \hat{\alpha} + \hat{\alpha} \beta\right) \gamma + \hat{\alpha}
\]

and so the non-explosive root is

\[
\gamma^2 = \frac{1 + \hat{\alpha} + \hat{\alpha} \beta \left(1 + \hat{\alpha} + \hat{\alpha} \beta\right)^2 - 4\hat{\alpha}^2 \beta}{2\hat{\alpha} \beta}
\]

Now, we can coefficient match on the constant,

\[
\gamma^0 = \frac{\hat{\alpha}}{1 + \hat{\alpha} - \hat{\alpha} \beta (\gamma^2 - 1)} \frac{\hat{\alpha} \beta \gamma^0 + \hat{\lambda}}{\hat{\alpha}}
\]

\[
\gamma^0 = \frac{\gamma^2}{1 - \beta \gamma^2} \frac{\hat{\lambda}}{\hat{\alpha}}
\]

Finally, coefficient matching on \( \gamma^1 \),

\[
\gamma^1 = \frac{\hat{\alpha}}{1 + \hat{\alpha} - \hat{\alpha} \beta (\gamma^2 - 1)} \left[ \hat{\alpha} \beta \left( \gamma^1 + \frac{1}{\beta} \right) \rho - \frac{1 + \hat{\alpha}}{\beta} \right]
\]

\[
\gamma^1 = \frac{\gamma^2}{1 - \beta \gamma^2 \rho} \left[ \rho - \frac{1 + \hat{\alpha}}{\beta} \right]
\]

### A.5 Proof of Proposition 6

Consider reduced-form preferences,

\[
U_t(\pi_t, \pi_t \pi_{t+1}, \theta_t) = -\frac{1}{2} \pi_t^2 - \frac{1}{2} \alpha \left( \pi_t - \beta \pi_{t+1} \pi_t \pi_{t+1} \right)^2 + v(\pi_{t+1} + \theta_t)
\]

where for notational convenience we use \( \alpha \) in place of \( \hat{\alpha} \) in the derivations (and then simply replace \( \alpha \) with \( \hat{\alpha} \) at the end). Thus, we have derivatives

\[
\frac{\partial U_t}{\partial \pi_t} = -\pi_t - \alpha \left( \pi_t - \beta \pi_{t+1} \pi_t \pi_{t+1} \right)
\]

\[
\frac{\partial U_t}{\partial \pi_{t+1}} = \alpha \beta \left( \pi_t - \beta \pi_{t+1} \pi_t \right) + v' \left( \pi_t \pi_{t+1} \right)
\]

Under the usual definitions of \( \nu_t \), we then have

\[
\nu_{t+1} = -\pi_t - \alpha \left( \pi_t - \beta \pi_{t+1} \pi_t \pi_{t+1} \right)
\]

(22)
\[ v_t = -\alpha \left( \pi_t - \beta E_t \pi_{t+1} \right) - v_0 + v_1 E_t \pi_{t+1} + v_1 \theta_t \]  

(23)

where we have used \( v'(i_t) = \beta v_0 - \beta v_1 i_t \) and \( i_t^* = E_t \pi_{t+1} + \theta_t \).

We now guess and verify a linear solution of the form

\[ v_t = \gamma_0 + \gamma_1 v_{t-1} + \gamma_2 \theta_t. \]

Rearranging equation (22), we get

\[ \beta E_t \pi_{t+1} = \frac{1}{\alpha} v_{t-1} + \frac{1 + \alpha}{\alpha} \pi_t, \]  

(24)

and substituting into equation (23) we get

\[ v_t = -v_0 + \left( \alpha \beta + v_1 \right) \left( 1 + \alpha \right) - \alpha^2 \beta \pi_t + \frac{\alpha \beta + v_1}{\alpha \beta} v_{t-1} + v_1 \theta_t. \]

From here, we denote \( \frac{1}{\zeta} = \frac{(\alpha \beta + v_1)(1 + \alpha) - \alpha^2 \beta}{\alpha \beta} > 0 \). Thus rearranging the above equation, we have

\[ \frac{1}{\zeta} \pi_t = v_t + v_0 - \frac{\alpha \beta + v_1}{\alpha \beta} v_{t-1} - v_1 \theta_t \]  

(25)

We now lead this equation forward one period and take expectations,

\[ \frac{1}{\zeta} E_t \pi_{t+1} = E_t v_{t+1} + v_0 - \frac{\alpha \beta + v_1}{\alpha \beta} E_t v_{t-1} - v_1 E_t \theta_{t+1} \]

and now, we can use the guess for \( v_t \) along with the property \( E_t \theta_{t+1} = \rho \theta_t \) to obtain

\[ \frac{1}{\zeta} E_t \pi_{t+1} = \gamma_0 + v_0 + \left( \gamma_1 - \frac{\alpha \beta + v_1}{\alpha \beta} \right) v_t + (\gamma_2 - v_1) \rho \theta_t. \]

Now, equations (24) and (25) jointly imply

\[ \frac{1}{\zeta} E_t \pi_{t+1} = \frac{1}{\zeta} \frac{1}{\alpha \beta} v_{t-1} + \frac{1 + \alpha}{\alpha \beta} \left( v_t + v_0 - \frac{\alpha \beta + v_1}{\alpha \beta} v_{t-1} - v_1 \theta_t \right) \]

and so substituting in, we obtain

\[ \gamma_0 + v_0 + \left( \gamma_1 - \frac{\alpha \beta + v_1}{\alpha \beta} \right) v_t + (\gamma_2 - v_1) \rho \theta_t = \frac{1}{\zeta} \frac{1}{\alpha \beta} v_{t-1} + \frac{1 + \alpha}{\alpha \beta} \left( v_t + v_0 - \frac{\alpha \beta + v_1}{\alpha \beta} v_{t-1} - v_1 \theta_t \right) \]

which rearranges and simplifies to

\[ \left( \gamma_1 - \frac{1 + \alpha + \alpha \beta + v_1}{\alpha \beta} \right) v_t = \left( \frac{1 + \alpha - \alpha \beta}{\alpha \beta} v_0 - \gamma_0 \right) - \frac{1}{\beta} v_{t-1} - \left( \frac{1 + \alpha - \alpha \beta \rho}{\alpha \beta} v_1 + \gamma_2 \rho \right) \theta_t. \]
The LHS is linear, so using our guess $\nu_t = \gamma_0 + \gamma_1 \nu_{t-1} + \gamma_2 \theta_t$ and coefficient matching, we have the system

$$
\begin{align*}
\gamma_0 &= \frac{1 + \alpha (1 - \beta)}{\alpha \beta} \nu_0 - \gamma_0 \\
\gamma_1 &= -\frac{1}{\beta} \frac{1 + \alpha + \alpha \beta + \nu_1}{\gamma_1 - \frac{1 + \alpha + \alpha \beta + \nu_1}{\alpha \beta}} \\
\gamma_2 &= -\frac{\left(1 + \alpha (1 - \beta) \nu_1 + \gamma_2 \theta_1\right)}{\gamma_1 - \frac{1 + \alpha + \alpha \beta + \nu_1}{\alpha \beta}}
\end{align*}
$$

The second equation rearranges to a quadratic $\beta \gamma_1^2 - \frac{1 + \alpha + \alpha \beta + \nu_1}{\alpha} \gamma_1 + 1 = 0$ in $\gamma_1$. We choose the non-explosive lower root to maintain consistency with the transversality condition, which yields

$$
\gamma_1 = \frac{1 + \alpha (1 + \beta) + \nu_1 - \sqrt{(1 + \alpha (1 + \beta) + \nu_1)^2 - 4\alpha^2 \beta}}{2 \alpha \beta}
$$

From here, the equation for $\gamma_0$ can be rewritten as $\gamma_0 = -\beta \gamma_1 \left(\frac{1 + \alpha (1 - \beta)}{\alpha \beta}\nu_0 - \gamma_0\right)$, and rearranging yields

$$
\gamma_0 = -\gamma_1 \frac{1 + \alpha (1 - \beta)}{\alpha (1 - \beta \gamma_1)} \nu_0
$$

Similarly, the equation for $\gamma_2$ is rewritten as $\gamma_2 = \beta \gamma_1 \left(\frac{1 + \alpha (1 - \beta)}{\alpha \beta} \nu_1 + \gamma_2 \rho_1\right)$, which rearranges to

$$
\gamma_2 = \frac{1 + \alpha (1 - \beta) \rho_1}{\alpha} \gamma_1 \nu_1
$$

Thus, we have our solution.

Inflation is given by

$$
\frac{1}{\xi} \pi_t = \nu_t - \frac{\alpha \beta + \nu_1}{\alpha \beta} \nu_{t-1} + \nu_0 - \nu_1 \theta_t
$$

**A.6 Proof of Proposition 7**

Given reduced form preferences $U_t = -\frac{1}{2} \pi_t^2 + \theta_t \frac{\pi_{t-1} - \beta \pi_t}{\kappa}$, then we have

$$
\frac{\partial U_t}{\partial \pi_t} = -\pi_t + \frac{1}{\kappa} \theta_t
$$

$$
\frac{\partial U_{t-1}}{\partial \pi_{t-1} \pi_t} = -\frac{\beta}{\kappa} \theta_{t-1}
$$
Thus substituting in the definitions,

\[ v_{t-1} = -\pi_t + \frac{1}{\kappa/\theta_t} \]

\[ v_{t-1} = \frac{1}{\kappa/\theta_{t-1}} \]

Thus putting them together, we get\( \pi_t = \frac{1}{\kappa/\theta_t} - \frac{1}{\kappa/\theta_{t-1}} \). Finally, using \( \mathbb{E}_t \pi_{t+1} = 1 - \rho + \rho \theta_t \) we get

\[ \mathbb{E}_t \pi_{t+1} = \frac{\mathbb{E}_t \theta_{t+1} - \theta_t}{\kappa} = (1 - \rho) \frac{1}{\kappa} - (1 - \rho) \frac{\theta_t}{\kappa} \]

which gives the result.

**A.7 Proof of Proposition 8**

The proof follows the same steps as in Proposition 3. The envelope condition is the same, given that the additional term \(-c(\pi_t - \mathbb{E}_{t-1}[\pi_t|\tilde{\theta}_t])\) in \( V_t \) depends on reported types and not true types. From here, the value function at date \( t \) under our proposed mechanism given by

\[ \mathcal{W}_t(\theta^t) = -c(\pi_t - \mathbb{E}_{t-1}[\pi_t]) + V_t + \beta \mathbb{E}_t \left[ \mathcal{W}_t(\theta^{t+1}) \mid \theta_t \right] \]

\[ = -(c + b_{t-1})(\pi_t - \mathbb{E}_{t-1}[\pi_t]) + U_t + \beta \mathbb{E}_t \left[ \mathcal{W}_t(\theta^{t+1}) \mid \theta_t \right] \]

\[ = -v_{t-1}(\pi_t - \mathbb{E}_{t-1}[\pi_t]) + U_t + \beta \mathbb{E}_t \left[ \mathcal{W}_t(\theta^{t+1}) \mid \theta_t \right] \]

which is the same value function as in the proof of Proposition 3 when evaluated at the constrained efficient allocation. Thus the result follows using the same proof as for Proposition 3.

**A.7.1 Proof of Proposition 9**

Consider the Ramsey problem,

\[ \max_{\pi} \sum_{t=0}^{\infty} \beta^t U_t(\pi_t, \mathbb{E}_t[\pi_{t+1}|\tilde{\theta}_t], ..., \mathbb{E}_t[\pi_{t+K}|\tilde{\theta}_t], \theta_t) \]

It is expositionally helpful to extend the sum to include \( U_{-1}, ..., U_{-K} = 0 \). Under this extended sum, differentiating in \( \pi_t(\theta^t) \) for \( t \geq 0 \), we have

\[ 0 = \sum_{s=t-K}^{t-1} \beta^s \frac{\partial U_s}{\partial \mathbb{E}_s[\pi_s|\theta_s]} \frac{\partial \mathbb{E}_s[\pi_s|\theta_s]}{\partial \pi_t(\theta^t)} f(\theta^t) + \beta^t \frac{\partial U_t}{\partial \pi_t} f(\theta^t). \]
From here, note that we have
\[
\frac{\partial E_s[\pi_t|\theta_s]}{\partial \pi_t} f(\theta^s) = f(\theta^t|\theta^s) f(\theta^s) = f(\theta^t)
\]
Thus rearranging and dividing through, we have
\[
\frac{\partial U_t}{\partial \pi_t} = -\sum_{s=t-K}^{t-1} \beta^{s-t} \frac{\partial U_s}{\partial E_s[\pi_t|\theta_s]}.
\]
Substituting in the definition of \(\nu_{t,k}\) gives the result.

A.8 Proof of Proposition 11

The proof strategy is as follows. First, we derive the relevant envelope condition associated with local incentive compatibility, which defines necessary conditions on the value function associated with an incentive compatible mechanism. We then show that the value function generated by our proposed mechanism satisfies this envelope condition.

Envelope Condition. Suppose that the central bank has a history \(\hat{\theta}^{t-1}\) of reports and a history \(\theta^t\) of true types at date \(t\). Given a mechanism with transfer rule \(T_t(\hat{\theta}_t)\) and allocation rule \(\pi_t(\hat{\theta}_t)\), the value function of a central bank that has truthfully reported in the past, assuming truthful reporting in the future, is given by
\[
W_t(\theta^t) = \max_{\hat{\theta}_t} T_t + U_t(\pi_t, E_t[\pi_{t+1}|\hat{\theta}_t], ..., E_t[\pi_{t+K}|\hat{\theta}_t], \theta_t) + \beta E_t\left[W_{t+1}(\theta^t, \hat{\theta}_t, \theta_{t+1})|\theta_t\right]
\]
Notice that the expectations at date \(t\) are based on the date \(t\) reported type, not the date \(t\) true type. Furthermore, notice that \(W_{t+1}\) depends on the reported type \(\hat{\theta}_t\), but not on the true type \(\theta_t\). This is because flow utility at dates \(t + s\) (\(s \geq 0\)) do not depend on past true types and because the shock structure is Markov. This implies that we can in fact write \(W_{t+1}(\theta^{t-1}, \hat{\theta}_t, \theta_{t+1})\). As a result, the Envelope Condition in the true type \(\theta_t\), evaluated at truthful reporting \(\hat{\theta}_t = \theta_t\), is
\[
\frac{\partial W_t(\theta^t)}{\partial \theta^t} = \frac{\partial U_t}{\partial \theta^t} + \beta \frac{\partial E_t\left[W_{t+1}(\theta^{t-1}, \hat{\theta}_t, \theta_{t+1})|\theta_t\right]}{\partial \theta^t}
\]
where we have
\[
\frac{\partial}{\partial \theta_t} \left[ \mathcal{W}_{t+1}(\theta^{t-1}, v_t, \theta_{t+1}) \right] = \frac{\partial}{\partial \theta_t} \int_0^\pi \mathcal{W}_{t+1}(\theta^{t-1}, v_t, \theta_{t+1}) f(\theta_{t+1}|\theta_t) d\theta_{t+1} = \mathbb{E}_t \left[ \mathcal{W}_{t+1}(\theta^{t-1}, v_t, \theta_{t+1}) \frac{\partial f(\theta_{t+1}|\theta_t)}{f(\theta_{t+1}|\theta_t)} | \theta_t \right]
\]

Substituting in and evaluating at truthful reporting, we obtain
\[
\frac{\partial}{\partial \theta_t} \mathcal{V}_t(\theta^t) = \frac{\partial}{\partial \theta_t} U_t(\pi_t, \mathbb{E}_t \pi_{t+1}, ..., \mathbb{E}_t \pi_{t+k}, \theta_t) + \beta \mathbb{E}_t \left[ \mathcal{W}_{t+1}(\theta^{t+1}) | \theta_t \right]
\]

which provides a conventional envelope condition for incentive compatibility. For clarity, note that \( \frac{\partial U_t}{\partial \theta_t} \) is the derivative of \( U_t \) in the direct type \( \theta_t \), but not including the Phillips curve expectation, which is the derivative in the reported type.

Verifying the Envelope Condition. We now verify the value function under our mechanism satisfies the envelope condition. Our mechanism has a transfer rule
\[
T_t = -\sum_{k=1}^K v_{t,k} (\pi_t - \mathbb{E}_{t-k} \pi_t)
\]

and an allocation rule given by the constrained efficient allocation of Proposition 9. It will at times be helpful to define
\[
v_{t-1} = \sum_{k=1}^K v_{t,k}.
\]

The value function associated with this mechanism is
\[
\mathcal{V}_t(\theta^t) = -\sum_{k=1}^K v_{t,k} (\pi_t - \mathbb{E}_{t-k} \pi_t)
\]

\[
+ \mathbb{E}_t \left( \sum_{k=1}^K \frac{d\mathbb{E}_t \pi_{t+k}}{d\theta_t} \mathcal{W}_{t+1}(\theta^{t+1}) | \theta_t \right)
\]

where all objects are evaluated at their constrained efficient values associated with Proposition 9, given the realized shock history. Differentiating the value function in \( \theta_t \), we have
\[
\frac{\partial \mathcal{V}_t(\theta^t)}{\partial \theta_t} = \frac{\partial U_t}{\partial \theta_t} + \beta \mathbb{E}_t \left[ \mathcal{W}_{t+1}(\theta^{t+1}) \frac{\partial f(\theta_{t+1}|\theta_t)}{f(\theta_{t+1}|\theta_t)} | \theta_t \right]
\]

\[
- v_{t-1} \frac{\partial \pi_t}{\partial \theta_t} + \sum_{k=1}^K \frac{d\mathbb{E}_t \pi_{t+k}}{d\theta_t} \mathcal{W}_{t+1}(\theta^{t+1}) | \theta_t \]

\[
+ \beta \mathbb{E}_t \left[ \frac{\partial \mathcal{W}_{t+1}(\theta^{t+1})}{\partial \theta_t} | \theta_t \right]
\]
The first line on the RHS are the terms associated with the envelope condition. The second line are derivatives that arise because in equilibrium, the reported type equals the true type, and we have evaluated the value function given truthful reporting. It therefore remains to show that the second line sums to zero and hence our mechanism satisfies the required envelope condition.

We begin by noting that the first two terms on the second line sum to zero, that is

\[ -v_{t-1} \frac{\partial \pi_t}{\partial \theta_t} + \frac{\partial U_t}{\partial \pi_t} \frac{\partial \pi_t}{\partial \theta_t} = 0. \]

This follows immediately from Proposition 9 given the definition of \( v_{t-1} \). We are therefore left to study the final two terms, and so we write

\[
\frac{\partial \mathcal{W}_t(\theta^t)}{\partial \theta_t} = \frac{\partial U_t}{\partial \theta_t} + \beta \mathbb{E}_t \left[ \mathcal{W}_{t+1}(\theta^{t+1}) \frac{\partial f(\theta_{t+1} | \theta_t)}{f(\theta_{t+1} | \theta_t)} \right]_{\theta_t} \\
+ \sum_{k=1}^{K} \frac{\partial U_t}{\partial \mathbb{E}_t \pi_{t+k}} \frac{d \mathbb{E}_t \pi_{t+k}}{d \theta_t} + \beta \mathbb{E}_t \left[ \frac{\partial \mathcal{W}_{t+1}(\theta^{t+1})}{\partial \theta_t} \right]_{\theta_t}
\]

It is helpful to write out the continuation value function \( \mathcal{W}_{t+1} \) in sequence notation. Iterating forward, we obtain

\[
\mathcal{W}_{t+1}(\theta^{t+1}) = \mathbb{E}_{t+1} \sum_{s=0}^{\infty} \beta^s \left[ - \sum_{k=1}^{K} v_{t+1+s,k} (\pi_{t+1+s} - \mathbb{E}_{t+1+s-k} \pi_{t+1+s}) + U_{t+1+s} \right]
\]

Now, we differentiate in \( \theta_t \). Here, we obtain

\[
\frac{\partial \mathcal{W}_{t+1}(\theta^{t+1})}{\partial \theta_t} = \mathbb{E}_{t+1} \sum_{s=0}^{\infty} \beta^s \left[ - \sum_{k=1}^{K} v_{t+1+s,k} \pi_{t+1+s} \frac{\partial v_{t+1+s,k}}{\partial \theta_t} \right] \\
+ \mathbb{E}_{t+1} \sum_{s=0}^{\infty} \beta^s \left[ \sum_{k=1}^{K} \frac{d \mathbb{E}_{t+1+s-k} \pi_{t+1+s}}{d \theta_t} \right] \\
+ \mathbb{E}_{t+1} \sum_{s=0}^{\infty} \beta^s \left[ - \sum_{k=1}^{K} v_{t+1+s,k} \frac{\partial \pi_{t+1+s}}{\partial \theta_t} + \frac{\partial U_{t+1+s}}{\partial \pi_{t+1+s}} \frac{\partial \pi_{t+1+s}}{\partial \theta_t} \right] \\
+ \mathbb{E}_{t+1} \sum_{s=0}^{\infty} \beta^s \left[ \sum_{k=1}^{K} \frac{\partial U_{t+1+s}}{\partial \mathbb{E}_{t+1+s-k} \pi_{t+1+s}} \frac{\partial \pi_{t+1+s-k}}{\partial \theta_t} \right]
\]

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To begin with, note that the third line is zero, from Proposition 9. Thus we can write,

\[
\frac{\partial \mathcal{W}_{t+1}(\theta^{t+1})}{\partial \theta_t} = \mathbb{E}_{t+1} \sum_{s=0}^{\infty} \beta^s \left[ -\sum_{k=1}^{K} \frac{\partial v_{t+1+s,k}}{\partial \theta_t} (\pi_{t+1+s} - \mathbb{E}_{t+1+s-k} \pi_{t+1+s}) \right]
\]

\[
+ \mathbb{E}_{t+1} \sum_{s=0}^{\infty} \beta^s \left[ \sum_{k=1}^{K} \nu_{t+1+s,k} \frac{d\mathbb{E}_{t+1+s-k} \pi_{t+1+s}}{d\theta_t} \right]
\]

\[
+ \mathbb{E}_{t+1} \sum_{s=0}^{\infty} \beta^s \left[ \sum_{k=1}^{K} \frac{\partial U_{t+1+s}}{\partial \mathbb{E}_{t+1+s} \pi_{t+1+s+k}} \mathbb{E}_{t+1+s} \frac{\partial \pi_{t+1+s+k}}{\partial \theta_t} \right]
\]

Next, recall that we can write

\[
\nu_{t,k} = -\frac{1}{\beta^k} \frac{\partial U_{t-k}}{\partial \mathbb{E}_{t-k} \pi_t}
\]

Therefore, we can equivalently write

\[
\frac{\partial U_{t+1+s}}{\partial \mathbb{E}_{t+1+s} \pi_{t+1+s+k}} = -\beta^k \nu_{t+1+s+k,k}
\]

Thus substituting into the third line,

\[
\frac{\partial \mathcal{W}_{t+1}(\theta^{t+1})}{\partial \theta_t} = \mathbb{E}_{t+1} \sum_{s=0}^{\infty} \beta^s \left[ -\sum_{k=1}^{K} \frac{\partial v_{t+1+s,k}}{\partial \theta_t} (\pi_{t+1+s} - \mathbb{E}_{t+1+s-k} \pi_{t+1+s}) \right]
\]

\[
+ \mathbb{E}_{t+1} \sum_{s=0}^{\infty} \beta^s \left[ \sum_{k=1}^{K} \nu_{t+1+s,k} \frac{d\mathbb{E}_{t+1+s-k} \pi_{t+1+s}}{d\theta_t} \right]
\]

\[
+ \mathbb{E}_{t+1} \sum_{s=0}^{\infty} \beta^s \left[ \sum_{k=1}^{K} -\beta^k \nu_{t+1+s+k,k} \mathbb{E}_{t+1+s} \frac{\partial \pi_{t+1+s+k}}{\partial \theta_t} \right]
\]

From here, let us compare the second and third lines. When \( s \geq k \), we know that \( t + 1 + s - k \geq t + 1 \) and so we have

\[
\frac{d\mathbb{E}_{t+1+s-k} \pi_{t+1+s}}{d\theta_t} = \mathbb{E}_{t+1+s-k} \frac{\partial \pi_{t+1+s}}{\partial \theta_t}.
\]

We also know that all terms with \( t + 1 + s - k < t \), that is \( k > 1 + s \), drop out of the second line (since they are date \( t - 1 \) or lower adapted constants). What this leaves us with is that the second line cancels out with the third line except for the points where \( t + 1 + s - k = t \), that is precisely the points where there is also a probability measure derivative. Put together and taking the expectation at date \( t \), this gives us

\[
\mathbb{E}_t \frac{\partial \mathcal{W}_{t+1}(\theta^{t+1})}{\partial \theta_t} = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[ -\sum_{k=1}^{K} \frac{\partial v_{t+1+s,k}}{\partial \theta_t} (\pi_{t+1+s} - \mathbb{E}_{t+1+s-k} \pi_{t+1+s}) \right]
\]

\[
+ \sum_{s=0}^{K-1} \beta^s \nu_{t+1+s,1+s} \frac{d\mathbb{E}_t \pi_{t+1+s}}{d\theta_t}
\]

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where we note that all terms on the second line are \( t \)-adapted, so the expectation operator drops out. Now consider the first line. Here, we know that \( \nu_{t+1+s,k} \) is date \( t+1+s-k \) adapted. Therefore, it drops out for all \( s < k \). When \( s \geq k \), we know that \( E_{t+1+s-k} \pi_{t+1+s} \) is a date \( t+1+s-k \geq t+1 \) adapted constant, which is the same as \( \nu_{t+1+s,k} \). Therefore by law of iterated expectations for \( s \geq k \),

\[
E_t \frac{\partial \nu_{t+1+s,k}}{\partial \theta_t} \left( \pi_{t+1+s} - E_{t+1+s-k} \pi_{t+1+s} \right) = E_t \frac{\partial \nu_{t+1+s,k}}{\partial \theta_t} \left( \pi_{t+1+s} - E_{t+1+s-k} \pi_{t+1+s} \right) = 0
\]

Therefore, the entire first line is zero, and we are left with

\[
E_t \frac{\partial \mathcal{W}_{t+1}(\theta^{t+1})}{\partial \theta_t} = \sum_{s=0}^{K-1} \beta^s \nu_{t+1+s,1+s} \frac{dE_t \pi_{t+1+s}}{d\theta_t}.
\]

Finally, we can now go back and substitute in for our equation for the derivative of \( \mathcal{W}_t \). Substituting in,

\[
\frac{\partial \mathcal{W}_t(\theta^t)}{\partial \theta_t} = \frac{\partial U_t}{\partial \theta_t} + \beta E_t \left[ \mathcal{W}_{t+1}(\theta^{t+1}) \frac{\partial f(\theta_{t+1} | \theta_t)}{f(\theta_{t+1} | \theta_t)} \right] \frac{\partial \theta_t}{\partial \theta_t} + \beta \sum_{k=1}^{K} \frac{\partial U_t}{\partial \mathcal{W}_t \pi_{t+k}} \frac{dE_t \pi_{t+k}}{d\theta_t} + \beta \sum_{s=0}^{K-1} \beta^s \nu_{t+1+s,1+s} \frac{dE_t \pi_{t+1+s}}{d\theta_t}
\]

Substituting in \( \nu_{t+1+s,1+s} = -\frac{1}{\beta^{t+s}} \frac{\partial U_t}{\partial \mathcal{W}_t \pi_{t+1+s}} \), we get

\[
\frac{\partial \mathcal{W}_t(\theta^t)}{\partial \theta_t} = \frac{\partial U_t}{\partial \theta_t} + \beta E_t \left[ \mathcal{W}_{t+1}(\theta^{t+1}) \frac{\partial f(\theta_{t+1} | \theta_t)}{f(\theta_{t+1} | \theta_t)} \right] \frac{\partial \theta_t}{\partial \theta_t} + \beta \sum_{k=1}^{K} \frac{\partial U_t}{\partial \mathcal{W}_t \pi_{t+k}} \frac{dE_t \pi_{t+k}}{d\theta_t} - \beta \sum_{s=0}^{K-1} \beta^s \nu_{t+1+s,1+s} \frac{dE_t \pi_{t+1+s}}{d\theta_t}
\]

and the second line drops to zero (the two sums are equivalent replacing \( k = 1+s \)). Thus, we obtain

\[
\frac{\partial \mathcal{W}_t(\theta^t)}{\partial \theta_t} = \frac{\partial U_t}{\partial \theta_t} + \beta E_t \left[ \mathcal{W}_{t+1}(\theta^{t+1}) \frac{\partial f(\theta_{t+1} | \theta_t)}{f(\theta_{t+1} | \theta_t)} \right] \frac{\partial \theta_t}{\partial \theta_t}
\]

which is the required envelope condition. This completes the proof.

A.9 Proof of Proposition 14

Integrating the Envelope Condition (equation 6), we obtain integral incentive compatibility

\[
\mathcal{W}_t(\theta^t) = \int_{\theta_t}^{\theta_{t+1}} \frac{\partial U_t(\theta_{t+1}-s_t)}{\partial s_t} ds_t + \beta \int_{\theta_t}^{\theta_{t+1}} E_t \left[ \mathcal{W}_{t+1} \frac{\partial f_{t+1}(\theta_{t+1} | s_t)}{\partial s_t} | s_t \right] ds_t
\]

(26)

Integral incentive compatibility relates the total date-\( t \) utility to the central bank to two information rents. Note that due to shock persistence, the central bank earns information rents not only due to
Suppose that we take the Bellman equation:

\[ \mathcal{W}_t(\theta^t) = \mathbb{E}_t \left[ \sum_{s=0}^{\infty} \beta^s B^s_t(\theta^{t+s}) \right] \forall t, \]

where \( B^s_t \) is given by

\[ B^s_t(\theta^{t+s}) = \prod_{k=0}^{s-1} \frac{1}{f_{t+k}(\theta_{t+k+1} | \theta_{t+k})} \times \int_{s_t \leq \theta_{t+s}, s_{t+s} \leq \theta_{t+s}} \frac{\partial U_{t+s}(\theta^{t-1}, s_t, ..., s_{t+s})}{\partial s_{t+s}} \prod_{k=0}^{s-1} \frac{\partial f_{t+k}(\theta_{t+k+1} | s_{t+k})}{\partial s_{t+k}} \ ds_{t+s} ds_t. \]

Proof. Suppose that we take the Bellman equation:

\[ \mathcal{W}_t(\theta^t) = \int_\theta^{\theta_t} \frac{\partial U_t(\theta^t, s_t)}{\partial s_t} ds_t + \beta \int_\theta^{\theta_t} \mathbb{E}_t \left[ \mathcal{W}_{t+1} \frac{\partial f_t(\theta_{t+1} | s_t)}{f_t(\theta_{t+1} | s_t)} \bigg| s_t \right] \]

And iterate it forward once. Iterating forward once, we obtain:

\[ \mathcal{W}_t(\theta^t) = \int_\theta^{\theta_t} \mathbb{E}_t \left[ \frac{\partial U_t(\theta^t, s_t)}{\partial s_t} \right] ds_t + \frac{\partial f_t(\theta_{t+1} | s_t)}{f_t(\theta_{t+1} | s_t)} \left( \beta \int_\theta^{\theta_t} \mathbb{E}_t \left[ \mathcal{W}_{t+1} \frac{\partial f_{t+1}(\theta_{t+2} | s_{t+1})}{f_{t+1}(\theta_{t+2} | s_{t+1})} \bigg| s_{t+1} \right] \right) \]

Iterating forward, suppose that we define the following recursive operator. In particular, we define:

\[ B^0_t(g, \theta) = \int_\theta^\theta g ds_t \]

Note that for the function \( g^0_t = \frac{\partial U_t(\theta^t, s_t)}{\partial s_t} \), we have that \( B^0_t \) is the first term in the infinite series defining \( \mathcal{W}_t \).

And suppose we define next:

\[ B^1_t(g, \theta) = \int_\theta^\theta \mathbb{E}_t \left[ \frac{\partial f_t(\theta_{t+1} | s_t)}{f_t(\theta_{t+1} | s_t)} \bigg| g_s \right] ds_t \]

Consider the function \( g^1_t = \int_\theta^{\theta_t} \frac{\partial U_{t+1}(\theta^{t+1} | s_{t+1})}{\partial s_{t+1}} ds_{t+1} \). Taking the function \( B^1_t(g^1_t, \theta_t) \) and multiplying by \( \beta \), we obtain the second term in the infinite series for \( \mathcal{W}_t \).

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From here, we define a recursive operator. Consider a function $g_t^i$ that is a date $t + s$ adapted function. We define the operator:

$$B_t^2 (g_t^2, \theta_t) = B_t^1 \left( B_{t+1}^1 (g_{t+1}^2, \theta_{t+1}), \theta_t \right)$$

So that we have:

$$B_t^2 (g_t^2, \theta_t) = \int_\mathcal{G} E_t \left[ \frac{\partial f_t (\theta_{t+1} | s_t)}{f_t (\theta_{t+1} | s_t)} \right] \int_\mathcal{G} E_{t+1} \left[ \frac{\partial f_{t+1} (\theta_{t+2} | s_{t+1})}{f_{t+1} (\theta_{t+2} | s_{t+1})} \right] g_{t+1}^2 (s_{t+1}, \theta_{t+2}) \bigg| s_{t+1} \bigg| s_t \bigg| ds_{t+1} \bigg| s_t \bigg| ds_t$$

Which, when $g_t^2 (s_t, s_{t+1}, \theta_{t+2}) = \int_\mathcal{G} \frac{\partial U_{t+2} (\theta_{t+1}, s_{t+1}, \theta_{t+2})}{ds_{t+2}} ds_t$, gives us the next term in the infinite series defining $W_t$.

Continuously defining these recursive operators as such, and defining functions $g_t^i (s_t, ..., s_{t+s-1}, \theta_{t+s}) = \int_\mathcal{G} \frac{\partial U_{t+s} (\theta_{t+s}, s_t, ..., s_{t+s-1})}{ds_{t+s}} ds_t$, we obtain the infinite series that characterizes $W_t$.

In other words, we can construct such recursive operators. From here, we look to simplify these operators. Let us start from the operator $B_t^1 (g, \theta_t)$. In particular, we have:

$$B_t^1 (g, \theta_t) = \int_\mathcal{G} E_t \left[ \frac{\partial f_t (\theta_{t+1} | s_t)}{f_t (\theta_{t+1} | s_t)} g (s_t, \theta_{t+1}) \bigg| s_t \bigg| ds_t \right] ds_t$$

$$= \int_\mathcal{G} \int_{\theta_{t+1}} \frac{\partial f_t (\theta_{t+1} | s_t)}{\partial s_t} g (s_t, \theta_{t+1}) d\theta_{t+1} ds_t$$

$$= \int_{\theta_{t+1}} \left[ \int_\mathcal{G} \frac{\partial f_t (\theta_{t+1} | s_t)}{\partial s_t} g (s_t, \theta_{t+1}) ds_t \right] d\theta_{t+1}$$

$$= \int_{\theta_{t+1}} \left[ \int_\mathcal{G} \frac{\partial f_t (\theta_{t+1} | s_t)}{\partial s_t} g (s_t, \theta_{t+1}) ds_t \right] \left[ \frac{1}{f_t (\theta_{t+1} | \theta_t)} \right] \left[ \int_\mathcal{G} \frac{\partial f_t (\theta_{t+1} | s_t)}{\partial s_t} g (s_t, \theta_{t+1}) ds_t \right] \left[ \frac{\partial U_{t+1} (\theta_{t+1}, s_t, \theta_{t+1})}{ds_{t+1}} \right] ds_{t+1}$$

In particular, as applied to the function $g_t^1 = \int_\mathcal{G} \frac{\partial U_{t+1} (\theta_{t+1}, s_t, \theta_{t+1})}{ds_{t+1}} ds_{t+1}$, we obtain:

$$B_t^1 (g, \theta_t) = E_t \left[ \frac{1}{f_t (\theta_{t+1} | \theta_t)} \right] \left[ \int_\mathcal{G} \int_\mathcal{G} \frac{\partial U_{t+1} (\theta_{t+1}, s_t, \theta_{t+1})}{ds_{t+1}} \frac{\partial f_t (\theta_{t+1} | s_t)}{\partial s_t} \bigg| ds_{t+1} \bigg| ds_t \bigg| \theta_t \bigg]$$

Which is of the form in the Lemma.

Now, let us consider the second operator. We have:

$$B_t^2 (g, \theta_t) = B_t^1 \left( B_{t+1}^1 (g, \theta_{t+1}), \theta_t \right)$$

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Recall that the simplified operator above expresses:

\[ B_t^1(g, \theta_t) = E_t \left[ \frac{1}{f_t(\theta_{t+1}|\theta_t)} \left[ \int_\theta^{\theta_t} \frac{\partial f_t(\theta_{t+1}|s_t)}{\partial s_t} g(s_t, \theta_{t+1}) ds_t \right] \left| \theta_t \right. \right] \]

In other words, we have along history \((\theta^t-1, s_t)\):

\[ B_{t+1}^1(g, \theta_{t+1}) = E_{t+1} \left[ \frac{1}{f_{t+1}(\theta_{t+2}|\theta_{t+1})} \left[ \int_\theta^{\theta_{t+1}} \frac{\partial f_{t+1}(\theta_{t+2}|s_{t+1})}{\partial s_{t+1}} g(s_{t+1}, \theta_{t+1}, \theta_{t+2}) ds_{t+1} \right] \left| \theta_{t+1} \right. \right] \]

And applying this into the operator defining \(B_t^2\), we obtain:

\[ B_t^2(g, \theta_t) = E_t \left[ \frac{1}{f_t(\theta_{t+1}|\theta_t)} \left[ \int_\theta^{\theta_t} \frac{\partial f_t(\theta_{t+1}|s_t)}{\partial s_t} B_{t+1}^1(g, \theta_{t+1}) ds_t \right] \left| \theta_t \right. \right] \]

\[ = E_t \left[ \frac{1}{f_t(\theta_{t+1}|\theta_t)} \left[ \int_\theta^{\theta_t} \frac{\partial f_t(\theta_{t+1}|s_t)}{\partial s_t} \frac{1}{f_{t+1}(\theta_{t+2}|\theta_{t+1})} \left[ \int_\theta^{\theta_{t+1}} \frac{\partial f_{t+1}(\theta_{t+2}|s_{t+1})}{\partial s_{t+1}} g(s_{t+1}, \theta_{t+1}, \theta_{t+2}) ds_{t+1} \right] \left| \theta_{t+1} \right. \right] \left| \theta_t \right. \right] \]

\[ \overset{\text{LIE}}{=} E_t \left[ \frac{1}{f_t(\theta_{t+1}|\theta_t)} \frac{1}{f_{t+1}(\theta_{t+2}|\theta_{t+1})} \left[ \int_\theta^{\theta_t} \frac{\partial f_t(\theta_{t+1}|s_t)}{\partial s_t} \frac{\partial f_{t+1}(\theta_{t+2}|s_{t+1})}{\partial s_{t+1}} g(s_{t+1}, \theta_{t+1}, \theta_{t+2}) ds_{t+1} ds_t \right] \left| \theta_t \right. \right] \]

And substituting in \(g_t^2 = \int_\theta^{\theta_t} \frac{\partial U_{t+2}(\theta^t-1, s_{t+1}, s_{t+2})}{\partial s_{t+2}} ds_{t+2}\), we get the next expression from the Lemma. From here, the result follows from repeated iteration.

Lemma 17 allows us to represent the principal’s optimization problem in a tractable way. Given an allocation rule for inflation, we use the characterization of the value function in Lemma 17 as well as the Bellman equation to characterize the transfer rule which implements the allocation,

\[ T_t = W_t - U_t - \beta E_t[W_{t+1}|\theta_t]. \]

We can then substitute the implementing taxes into the government’s utility function, and obtain the following result characterizing the relaxed social planning problem.

Lemma 18. The relaxed social planning problem can be written as

\[ \max_{(\pi_t)} \mathbb{E}_{-1} \left[ \sum_{t=0}^{\infty} \beta^t \left[ -\frac{\kappa}{1+\kappa} B_t^0 + U_t \right] \right], \]

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where $B_0^t$ is given as in Lemma 17. The implementing transfer rule is given by

$$T_t = W_t - U_t - \beta \mathbb{E}_t[W_{t+1}|\theta_t],$$

where $W_t$ is given as a function of the allocation rule as in Lemma 17.

Proof. For any allocation rule, $T_t$ provides the implementation. Recall that the government’s welfare is given by:

$$\max E_{-1} \left[ \sum_{t=0}^{\infty} \beta^t U_t - \kappa T_t \right],$$

Recall that bank welfare is given by:

$$W_0 = E_0 \sum_{t=0}^{\infty} \left[ \beta^t U_t + T_t \right]$$

In other words, we always have:

$$-E_0 \sum_{t=0}^{\infty} T_t = E_0 \sum_{t=0}^{\infty} \beta^t U_t - W_0$$

Substituting in above, by Law of Iterated Expectations we obtain the planning problem:

$$\max E_{-1} \left[ -\kappa W_0 + \sum_{t=0}^{\infty} \beta^t (1 + \kappa) U_t \right],$$

and where lastly, we use Lemma 4 substitute in for $W_0$ to obtain the result. ■

Lemma 18 provides a characterization of the relaxed social planning problem, subject to integral incentive compatibility. We are now ready to characterize the optimal allocation in Proposition 14.\footnote{We characterize the optimal allocation assuming that $\pi_t$ is interior.}

Recall that our objective function for the second-best optimization problem was given by:

$$\max \int_{\theta_0} \left[ \sum_{t=0}^{\infty} \beta^t \left( \bar{s}_0^t, \theta_0 \right) + U_t \left( \pi_t, \pi_{t+1}, \theta_t, \theta_{t+1} \right) \right] dF_0(\theta_0)$$

Note that given the optimal mechanism implements truthful reporting, we may substitute in $\bar{\theta}_t = \theta_t$.

Recall further the simplified form of the operators:

$$B^t_s = E_t \left[ \prod_{k=0}^{s-1} \frac{1}{f_{t+k}(\theta_{t+k+1}|\theta_{t+k})} \int_{s_t \leq \theta_t, ..., s_{t+s} \leq \theta_{t+s}} \frac{\partial U_{t+s}(\theta^{t-1}, s_t, ..., s_{t+s})}{\partial s_{t+k}} \prod_{k=0}^{s-1} \frac{\partial f_{t+k}(\theta_{t+k+1}|s_{t+k})}{\partial s_{t+k}} ds_{t+s}...ds_t \right]$$
Now, denote the realized value of the operator $B_0^t$ by:

$$B_0^t(\theta^t) = \prod_{k=0}^{t-1} \frac{1}{f_k(\theta_{k+1} | \theta_k)} \int_{s_0 \leq \theta_0, s_t \leq \theta_t} \frac{\partial U_t(s_0, \ldots, s_t)}{\partial s_t} \prod_{k=0}^{t-1} \frac{\partial f_k(\theta_{k+1} | s_k)}{\partial s_k} ds_t \ldots ds_0$$

So that $B_0^t(\theta^t)$ is a random variable derived from the history $\theta^t$ of shocks. Given the definition of this random variable, denote $E_{-1}$ to be the beginning-of-period-0 expectation, not conditional on the information $\theta_0$. From here, we can rewrite the objective function of the government as:

$$\max E_{-1} \left[ \sum_{t=0}^{\infty} \beta^t \left[ -\frac{\kappa}{1+\kappa} B_0^t(\pi_t, \pi_{t+1}, \theta_t | \theta^{t-1}) + (1+\kappa) U_t(\pi_t, \pi_{t+1}, \theta_t) \right] \right]$$

From here, consider the optimal choice of inflation $\pi_t(z^t)$, for a realized history $\theta^t = z^t$ of shocks. Note that the solution can be written in the form (for $t \geq 1$):

$$\frac{\partial U_{t-1}}{\partial \pi_t(z^t)} \frac{\partial U_t}{\partial \pi_t(z^t)} = \frac{\kappa}{1+\kappa} E_{-1} \sum_{s=t-1}^{t} \beta^{s-(t-1)} \frac{d}{d \pi_t(z^t)} B_0^s(\pi_s, \pi_{s+1}, \theta_s | \theta^{s})$$

So that all that remains is to characterize the derivatives of $B_0^t$ with respect to $\pi_t(z^t)$. When $s = t$, we have:

$$\frac{d}{dz^t} B_0^t(\theta^t) = \frac{d}{\pi_t(z^t)} \left[ \prod_{k=0}^{t-1} \frac{1}{f_k(\theta_{k+1} | \theta_k)} \int_{s_0 \leq \theta_0, s_t \leq \theta_t} \frac{\partial U_t(s_0, \ldots, s_t)}{\partial s_t} \prod_{k=0}^{t-1} \frac{\partial f_k(\theta_{k+1} | s_k)}{\partial s_k} ds_t \ldots ds_0 \right]$$

Note that $\pi_t(z^t)$ appears in $\frac{\partial U_t(s_0, \ldots, s_t)}{\partial s_t}$ only along the path given by $s_0 = z_0, s_1 = z_1, \ldots, s_t = z_t$. Essentially then, this derivative at a single point $\pi_t(z^t)$ comes down to extracting the derivative along that path under the integral. The derivative along that path is then given by:

$$\frac{d}{dz^t} B_0^t(\theta^t) = (1)_{z_0 \leq \theta_0, z_t \leq \theta_t} \prod_{k=0}^{t-1} \frac{1}{f_k(\theta_{k+1} | \theta_k)} \frac{\partial^2 U_t}{\partial z_t \partial \pi_t(z^t)} \prod_{k=0}^{t-1} \frac{\partial f_k(\theta_{k+1} | z_k)}{\partial z_k}$$

Note the subtlety that the $\theta$’s are preserved, as the realization of the random history, whereas the $s$’s are replaced by $z$’s, as the path under the integrals that leads to the history $z^t$ under the integrals. It is worth remembering then, when we substitute into the expectation, that $\theta_t$ is a random variable, and $z^t$ is (fixed) the history being differentiated along, and so is not a random variable.

Note that by exactly the same logic, we obtain $\forall t \geq 2$

$$\frac{d}{dz^{t-1}} B_0^{t-1}(\theta^{t-1}) = (1)_{z_0 \leq \theta_0, z_{t-1} \leq \theta_{t-1}} \prod_{k=0}^{t-2} \frac{1}{f_k(\theta_{k+1} | \theta_k)} \frac{\partial^2 U_{t-1}}{\partial z_{t-1} \partial \pi_t(z^t)} \prod_{k=0}^{t-2} \frac{\partial f_k(\theta_{k+1} | z_k)}{\partial z_k}$$
As a result, the right-hand side of the first-order condition becomes $\forall t \geq 2$

\[
\frac{1 + \kappa}{\kappa} \text{RHS} = E_{-1} \sum_{s=t-1}^{t} \frac{d}{d\tau_i(z^t)} B_0^s(\pi_s, \pi_{s+1}, \theta_s | \theta^* ) \\
= E_{-1} \left[ 1_{z_0 \leq \theta_0, \ldots, z_{t-1} \leq \theta_{t-1}} \prod_{k=0}^{t-2} \frac{1}{f_k(\theta_{k+1} | \theta_k)} \frac{\partial^2 U_{t-1}}{\partial z_{k}} \right] \\
+ \beta E_{-1} \left[ 1_{z_0 \leq \theta_0, \ldots, z_{t-1} \leq \theta_{t-1}} \prod_{k=0}^{t-1} \frac{1}{f_k(\theta_{k+1} | \theta_k)} \frac{\partial f_k(\theta_{k+1})}{\partial z_{k}} \right] \\
= \frac{\partial^2 U_{t-1}}{\partial z_{t-1}} \prod_{k=0}^{t-1} \frac{1}{f_k(\theta_{k+1} | \theta_k)} \frac{\partial f_k(\theta_{k+1})}{\partial z_{k}} \\
+ \frac{\partial^2 U_{t}}{\partial z_{t}} \beta E_{-1} \left[ 1_{z_0 \leq \theta_0, \ldots, z_{t-1} \leq \theta_{t-1}} \prod_{k=0}^{t-1} \frac{1}{f_k(\theta_{k+1} | \theta_k)} \frac{\partial f_k(\theta_{k+1})}{\partial z_{k}} \right]
\]

Where here, we applied the fact that we have chosen a specific history $z^t$, so that the cross-partialials above are not random variables, but rather are specific realizations of those random variables. By contrast, the part inside the expectation corresponds to histories which contain these specific histories, and so are random variables.

Now, consider these two expectations. Now, we define $\Omega_t(z^t)$ by:

$$
\Omega_t(z^t) \equiv E_{-1} \left[ 1_{z_0 \leq \theta_0, \ldots, z_{t-1} \leq \theta_{t-1}} \prod_{k=0}^{t-2} \frac{1}{f_k(\theta_{k+1} | \theta_k)} \frac{\partial f_k(\theta_{k+1})}{\partial z_{k}} \right]
$$

$$
= \int_{z_t}^{\overline{\theta}} \int_{z_{t-1}}^{\overline{\theta}} \ldots \int_{z_0}^{\overline{\theta}} \prod_{k=0}^{t-1} \frac{\partial f_k(\theta_{k+1})}{\partial z_{k}} f(\theta_0) d\theta_t \ldots d\theta_0
$$

$$
= \int_{z_t}^{\overline{\theta}} \frac{\partial f_k(\theta_k | z_{t-1})}{\partial z_{k}} \Omega_{t-1}(z^{t-1}) d\theta_t
$$

$$
= \Omega_{t-1}(z^{t-1}) \int_{z_t}^{\overline{\theta}} \frac{\partial f_k(\theta_k | z_{t-1})}{\partial z_{k}} d\theta_t
$$

Which is well-defined for all $t \geq 1$. However, it requires an initial condition $\Omega_0(z^0)$. It is helpful to define this initial condition in the date 1 FOC. Note that at date 1, we have:

$$
B_0^{-1}(\theta^{t-1}) = B_0^0(\theta^0) = \int_{\theta_0}^{\overline{\theta}} \frac{\partial U_0}{\partial \theta_0} d\theta_0
$$
So that we have \( \frac{d}{dt} B_{0,t-1}^{t-1}(\theta^{t-1}) = 1_{\theta_0 \leq \theta_t} \frac{\partial U_0}{\partial \pi_t(z)} \). In particular then, the expectation is simply:

\[
E_{-1}[1_{\theta_0 \leq \theta_t}] = \int_{\theta_0}^{\theta_t} f(\theta_0) d\theta_0 = 1 - F(z_0)
\]

So that we have initial condition \( \Omega_0(z_0) = 1 - F(z_0) \).

This gives us a state space reduction property, where we can fully determine \( \Omega_t \) from \( \Omega_{t-1} \) and \( z_{t-1} \) by a recursive sequence, where the initial value is \( \Omega_0(z_0) = 1 - F(z_0) \).

From here, we can substitute back into the FOCs:

\[
(1 + \kappa) \left[ \frac{\partial U_{t-1}}{\partial \pi_t(z)} f(z^{t-1}) + \beta \frac{\partial U_t}{\partial \pi_t(z)} f(z^t) \right] = \kappa \left[ \Omega_{t-1}(z^{t-1}) \frac{\partial^2 U_{t-1}}{\partial z_{t-1} \partial \pi_t(z)} + \beta \Omega_t(z^t) \frac{\partial^2 U_t}{\partial z_t \partial \pi_t(z)} \right]
\]

From here, it is helpful to divide through by \( f(z^{t-1}) \):

\[
(1 + \kappa) \left[ \frac{\partial U_{t-1}}{\partial \pi_t(z)} + \beta \frac{\partial U_t}{\partial \pi_t(z)} f(z_t|z_{t-1}) \right] = \kappa \left[ \Omega_{t-1}(z^{t-1}) \frac{\partial^2 U_{t-1}}{\partial z_{t-1} \partial \pi_t(z)} + \beta \Omega_t(z^t) \frac{\partial^2 U_t}{\partial z_t \partial \pi_t(z)} f(z_t|z_{t-1}) \right]
\]

And from here, we define \( \Gamma_t(z^t) = \frac{\Omega_t(z^t)}{f(z^t)} \). Note that we have:

\[
\Gamma_t(z^t) = \frac{\Omega_t(z^t)}{f(z^t)} = \frac{\Omega_{t-1}(z^{t-1})}{f(z^{t-1})} \int_{z_t}^{\theta_t} \frac{\partial U_0(\theta_t|z_{t-1})}{\partial \pi_t(z)} d\theta_t = \Gamma_{t-1}(z^{t-1}) \int_{z_t}^{\theta_t} \frac{\partial U_0(\theta_t|z_{t-1})}{\partial \pi_t(z)} d\theta_t
\]

Giving us our key result for \( t \geq 1 \).

Note that the relevant initial condition is \( \Gamma_0 = 1 - F(z_0) \). This is the standard term in evaluating the virtual value in static mechanism design problems, and it is not surprising that it appears here. What it notable is that this term appears in the date 1 optimality condition, in addition (as we will see) to the date-0 one. This is because of the time consistency problem.

Lastly, we can evaluate the FOC for \( \pi_0 \). In \( \pi_0 \), there is no time consistency element, and we are left with the simple tradeoff between current \( \pi \) and transfers. Repeating the steps from above, we obtain the simple condition

\[
\frac{\partial U_0}{\partial \pi_0} = \kappa \frac{\partial^2 U_0}{\partial z_0 \partial \pi_0}
\]

which is a standard virtual value condition. This gives the full result.

This concludes the proof.
A.9.1 Second best with Average Transfers

In the baseline model, we impose the assumption that the outside option takes the form \( \psi_0(\theta^0) \geq 0 \). We might alternatively have expressed this in the form

\[
\int_{\theta_0} \psi_0(\theta^0) f(\theta_0|\theta_{-1}) \, d\theta_0 \geq 0
\]

The core difference between these two assumptions from a modeling perspective is on the timing of information arrival versus the participation decision. Under the baseline assumption, either \( \theta_0 \) is already known to the central bank, or the central bank has the opportunity to revert to the outside option after learning \( \theta_0 \). Under the second assumption, \( \theta_0 \) is not known to the central bank, and the central bank does not have the option to revert to the outside option after learning it.

Under this alternative structure, the optimality of the dynamic inflation target returns. In particular, implementable allocations are still defined as in Lemma 17, while the transfer rule is

\[
T_t(\theta^t) = \psi_t - U_t - \beta E_t[\psi_{t+1}|\theta_t]
\]

The average participation constraint implies that we have

\[
0 = E_{-1} \psi_0 = E_{-1} \sum_{t=0}^{\infty} \beta^t \psi_t + \sum_{t=0}^{\infty} \beta^t \frac{1}{1+\kappa} U_t,
\]

which is markedly different from the baseline model. In particular, substituting this expression into social welfare, we obtain the social optimization problem

\[
\max_{\pi_t} \sum_{t=0}^{\infty} \beta^t (1+\kappa) U_t
\]

implying that the optimal allocation rule is constrained efficient. From here, we obtain the optimality of the dynamic inflation target.

**Proposition 19.** Suppose that the participation constraint takes the form

\[
\int_{\theta_0} \psi_0(\theta^0) f(\theta_0|\theta_{-1}) \, d\theta_0 \geq 0
\]

Then, the optimal mechanism is a dynamic inflation target, and yields the constrained efficient allocation.

**Proof.** The proof follows immediately. The objective function is to maximize social welfare and hence the optimal allocation is the full-information Ramsey allocation. The mechanism that implements this is the dynamic inflation target, with a lump sum transfer at date 0 to achieve a binding participation constraint.

The intuition behind Proposition 19 is straightforward: under the average constraint, the government can capture the full social surplus and simply reduce the average transfer to the central
bank at date 0 to satisfy the participation constraint. This implies that the government chooses the mechanism and allocation that maximize social surplus, which is the dynamic inflation target.

**A.10 Proof of Corollary 15**

The proof follows immediately from the definition of $\Gamma_t$, which is equal to zero if $\theta_t \in \{\theta, \bar{\theta}\}$. When $\Gamma_t = 0$, the allocation rule is constrained efficient for all $\Gamma_{t+k}, k \geq 1$, so the optimal mechanism reverts to constrained efficiency, which is implemented by the dynamic inflation target.

**A.11 Proof of Proposition 16**

Observe that the integral envelope condition (26) still holds and implies Lemma 17 characterizes the central bank’s value function, given central bank preferences have not changed. Thus the transfer rule is still given by $T_t = W_t - U_t - \beta \mathbb{E}_t [W_{t+1}|\theta_t]$. Thus we still have

$$-\mathbb{E} \sum_{t=0}^{\infty} T_t = \mathbb{E} \sum_{t=0}^{\infty} \beta^t U_t - W_0$$

where $W_0 = \mathbb{E}_0 \left[ \sum_{s=0}^{\infty} \beta^s B_0^s (\theta^s) \right]$. Given the change in preferences, the government’s objective function is now

$$\mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t V_t - \kappa T_t \right]$$

thus substituting in the transfer rule and definition of $W_0$, the government’s objective function is

$$\mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t \left( V_t + \kappa U_t - B_0^t \right) \right]$$

Finally dividing through by $1 + \kappa$ and defining $K = \frac{\kappa}{1+\kappa} (1 - K = \frac{1}{1+\kappa})$, we obtain

$$\mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t \left( (1 - K) V_t + K U_t - KB_0^t \right) \right]$$

Thus we simply define $Z_t = (1 - K) V_t + K U_t$ and the derivation proceeds exactly the same as before with $Z_t$ replacing $U_t$ as the government’s effective utility function. This recovers the first order condition given and completes the proof.
B Additional Applications

In this Appendix, we develop several additional applications. We study persistent cost-push shocks in Appendix B.1 and revisit the canonical New Keynesian consensus on inflation targets in this setting. In Appendix B.2, we revisit our discussion of persistent changes in the natural interest rate $r_t^*$ studied in Section 4.2, but now allowing for an arbitrary EIS, $\sigma > 0$, for which we leverage the theoretical framework developed in Section 5.

B.1 Cost-Push Shocks, Flexible Inflation Targeting, and Price-Level Targeting

In our first application, we study a persistent cost-push shock. This revisits the related full-information environment of Svensson and Woodford (2004) and studies the properties of the optimal mechanism relative to the dynamic inflation target. Social welfare is characterized by a New Keynesian loss function around a non-distorted steady state,

$$U_t(\pi_t, y_t, \theta_t) = -\frac{1}{2} \pi_t^2 - \frac{1}{2} \alpha(\pi_t - \theta_t)^2.$$ 

For simplicity, we set the slope of the Phillips curve to be $\kappa = 1$. Internalizing the NKPC (11) into the loss function yields reduced-form preferences

$$U_t(\pi_t, E_t\pi_{t+1}, \theta_t) = -\frac{1}{2} \pi_t^2 - \frac{1}{2} \alpha(\pi_t - \beta E_t\pi_{t+1} - \theta_t)^2.$$ (27)

Note that $\theta_t$ is a cost-push shock in the usual sense: higher $\theta_t$ means higher current inflation is needed in order to maintain the same output loss. We assume the cost-push shock satisfies $E_t\theta_{t+1} = \rho \theta_t$, where $0 \leq \rho \leq 1$ is its persistence. The following result characterizes the dynamic inflation target.

**Proposition 20.** The dynamic inflation target that implements the full-information Ramsey allocation is

$$\nu_t = \gamma_1 \nu_{t-1} + \gamma_2 \theta_t,$$

$$\tau_t = -(1 - \gamma_1) \gamma_1 \nu_{t-1} + \gamma_2 (\gamma_1 - 1 + \rho) \theta_t,$$

where $0 \leq \gamma_1 \leq 1$ does not depend on $\rho$, and $\gamma_2 \geq 0$ increases in $\rho$. Optimal inflation sets $\pi_t = \nu_t - \nu_{t-1}$.

Proposition 20 specializes the dynamic inflation target of Proposition 3 to our cost-push shock application. In response to a positive and persistent innovation in the shock, i.e., a high $\theta_t$ realization, the central bank updates both parameters of the target for the next period. First, the target flexibility decreases in the sense that $\nu_t$ rises. This happens because the cost-push shock leads to a larger output gap today, increasing the inflationary bias of the central bank.

Second, the response of the target level is ambiguous and depends on the shock persistence. When shocks are not persistent, a cost-push shock is followed by a lower target level. As shocks become more persistent, there is a critical level $\rho^* = 1 - \gamma_1$ after which the central bank raises...
the target level instead. This result reflects the common intuition of the cost-push shock model: The central bank would like to promise low future inflation to improve the contemporaneous inflation-output trade-off; as shocks become more persistent, however, it also wants to promise higher future inflation to mitigate future expected cost-push shocks.

The target also decreases as the previous period’s target flexibility parameter \( \nu_{t-1} \) rises. This reflects the history dependency: a high past inflationary bias leads to a desire for low inflation today, which in turn leads to a desire for low inflation tomorrow. This means that the increase in \( \nu_t \) serves as a force for future deflationary pressures. Finally, contemporaneous inflation unambiguously rises in response to a positive cost-push shock. It is interesting to note that the target flexibility is always more responsive to a contemporaneous cost-push shock than its flexibility, since we have

\[-1 < \gamma_1 - 1 + \rho < 1.\]

**Cost-push shocks with costly mechanism enforcement.** It is interesting to see how costly enforcement affects the allocation rule under the optimal mechanism. Note that we have \( \frac{\partial U_t}{\partial \pi_t} = \frac{1}{2} \alpha \) and \( \frac{\partial U_t}{\partial E_t \pi_t + 1} = -\frac{1}{2} \alpha \beta \). This is analogous to the flattening Philips curve example, and means we can write

\[ \frac{\partial U_t}{\partial \pi_t} = \nu_{t-1} + \frac{1}{2} K \Delta \Gamma_t \]

Thus relative to the Ramsey solution, the optimal mechanism adjusts the allocation trading off two effects on information rents. On the one hand, higher expected inflation reduces past information rents by increasing costs of inflation for central banks that experience large past cost push shocks. On the other hand, higher contemporaneous inflation increases current information rents by reducing costs of large contemporaneous cost push shocks. The optimal allocation rule trades off these two effects. As once again \( \Delta \Gamma_t < 0 \) local to the boundaries of the shock distribution, particularly large or particularly small cost push shocks at date \( t \) lead past information rents to dominate, and calls for a more aggressive inflation response today in order to reduce historical information rents. Interestingly, this amplifies the response of inflation to a large cost push shock, pushing the allocation rule closer to the policy under discretion.

**B.1.1 Proof of Proposition 20**

Given reduced from preferences are

\[ U(\pi_t, E_t \pi_{t+1}, \theta_t) = -\frac{1}{2} \pi_t^2 - \frac{1}{2} \alpha (\pi_t - \beta E_t \pi_{t+1} - \theta_t)^2 \]

then we have

\[ \frac{\partial U_t}{\partial \pi_t} = -\pi_t - \alpha (\pi_t - \beta E_t \pi_{t+1} - \theta_t) \]

\[ \frac{\partial U_{t-1}}{\partial E_{t-1} \pi_t} = \beta \alpha (\pi_{t-1} - \beta E_{t-1} \pi_t - \theta_{t-1}). \]
By definition, we have
\[ \nu_{t-1} = -\frac{1}{\beta} \frac{\partial U_{t-1}}{\partial \pi_{t-1}} = -\alpha(\pi_{t-1} - \beta E_{t-1} \pi_t - \theta_{t-1}). \]

Therefore, we can write the FOC for the full-information Ramsey allocation, \( \frac{\partial U_t}{\pi_t} = \nu_{t-1}, \) equivalently as
\[ -\pi_t - \nu_t = \nu_{t-1} \]
or in other words, \( \pi_t = \nu_t - \nu_{t-1}. \) Combined with the definition of \( \nu_{t-1} \) and the initial condition \( \nu_{-1} = 0, \) this gives us a complete system.

Suppose that \( \mathbb{E}_t \theta_{t+1} = \rho \theta_t, \) where \( \rho = 1 \) corresponds to full persistence. We thus think of cost push shocks as reverting towards zero. We guess and verify a linear solution
\[ \nu_t = \gamma_1 \nu_{t-1} + \gamma_2 \theta_t. \]

Given this conjecture, we know from the FOC that
\[ \pi_t = (\gamma_1 - 1) \nu_{t-1} + \gamma_2 \theta_t. \]

Using the definition of \( \nu_t, \)
\[ \nu_t = -a \pi_t + a \beta \mathbb{E}_t \pi_{t+1} + a \theta_t, \]
we substitute in the expression for \( \pi_t \) and our conjecture for \( \nu_{t+1} \) to obtain
\[ \nu_t = -a \left( \nu_t - \nu_{t-1} \right) + a \beta \left( (\gamma_1 - 1) \nu_t + \gamma_2 \mathbb{E}_t \theta_{t+1} \right) + a \theta_t. \]

Now using that \( \mathbb{E}_t \theta_{t+1} = \rho \theta_t \) and rearranging, we get
\[ \nu_t = \frac{a}{1 + a + (1 - \gamma_1) a \beta} \nu_{t-1} + \frac{a \left( \beta \gamma_2 \rho + 1 \right)}{1 + a + (1 - \gamma_1) a \beta} \theta_t. \]

Thus coefficient matching, we have the system of equations
\[ \frac{a}{1 + a + (1 - \gamma_1) a \beta} = \gamma_1 \]
\[ \frac{a \left( \beta \gamma_2 \rho + 1 \right)}{1 + a + (1 - \gamma_1) a \beta} = \gamma_2 \]
The first equation is defined solely in terms of \( \gamma_1. \) Thus taking it and rearranging, we obtain the
quadratic
\[ a\beta\gamma_1^2 - \gamma_1(1 + \alpha + \alpha\beta) + \alpha = 0. \]

This quadratic has two roots, with the upper root being explosive since \( \beta < 1 \) implies \( \gamma_1^+ > 1 \). Thus selecting the non-explosive root gives \( 0 \leq \gamma_1 \leq 1 \), where
\[ \gamma_1 = \frac{1 + \alpha + \alpha\beta - \sqrt{(1 + \alpha + \alpha\beta)^2 - 4\alpha^2\beta}}{2\alpha\beta}. \]

Note that to see why this root lies between 0 and 1, the quadratic above equals \( \alpha > 0 \) for \( \gamma_1 = 0 \) and equals \(-1 < 0 \) when \( \gamma_1 = 1 \).

Given that \( 0 \leq \gamma_1 \leq 1 \), we can solve for \( \gamma_2 \) using the second equation, which gives
\[ \gamma_2 = \frac{\gamma_1}{1 - \beta\rho\gamma_1}, \]
which is positive since \( \beta\rho\gamma_1 \leq 1 \). Thus we have our solution. Given this solution, the parameters of the target are
\[ \nu_t = \gamma_1\nu_{t-1} + \gamma_2\theta_t \]
and
\[ \tau_t = \mathbb{E}_t\pi_{t+1} \]
\[ = (\gamma_1 - 1)\nu_t + \gamma_2\theta_t \]
\[ = -(1 - \gamma_1)\gamma_1\nu_{t-1} + \gamma_2(\gamma_1 - 1 + \rho)\theta_t \]

### B.2 Application: \( r^* \) Revisited and the Commitment Curve

We now revisit the application to persistent changes in the natural interest rate \( r^*_t \) studied in Section 4.2. We now allow for \( \sigma > 0 \). The realized nominal interest rate is
\[ i_t = \mathbb{E}_t\pi_{t+1} + \theta_t + \sigma \left[ \mathbb{E}_t y_{t+1} - y_t \right] - \rho_t. \]

Intuitively, an expected rise in the output gap means household consumption is expected to rise, raising the nominal interest rate and pushing the central bank away from the ELB. Similar to Section 4.2, we can write \( i_t = i^*_t - \rho_t \) and write the welfare losses \( v(i^*_t) \) from the ELB. In this case with \( \sigma > 0 \), we have a change in the definition of \( i^*_t \) to
\[ i^*_t = -\sigma\pi_t + \left( 1 + \sigma(1 + \beta) \right) \mathbb{E}_t\pi_{t+1} - \sigma\beta\mathbb{E}_t\pi_{t+2} + \theta_t. \]
which reflects internalizing the NKPC to substitute out the output gap. Intuitively, higher inflation today, $\pi_t$, increases output today and so reduces the required nominal rate. Higher inflation $\pi_{t+1}$ both directly increases the nominal rate and indirectly increases it by stimulating output $y_{t+1}$. Conversely, higher inflation $\pi_{t+1}$ depresses output $y_{t+1}$ and so reduces the nominal rate.

Let us consider the shape of the commitment curve. Recall that we have

$$U_t = -\frac{1}{2}\pi_t^2 - \frac{1}{2}\hat{\alpha} \left( \pi_t - \beta E_t \pi_{t+1} \right)^2 + v(i^*_t).$$

We can write

$$v_{t+1,1} = v^Y_{t+1,1} + v^i_{t+1,1},$$

where $v^Y_{t+1,1} = -\frac{1}{2}\hat{\alpha} \left( \pi_t - \beta E_t \pi_{t+1} \right)$ is the usual output gap component, and where $v^i_{t+1,1} = -\left( v_0 - \beta v_1 i^*_t \right) \left( 1 + \sigma (1 + \beta) \right) < 0$ is the component coming from the effective lower bound. From here, we can show that

$$v_{t+2,2} = -\beta^* v^i_{t+1,1},$$

where $\beta^* = \frac{\sigma}{1 + \sigma (1 + \beta)} < 1$ is increasing in $\sigma$.

Intuitively, in this case the commitment curve can be decomposed into two components. The first component is the output gap commitment curve, where we have $v^Y_{t+1,1} > 0$ and $v^Y_{t+k,k} = 0$ for all $k > 1$. This corresponds to the standard one period commitment to stabilize the output gap. The second component is the effective lower bound commitment curve, where $v^i_{t+1,1} < 0$ and $v^i_{t+2,2} = -\beta^* v^i_{t+1,1} > 0$. The effective lower bound commitment curve switches signs precisely because of the different effects of inflation at different horizons.